A statistical approach to describe highly excited heavy and superheavy nuclei *

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Abstract: A statistical approach based on the Weisskopf evaporation theory has been developed to describe the de-excitation process of highly excited heavy and superheavy nuclei, in particular for the proton-rich nuclei. The excited nucleus is cooled by evaporating γ-rays, light particles (neutrons, protons, α etc) in competition with binary fission, in which the structure effects (shell correction, fission barrier, particle separation energy) contribute to the processes. The formation of residual nuclei is evaluated via sequential emission of possible particles above the separation energies. The available data of fusion-evaporation excitation functions in the ^{28}Si+^{198}Pt reaction can be reproduced nicely within the approach.

Keywords: excited nucleus, structure effects, Weisskopf evaporation theory, fusion-evaporation reactions

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1 Introduction

The synthesis of very heavy (superheavy) nuclei is a very important subject in nuclear physics, motivated by reaching the island of stability predicted theoretically. There has been much progress in experiments with the fusion-evaporation reaction mechanism [1–3]. The existence of superheavy nuclei (SHN) ($Z \geqslant 106$) is due to a strong shell effect against the large Coulomb repulsion. However, the shell effect will be reduced by the increasing excitation energy, which leads to the decrease of the survival of residue nucleus. The fusion-evaporation reaction to form superheavy compound nuclei can be understood as three stages, in accordance with the evolution of two heavy colliding nuclei, namely the capture process of the colliding system to overcome the Coulomb barrier, the formation of the compound nucleus to pass over the inner fusion barrier, and the de-excitation of the thermal compound nucleus against fission [4]. The survival probability of the last stage is particularly important in the evaluation of production cross sections of heavy and superheavy nuclei. Besides the synthesis of SHN, one can produce proton-rich nuclei close to the proton drip line using the fusion-evaporation reactions or multi-nucleon transfer mechanism.

2 Description of statistical theory

In this work, we extend the statistical approach based on the Weisskopf evaporation theory [5] to describe the de-excitation process of highly excited proton-rich heavy nuclei. Evaporation of light charged particles in the statistical model has been implemented. It is well known that the evaporation residue cross section in the fusion reactions is evaluated as a sum over partial angular momentum J at the centre-of-mass energy $E_{\rm c.m.}$ in the evaporation channel s,

$$\begin{aligned}&\sigma^s_{\rm ER}(E_{\rm c.m.})\\&=\frac{\pi\hbar^2}{2\mu E_{\rm c.m.}}\sum_{J=0}^{J_{\max}}(2J+1)T(E_{\rm c.m.},J)P_{\rm CN}(E_{\rm c.m.},J)\\&\quad\times W^s_{\rm sur}(E_{\rm c.m.},J).\end{aligned}\tag{1}$$

Here, $T(E_{\rm c.m.})$ is the penetration probability of the two colliding nuclei overcoming the Coulomb barrier to form the DNS, which is calculated using the empirical coupled channel model [4, 6]. The $P_{\rm CN}$ is the probability that the heavy system evolves from a touching configuration into the formation of compound nucleus in competition with quasi-fission and fission of the heavy fragments. However, for light reaction systems or projectile-target combinations with larger mass asymmetry ($A_{\rm P}/A_{\rm T} < 0.1$) the probability is close to unity, $P_{\rm CN}\approx 1$ [7].

