

# Self-consistent analysis of sub-barrier fusion enhancement effect in Ca + Ca and Ni + Ni<sup>\*</sup>

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**Abstract:** The fusion dynamic mechanism of heavy ions at energies near the Coulomb barrier is complicated and still not very clear up to now. Accordingly, a self-consistent method based on the CCFULL calculations has been developed and applied for an ongoing study of the effect of the positive  $Q$ -value neutron transfer (PQNT) channels in this work. The typical experimental fusion data of Ca + Ca and Ni + Ni is analyzed within the unified calculation scheme. The PQNT effect in near-barrier fusion is further confirmed based on the self-consistent analysis and extracted quantitatively.

**Keywords:** sub-barrier fusion enhancement, coupled-channels calculation, positive  $Q$ -value neutron transfer, residual enhancement

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## 1 Introduction

Low-energy nuclear fusion reactions with heavy ions offer a fine way to explore the dynamics of many-body systems. This process also relates to the current synthesis mechanism of the superheavy elements (SHE) [1] and has important implication for nucleosynthesis in astrophysics [2]. Therefore, this reaction process has attracted plenty of research in recent decades [3–20].

For this process, a feature is the involvement of the couplings between the relative motion of the colliding nuclei and the intrinsic degrees of freedom such as low-lying collective vibrations of the target and projectile as well as the nucleon transfers [5]. Up to now, the coupled-channels (CC) model has successfully described the fusion enhancement phenomenon correlated with the collective excitations [6, 7]. The coupling effect of the positive  $Q$ -value neutron transfer (PQNT) channels involved in this process, which was first discovered in the experimental study of near-barrier fusion excitation functions of  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$  [8], is complicated, however. Up to now, the relevant experimental conclusions for this effect are still inconsistent and the theoretical descriptions are still immature. There are some proposed reduction methods [3], but they cannot successfully give a consis-

tent systematic behavior for the relevant coupling effect.

To solve this problem, one is confronted with inconsistent experimental data for the same systems measured by different groups. At energies below the Coulomb barrier, the discrepancy of the measured fusion cross sections even reaches several multiples in some cases. Theoretically, problems also exists in the current analysis by different methods. For example, although the CC model gives a good account of the collective coupling effect, the CC method has limitations, such as a need for external parameters to describe the nucleus-nucleus potential and the couplings [21].

Taking these into account, very recently, a self-consistent residual enhancement (RE) method was proposed [22] for trying to disentangle the PQNT coupling effect from the inelastic coupling effect in fusion based on the CCFULL calculation, with the coupling schemes for the collective excitations determined from the experimental fusion data of the reference systems. Here,  $\text{RE} = \sigma_{\text{exp}}/\sigma_{\text{CC}}$ , where  $\sigma_{\text{exp}}$  is the experimental fusion cross section and  $\sigma_{\text{CC}}$  is the CC calculation result considering the major inelastic couplings. The proposed RE method reduces overwhelmingly the uncertainty caused by the inelastic coupling effect in the CC calculation and can obtain a reliable quantitative PQNT effect.

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## 2 Analysis procedure

In this work, for further studying the relevant problems in fusion reactions, some typical near-spherical systems,  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  and  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$ , measured by the same groups, optimally suited for this purpose, were selected. The corresponding  $Q_{\text{gg}}$ -values for the multi-neutron transfer channels from ground state to ground state are shown in Table 1.

Table 1. The  $Q_{\text{gg}}$ -values for the multi-neutron transfer channels. The unit is MeV.

system	$Q_{+1n}$	$Q_{+2n}$	$Q_{+3n}$	$Q_{+4n}$
$^{40}\text{Ca} + ^{40}\text{Ca}$	-7.28	-9.09	-18.12	-21.78
$^{40}\text{Ca} + ^{48}\text{Ca}$	-1.58	2.62	0.16	3.88
$^{48}\text{Ca} + ^{48}\text{Ca}$	-4.80	-5.72	-11.76	-14.45
$^{58}\text{Ni} + ^{58}\text{Ni}$	-3.22	-2.08	-10.90	-14.50
$^{58}\text{Ni} + ^{64}\text{Ni}$	-0.66	3.89	1.11	3.89
$^{64}\text{Ni} + ^{64}\text{Ni}$	-3.56	-1.45	-6.23	-6.26