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The survival probability is particularly important in evaluation of the cross section, which is usually calculated with the statistical approach. The physical process in understanding the excited nucleus is clear. But the magnitude strongly depends on the ingredients in the statistical model, such as level density, separation energy, shell correction, fission barrier etc. The excited nucleus is cooled by evaporating γ-rays, light particles (neutrons, protons, α etc) in competition with fission. Similar to neutron evaporation by itself [4], the probability in the channel of evaporating the x-th neutron, the y-th proton and the z-alpha is expressed as

$$W_{\rm sur}(E^*_{\rm CN},x,y,z,J) = P(E^*_{\rm CN},x,y,z,J) \times \prod_{i=1}^{x} \frac{\Gamma_{\rm n}(E^*_i,J)}{\Gamma_{\rm tot}(E^*_i,J)} \prod_{j=1}^{y} \frac{\Gamma_{\rm p}(E^*_j,J)}{\Gamma_{\rm tot}(E^*_j,J)} \times \prod_{k=1}^{z} \frac{\Gamma_{\alpha}(E^*_k,J)}{\Gamma_{\rm tot}(E^*_k,J)}. \quad (2)$$

Here the $E^*_{\rm CN}$, J are the excitation energy evaluated from the mass table in Ref. [8] and the spin of the excited nucleus, respectively. The total width $\Gamma_{\rm tot}$ is the sum of partial widths of particle evaporation, γ-emission and fission. The excitation energy E^*_s before evaporating the s-th particle is evaluated by

$$E^*_{s+1} = E^*_s - B^n_i - B^p_j - B^\alpha_k - 2T_s \quad (3)$$

with the initial condition $E^*_1 = E^*_{\rm CN}$ and $s = i+j+k$. The B^n_i, B^p_j, B^α_k are the separation energy of the i-th neutron, j-th proton, k-th alpha,respectively. The nuclear temperature T_i is given by $E^*_i = aT^2_i - T_i$ with a being the level density parameter.

Assuming the electric dipole radiation ($L=1$) dominates γ-emission, the decay width is calculated by

$$\Gamma_\gamma(E^*_{\rm CN},J) = \frac{3}{\rho(E^*,J)} \int_{\varepsilon=0}^{E^*-\delta-\frac{1}{a}} \rho(E^*-E_{\rm rot}-\varepsilon,J) f_{E_1}(\varepsilon){\rm d}\varepsilon, \quad (4)$$

and

$$f_{E_1}(\varepsilon) = \frac{4}{3\pi}\frac{1+\kappa}{mc^2}\frac{e^2}{\hbar c}\frac{NZ}{A}\frac{\Gamma_{\rm G}\varepsilon^4}{(\Gamma_{\rm G}\varepsilon)^2+(\Gamma^2_{\rm G}-\varepsilon^2)^2}. \quad (5)$$

Here, $\kappa = 0.75$, and $\Gamma_{\rm G}$ and $E_{\rm G}$ are the width and position of the electric dipole resonance respectively. For a heavy nucleus, $\Gamma_{\rm G} = 5$ MeV [9],

$$E_{\rm G} = \frac{167.23}{A^{1/3}\sqrt{1.959+14.074A^{-1/3}}}. \quad (6)$$

The particle decay widths are evaluated with the Weisskopf evaporation theory as [5]

$$\Gamma_\nu(E^*,J) = (2s_\nu+1)\frac{m_\nu}{\pi^2\hbar^2\rho(E^*,J)} \int_0^{E^*-B_\nu-\delta-\delta_{\rm n}-\frac{1}{a}} \varepsilon\rho(E^*-B_\nu-\delta_{\rm n}-E_{\rm rot}-\varepsilon,J)\sigma_{\rm inv}(\varepsilon){\rm d}\varepsilon. \quad (7)$$

Here, s_ν, m_ν and B_ν are the spin, mass and binding energy of the evaporating particle, respectively. The pairing correction energy δ is set to be $12/\sqrt{A}, 0, -12/\sqrt{A}$ for even-even, even-odd and odd-odd nuclei, respectively. The inverse cross section is given by $\sigma_{\rm inv} = \pi R^2_\nu T(\nu)$. The penetration probability is set to be unity for neutrons and $T(\nu) = (1+\exp(\pi(V_C(\nu)-\varepsilon)/\hbar\omega))^{-1}$ for charged particles with $\hbar\omega = 5$ and 8 MeV for proton and alpha, respectively. The fission width is calculated with a similar method as in Ref. [10, 11].

The level density is calculated from the Fermi-gas model [12] as,

$$\rho(E^*,J) = K_{\rm coll}\cdot\frac{2J+1}{24\sqrt{2}\sigma^3a^{1/4}(E^*-\delta)^{5/4}} \exp\left[2\sqrt{a(E^*-\delta)}-\frac{(J+1/2)^2}{2\sigma^2}\right], \quad (8)$$

with $\sigma^2 = 6\bar{m}^2\sqrt{a(E^*-\delta)}/\pi^2$ and $\bar{m} \approx 0.24A^{2/3}$. The $K_{\rm coll}$ is the collective enhancement factor, which includes the rotational and vibrational effects [11, 13]. The level density parameter is related to the shell correction energy $E_{\rm sh}(Z,N)$ and the excitation energy E^* of the nucleus as

$$a(E^*,Z,N) = \tilde{a}(A)[1+E_{\rm sh}(Z,N)f(E^*-\Delta)/(E^*-\Delta)]. \quad (9)$$

Here, $\tilde{a}(A) = \alpha A + \beta A^{2/3}b_s$ is the asymptotic Fermi-gas value of the level density parameter at high excitation energy. The shell damping factor is given by

$$f(E^*) = 1-\exp(-\gamma E^*) \quad (10)$$

with $\gamma = \tilde{a}/(\epsilon A^{4/3})$. The parameters α, β, b_s and ϵ are taken to be 0.114, 0.098, 1. and 0.4, respectively [11]. Shown in Fig. 1 is a comparison of the partial decay width for the proton-rich nucleus ^{226}U and the superheavy nucleus ^{272}Ds. The charged particles (p, α) have smaller widths for the superheavy nucleus in comparison to the proton-rich nucleus because of larger separation energies. The fission width increases rapidly in the excitation energy range of 10-30 MeV for the superheavy nucleus and the larger width leads to a smaller survival probability, which is because the fission barrier decreases exponentially with increasing excitation energy [4]. The collective enhancement factor increases the level density, but reduces the partial widths, in particular for particle evaporation.