The microscopic calculation for the fusion cross section is performed by using the code CCFULL [23]. For the systems studied in the present work, the deformation parameters of the collective excitation states are obtained from the  $B(E\lambda)$  transitions in NNDC. Only one-phonon states of the reactants are considered in the calculations. The relevant information is shown in Table 2, where  $\lambda^\pi$  is spin and parity,  $E_\lambda$  is excitation energy and  $\beta_\lambda$  is the deformation parameter. All the mutual excitations are considered in the calculations. The effect of

higher excitation states on sub-barrier fusion is expected to be small due to the adiabatic nature of the fusion process [24] and are ignored here. The coupling radius parameter  $r_{0c} = 1.10$  fm. The adopted Akyüz-Winther (AW) proximity potential [25] was parameterized into the Woods-Saxon (WS) form with three parameters  $V_0$ ,  $r_0$ , and  $a$ . The parameters for the WS potential and the uncoupled barrier for the studied systems are given in Table 3. The experimental data were renormalized according to the calculation results at energies above the Coulomb barriers. All the values of  $\beta_\lambda$  for the nuclei obtained by fitting the experimental fusion data of the reference systems are in agreement with those obtained from the reduced transition probability  $B(E\lambda)$ , except for  $^{40}\text{Ca}$ , which was deduced from Ref. [26].

Table 2. The parameters used for the considered low-lying collective excitation states in the CC calculations.

nucleus	$\lambda^\pi$	$E_\lambda/\text{MeV}$	$\beta_\lambda$
$^{40}\text{Ca}$	$3^-$	3.737	0.270
	$2^+$	3.904	0.119
$^{48}\text{Ca}$	$2^+$	3.832	0.104
	$3^-$	4.507	0.175
$^{58}\text{Ni}$	$2^+$	1.454	0.154
	$3^-$	4.475	0.135
$^{64}\text{Ni}$	$2^+$	1.346	0.158
	$3^-$	3.560	0.216

Table 3. The parameters for the WS potential and the uncoupled barrier for the studied systems.

system	$V_0/\text{MeV}$	$r_0/\text{fm}$	$a/\text{fm}$	$V_B/\text{MeV}$	$R_B/\text{MeV}$	$R_B/\text{MeV}$
$^{40}\text{Ca} + ^{40}\text{Ca}$	62.531	1.174	0.652	54.925	9.735	3.757
$^{40}\text{Ca} + ^{48}\text{Ca}$	64.925	1.174	0.657	53.189	10.080	3.503
$^{48}\text{Ca} + ^{48}\text{Ca}$	64.104	1.175	0.662	51.751	10.379	3.249
$^{58}\text{Ni} + ^{58}\text{Ni}$	72.816	1.177	0.671	99.413	10.553	3.791
$^{58}\text{Ni} + ^{64}\text{Ni}$	73.873	1.177	0.673	97.713	10.754	3.661
$^{64}\text{Ni} + ^{64}\text{Ni}$	73.845	1.177	0.676	96.178	10.939	3.524

## 3 Results and discussion

The calculation results for  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  with single-channel (SC) and coupled-channels by using the adopted coupling schemes are given in Fig. 1(a). The experimental fusion cross sections for all three systems show strong enhancement compared with the SC calculation results (thin lines) at near- and below-barrier energies. Among them, both  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$  have no PQNT channels and were used as a benchmark for the suitable inelastic coupling effects in the CC calculations. The CC calculations results (thick lines) reproduce qualitatively the experimental fusion excitation functions of the two symmetric  $^{40}\text{Ca} + ^{40}\text{Ca}$  (dotted line) and  $^{48}\text{Ca} + ^{48}\text{Ca}$  (solid line) systems. The CC calcula-

tion result, on the other hand, with the coupling schemes extracted from  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$ , greatly underestimates the experimental fusion cross sections of the asymmetric  $^{40}\text{Ca} + ^{48}\text{Ca}$  system (dashed line) at near- and below-barrier energies. Usually, the sub-barrier fusion enhancement is ascribed to coupling to both the low-lying collective excitations and PQNT for the stable tightly-bound systems. Here, the coupling to the excitations has already been considered in the CC calculation for  $^{40}\text{Ca} + ^{48}\text{Ca}$ . Therefore, this underestimation is a strong indication of the sub-barrier fusion enhancement correlated with PQNT for  $^{40}\text{Ca} + ^{48}\text{Ca}$ . The fusion excitation functions of  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$  in Fig. 1(b) also show similar behavior.