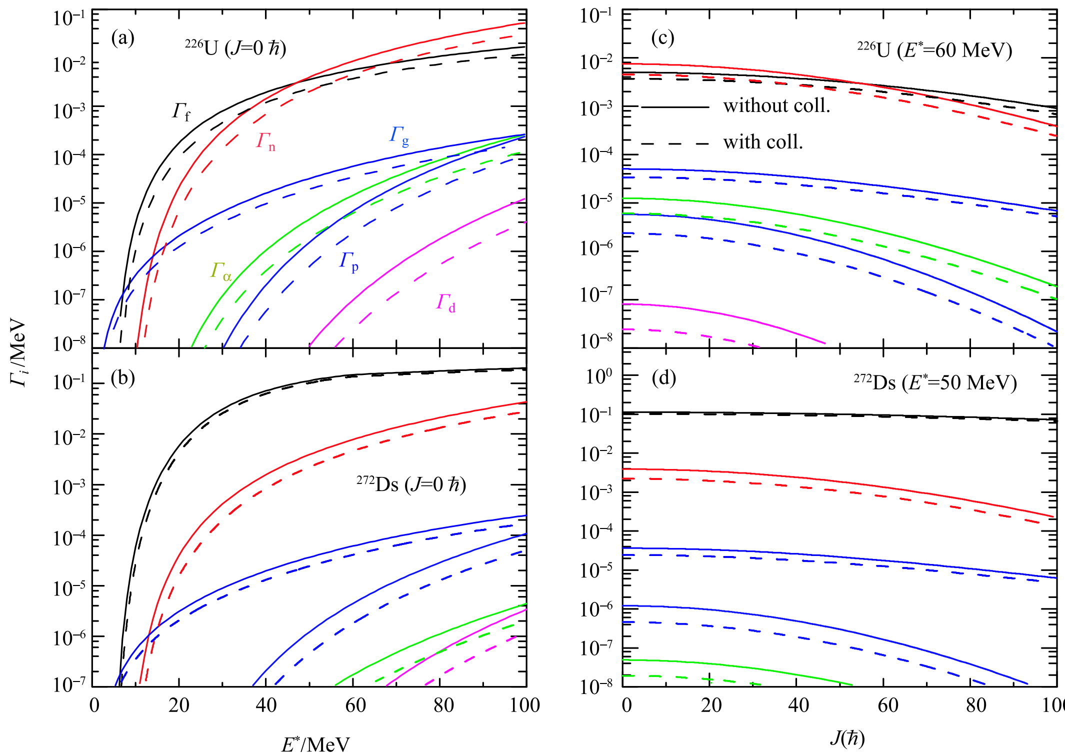

Fig. 1. (color online) Partial decay widths of proton-rich nucleus ^{226}U and superheavy nucleus ^{272}Ds as functions of excitation energy and angular momentum.

For one particle evaporation, the realization probability is given by

$$P(E^*_{\rm CN},J)=\exp\left(-\frac{(E^*_{\rm CN}-B_s-2T)^2}{2\sigma^2}\right). \quad (11)$$

The width σ is taken to fit the experimental width of fusion-evaporation excitation functions. The realization probability $P(E^*_{\rm CN},x,y,z,J)$ for evaporating x neutrons, y protons, z alphas at the excitation energy of $E^*_{\rm CN}$ and angular momentum of J is calculated by the Jackson formula [14] as

$$P(E^*_{\rm CN},s,J)=I(\Delta_s,2s-3)-I(\Delta_{s+1},2s-1), \quad (12)$$

where the quantities I and Δ are given by following:

$$I(z,m)=\frac{1}{m!}\int_0^z u^m{\rm e}^{-u}{\rm d}u, \quad (13)$$

$$\Delta_s=\frac{E^*_{\rm CN}-\sum\limits_{i=1}^{s}B_i^\nu}{T_i}. \quad (14)$$

The B_i^ν is the separation energy of evaporating the i-th particle and $s(x,y,z)=x+y+z$. The spectrum of the realization probability determines the structure of survival probability in each evaporation channel.

3 Results and discussion

We disintegrate the ingredients in evaluating the survival probability. Shown in Fig. 2 is a comparison of different channels in terms of the ratio of partial to total decay width, and the realization and survival probabilities of ^{226}U. The maximum position of the realization probability in each xn channel is similar and close to unity. However, the survival probabilities decrease with increasing neutron number. The collective enhancement on the level density reduces the survival probability of the excited nucleus. The channels with evaporating charged particles are shown in Fig. 3, i.e., 1αxn, 1pxn, 2αxn and 1α1pxn with $x=0-6$. The maximal probability is different for each channel with evaporating 1α, 1p, 2α and 1α1p, e.g., the optimal channels 1n1α , 4n1p, 6n1p, 4n2α, 5n2α and 6n1p1α. The maximal cross sections of evaporation residue nuclei with increasing incident energy are contributed from the survival probabilities of the compound nucleus for each channel and the fusion cross sections of the two colliding partners. The channels of evaporating charged particles have comparable values to pure neutron channels. Therefore, it is necessary to include the charged particle evaporation for the excited proton-rich nuclei. A similar approach for the decay of excited heavy nuclei is used in the multi-dimensional Langevin equations [15].

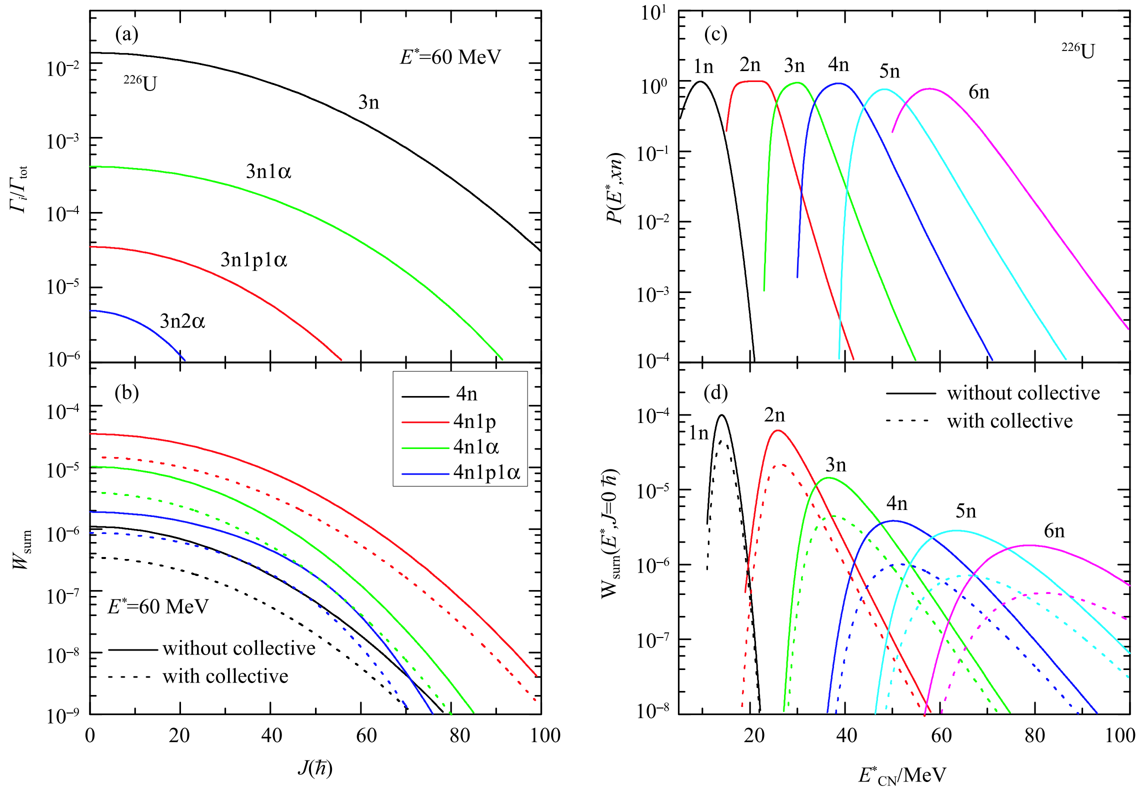

Fig. 2. (color online) (a) Ratio of partial to total decay width by evaporating different particles and (b) survival probability as a function of angular momentum. (c) Excitation energy dependence of realization probabilities and (d) survival probabilities in neutron channels.