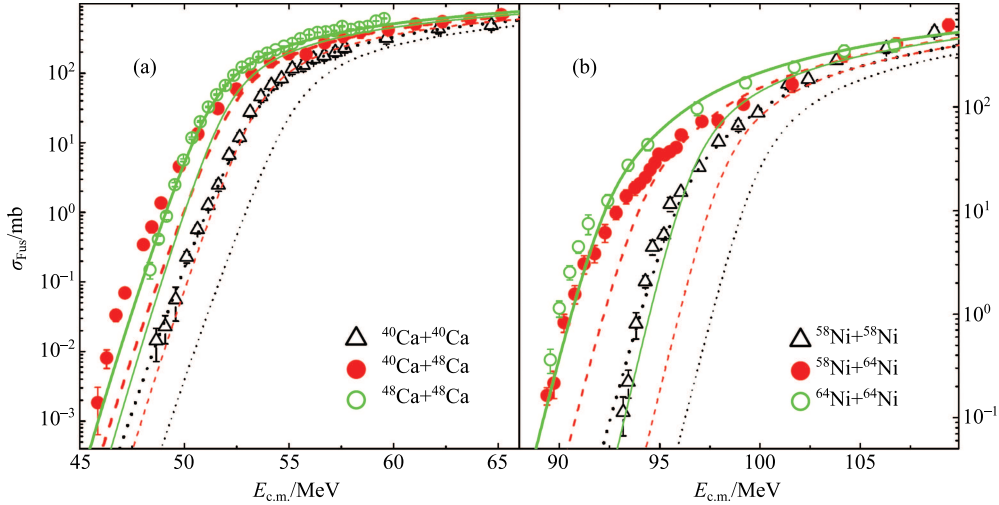


Fig. 1. (color online) The SC (thin lines) and CC (thick line) calculations for the fusion excitation functions of  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  (a) and  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$  (b). The dotted, dashed and solid lines in each panel represent the lighter symmetric, asymmetric and heavier symmetric systems, respectively. The original experimental fusion data are taken from Refs. [8–11].

Further, Fig. 2 shows the variation of RE with the reduced energy, the ratio of the energy in center-of-mass frame ( $E_{c.m.}$ ) to the Coulomb barrier energy ( $V_B$ ), for  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$  [22]. For  $^{40}\text{Ca} + ^{48}\text{Ca}$ , the RE deviates from unity with decreasing energy in the sub-barrier energy region. This isotopic dependence of RE is a strong sign for the PQNT effect in the sub-barrier fusion of  $^{40}\text{Ca} + ^{48}\text{Ca}$ , considering the fact that the inelastic couplings of the reactants which should be considered in the CC calculations have been calibrated by using the experimental data of  $^{40}\text{Ca} + ^{40}\text{Ca}$  and  $^{48}\text{Ca} + ^{48}\text{Ca}$ . A similar behavior also occurs for  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$  in Fig. 3, although the data quality of the experimental cross sections should be improved for such a quantitative analysis. This analysis gives a quantitative estimation for the effect of the sub-barrier fusion enhancement correlated with PQNT.

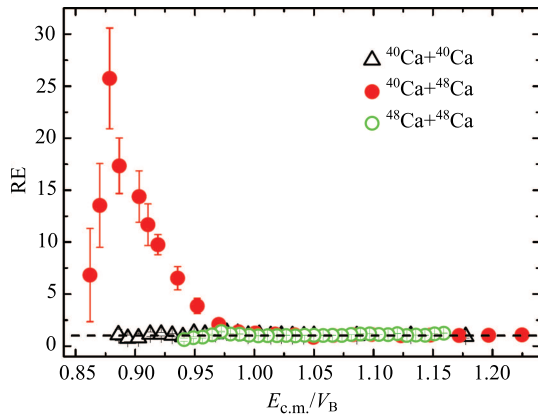


Fig. 2. (color online) RE for  $^{40,48}\text{Ca} + ^{40,48}\text{Ca}$ . The original experimental fusion data are taken from Refs. [9–11].

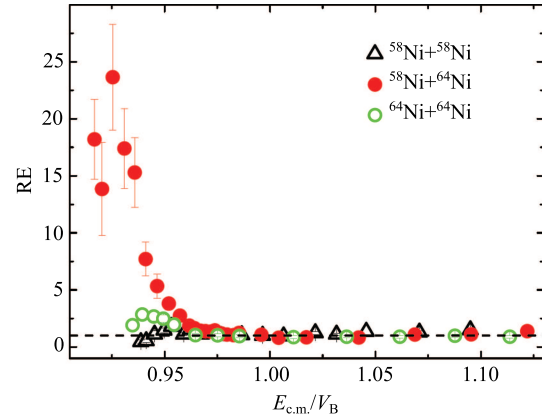


Fig. 3. (color online) RE for  $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$ . The original experimental fusion data are taken from Ref. [8].