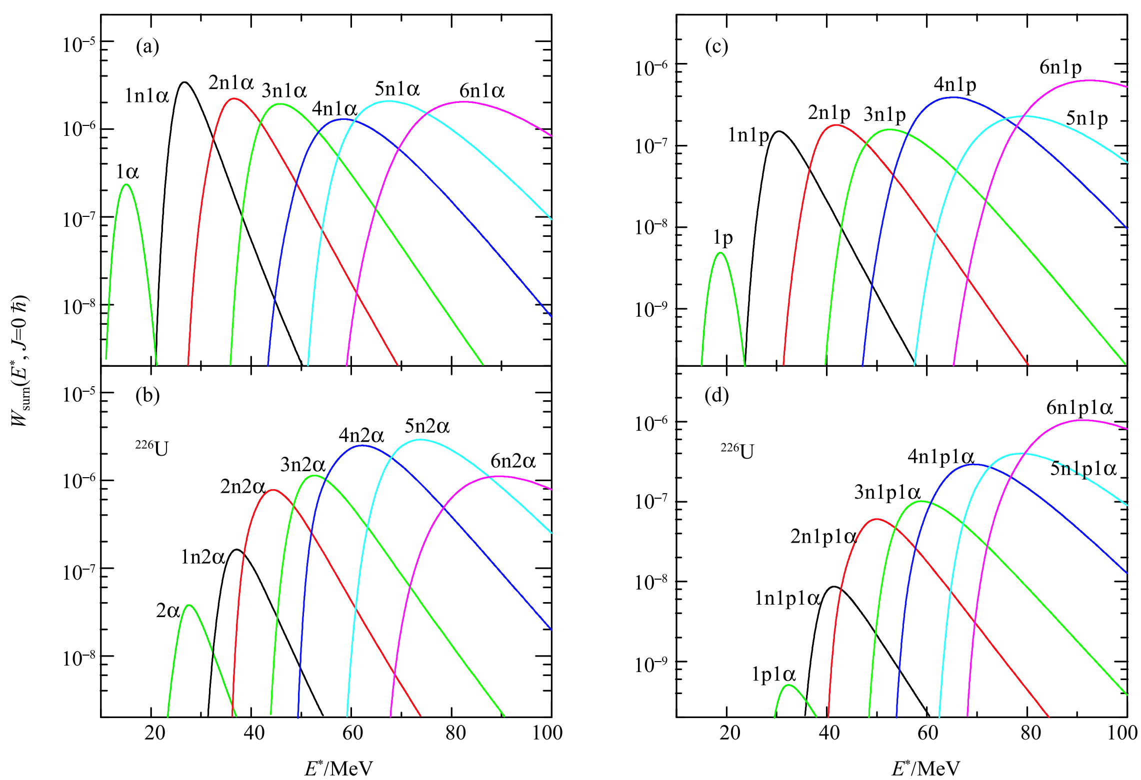

Fig. 3. (color online) The survival probabilities of proton-rich nucleus ^{226}U at the angular momentum of $J=0\hbar$ as functions of excitation energy in the channels $1\alpha x$n, 1pxn, $2\alpha x$n and 1α1pxn with $x=0-6$.

As a test of the approach, we calculated the fusion-evaporation excitation functions in the reaction ^{28}Si + ^{198}Pt as shown in Fig. 4. Production cross sections of the residue nuclei from the available experimental data [16] can be nicely reproduced. The solid lines are the total cross sections as the sum of each evaporation channel labeled in the figure. The capture cross sections are calculated by the empirical coupled channel model (EMCC) and we set the fusion probability $P_{\rm CN} \approx 1$. The quasi-fission reactions for the considered system are negligible. The channels with charged particle evaporation have larger cross sections than the pure neutron evaporation, i.e., 1αxn with x=3–6. Because of the smaller separation energy for the proton-rich nucleus, the α particles can easily be emitted from the excited mother nucleus in comparison to protons, despite the larger Coulomb barrier. At the excitation energy close to the separation energy of evaporated particles, the calculated residue cross sections are smaller than the experimental data. The underestimation is caused from the classical treatment in the evaluation of decay width for particle evaporation. It is still necessary to consider the quantum tunneling effect in the evaporation model. The higher fission barrier leads to the larger survival probability, which is favorable to produce proton-rich nuclei in experiments, even close to the proton drip line.

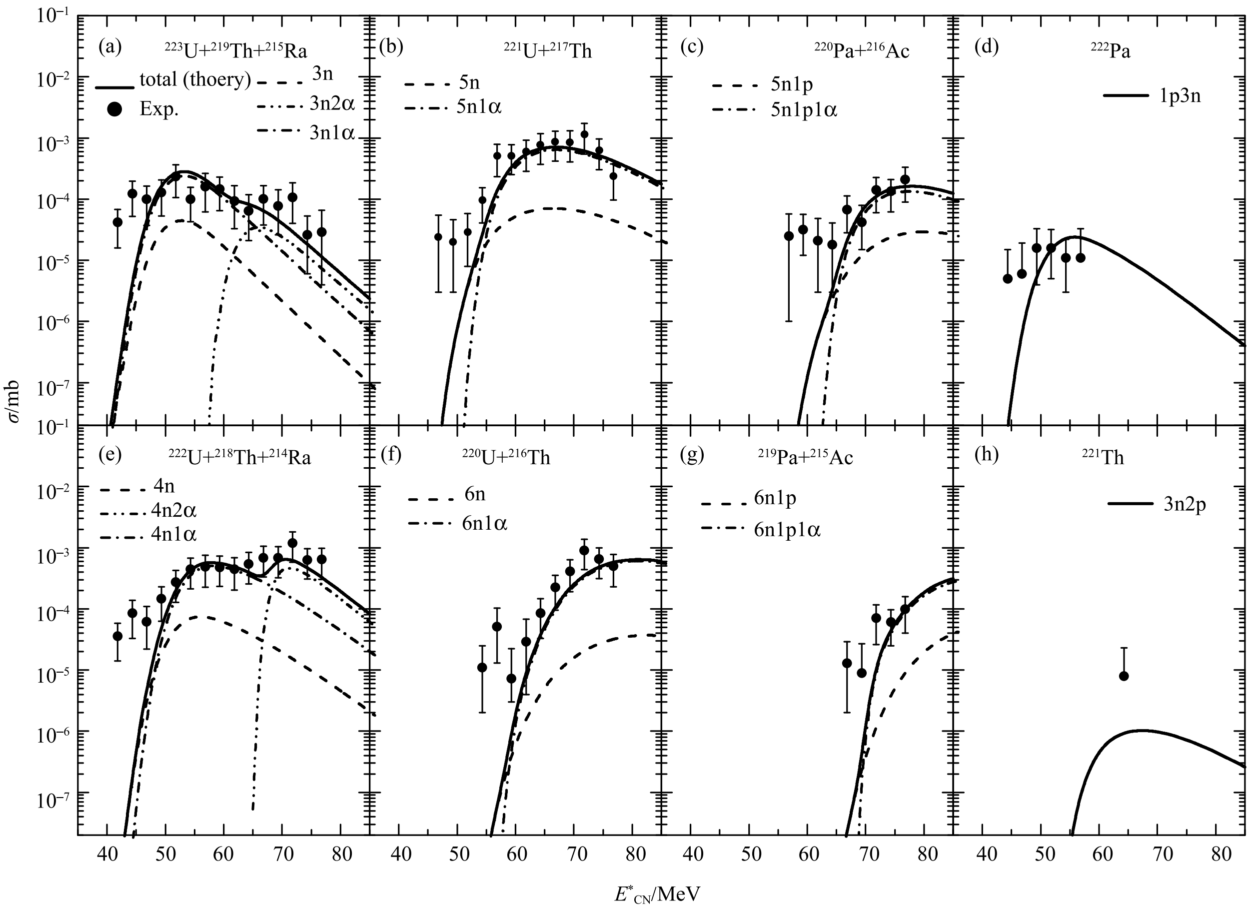

Fig. 4. Fusion-evaporation excitation functions for all possible channels in the reaction ^{28}Si + ^{198}Pt, compared with the experimental data [16].

4 Conclusions

In summary, a statistical approach based on the Weisskopf evaporation theory has been developed to describe the de-excitation process of highly excited heavy and superheavy nuclei, in particular for proton-rich nuclei, which is the most important stage in the evaluation of residue cross section. Light charged particle evaporation has been included in the model. The fusion-evaporation excitation functions of the available experimental data can be nicely reproduced. This approach is useful to evaluate the production cross section close to the proton (neutron) drip lines in fusion-evaporation reactions and multi-nucleon transfer reactions. This is of particular interest for experiments planned in the near future at the Heavy-Ion Accelerator Facility in Lanzhou (HIRFL).

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