Also, it seems that the WS potential works well in the whole measured energy region, at least, for the fusion of the studied symmetric systems of Ca + Ca and Ni + Ni.

Moreover, one matter which should be pointed out from Figs. 2 and 3 is that the RE for both  $^{40}\text{Ca} + ^{48}\text{Ca}$  and  $^{58}\text{Ni} + ^{64}\text{Ni}$  decreases below the peak energy with decreasing energy below the barriers. This decrease possibly means the fading out of the inelastic and/or PQNT coupling effect with decreasing reaction energy. This tendency is consistent with the qualitative physical image that the reaction channels will gradually shut down with decreasing reaction energy [20]. This phenomenon is highlighted here by using this method and may offer further constraints for the relevant theories. The deep-seated reaction dynamics should be explored further both experimentally and theoretically.

## 4 Summary

In summary, in order to extract the PQNT effect quantitatively in near-barrier heavy-ion fusion, the self-consistent inelastic-constrained analysis with the RE method is further applied to analyze the typical fusion excitation functions of Ca + Ca and Ni + Ni. The AW nuclear potential is used in the calculations. This analysis shows that the CCFULL calculation result can be

used as a scale for the RE, but still needs a reference for calibrating the inelastic coupling scheme in the CC calculations for the nuclei involved.

This analysis gives a further evidence for the sub-barrier fusion enhancement correlated with PQNT. Additionally, the quantitative fusion enhancement extent is also extracted by using the RE method, which should be helpful for checking and promoting the current theoretical models.

## References

- 1 X. J. Bao et al, Phys. Rev. C, **91**: 011603(R) (2015)
- 2 L. R. Gasques et al, Phys. Rev. C, **76**: 035802 (2007)
- 3 L. F. Canto et al, Phys. Rep., **596**: 1 (2015)
- 4 B. B. Back et al, Rev. Mod. Phys., **86**: 317 (2014)
- 5 M. Dasgupta et al, Annu. Rev. Nucl. Part. Sci., **48**: 401 (1998)
- 6 A. B. Balantekin, and N. Takigawa, Rev. Mod. Phys., **70**: 77 (1998)
- 7 K. Hagino, and N. Takigawa, Prog. Theor. Phys., **128**: 1001 (2012)
- 8 M. Beckerman et al, Phys. Rev. Lett., **45**: 1472 (1980)
- 9 G. Montagnoli et al, Phys. Rev. C, **85**: 024607 (2012)
- 10 M. Trotta et al, Phys. Rev. C, **65**: 011601 (2001)
- 11 C. L. Jiang et al, Phys. Rev. C, **82**: 041601(R) (2010)
- 12 H. Q. Zhang et al, Phys. Rev. C, **82**: 054609 (2010)
- 13 H. M. Jia et al, Phys. Rev. C, **86**: 044621 (2012)
- 14 H. M. Jia et al, Phys. Rev. C, **89**: 064605 (2014)
- 15 R. A. Broglia et al, Phys. Rev. C, **27**: R2433 (1983)
- 16 A. S. Umar and V. E. Oberacker, Phys. Rev. C, **74**: 021601(R) (2006)
- 17 G. Scamps, and K. Hagino, Phys. Rev. C, **92**: 054614 (2015)
- 18 N. Wang et al, Phys. Rev. C, **90**: 054610 (2014)
- 19 B. Wang et al, Sci. Chin. G, **59**: 642002 (2006)
- 20 C. J. Lin, Phys. Rev. Lett., **91**: 229201 (2003)
- 21 C. Simenel et al, Phys. Rev. C, **88**: 064604 (2013)
- 22 H. M. Jia et al, Phys. Lett. B, **755**: 43 (2016)
- 23 K. Hagino et al, Comput. Phys. Commun., **123**: 143 (1999)
- 24 K. Hagino et al, Phys. Rev. Lett., **79**: 2014 (1997)
- 25 Ö. Akyüz and A. Winther, in *Nuclear Structure and Heavy-Ion Reactions*, Proceedings of the International School of Physics "Enrico Fermi," Course LXXVII, Varenna, edited by R. A. Broglia and R. A. Ricci (North-Holland, Amsterdam, 1981)
- 26 H. Timmers et al, Nucl. Phys. A, **633**: 421 (1998)