Adaptive multitrack reconstruction for particle trajectories based on fuzzy c-regression models *

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Abstract: In this paper, an approach to straight and circle track reconstruction is presented, which is suitable for particle trajectories in an homogenous magnetic field (or 0 T) or Cherenkov rings. The method is based on fuzzy c-regression models, where the number of the models stands for the track number. The approximate number of tracks and a rough evaluation of the track parameters given by Hough transform are used to initiate the fuzzy c-regression models. The technique effectively represents a merger between track candidates finding and parameters fitting. The performance of this approach is tested by some simulated data under various scenarios. Results show that this technique is robust and could provide very accurate results efficiently.

Key words: track reconstruction, fuzzy c-regressions, Hough transform, track finding, track fitting

PACS: 29.40.Cs, 29.40.Gx, 29.40.Ka **DOI:** 10.1088/1674-1137/39/3/036201

1 Introduction

An important problem in the area of pattern recognition is curve recognition (here lines are considered as special curves). Such examples in the high energy physics area are: finding Cherenkov rings and 2D track reconstruction such as TPC. These problems can be considered as combinatorial optimization problems, i.e. given a set of detector signals one has to reconstruct several track subjects upon different constraints. Besides track finding, another important problem for track reconstruction is track fitting, i.e. estimating the parameters of the reconstructed tracks. The parameters of tracks are always related to the momentum, charge and the type of particles. Both the processes of track finding and track fitting are essential steps in the data analysis chain of high-energy physics experiments, especially for off-line data analysis.

Variety track reconstruction methods have been proposed for different experimental setups in the past. Some general review papers about the reconstruction methods can be found in Refs. [1–3]. All the methods can be classified into two classes, one is the classical method and the other is the adaptive method. The classical methods mainly contain track road, track following and the Hough transform [4]. Track road and track following need an initial track, which is always given by two or three random measurement points; however, the efficiency is limited because most of the initial tracks are wrong. As a global method, the Hough transform treats all measurements simultaneously; however the parameter fitting accuracy depends heavily on the quantization of the parameter space and the computational consumption increases dramatically in the high dimensions. In some track reconstruction methods [5], the Hough transform is used to provide initial values of the track parameters. The first attempt to apply adaptive methods for track reconstruction is the application of the Hopfield neural networks [6, 7]. Since then lots of methods such as elastic nets and deformable templates [5, 8], Gaussian-sum filter [9, 10], Kalman filter [11, 12] and cellular automaton [13] are suggested. These adaptive methods are used widely for track reconstruction in high energy physics, and each method has its advantages and disadvantages. In general, all the methods require numerical minimization of a complicated energy function. In the initial stage, the expectation–maximization (EM) algorithm [14] and simulated annealing algorithm [7, 8] are used and then the adaptive least-squares estimators and Kalman filter [15] are suggested.

In this work a new fuzzy c-regression models method (FCRM) for the straight and circle track reconstruction

Received 28 April 2014, Revised 8 August 2014
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is presented. Because those straight and circle tracks can be described as regression models, then the FCRM algorithm can be used to provide an effective way for multitrack reconstruction. The regression equations stand for the track models, the same as deformable templates, which are initialed by the Hough transform. The Hough transform provides the approximate number of regression models and a rough evaluation of the parameters of the regression equation. Instead of complex algorithms such as EM, simulation annealing and the Kalman Filter, we derived the minimum of energy function by solving regression equations with the weighted least squares method. Compared to other adaptive methods, the energy function of FCRM is simple and easy to understand. Because the fitting process in the FCRM is based on the least squares method, the fitting parameters are very accurate. Both the straight tracks and circle tracks from simulated data are reconstructed in this paper. Results show that FCRM is immune to noise and position uncertainties and provides very accurate fitting parameters.

2 Fuzzy c-regressions method

2.1 General concepts of FCRM

Assume that $S=\{(\boldsymbol{x}_1,\boldsymbol{y}_1),\cdots,(\boldsymbol{x}_n,\boldsymbol{y}_n)\}$ is a set of data where each independent observation $x_k\in\boldsymbol{R}^p$ has a corresponding dependent observation $y_k\in\boldsymbol{R}^t$. If we assume that the data to be drawn from c models:

$$y=f_i(\boldsymbol{X};\beta_i)+\varepsilon_i,\quad 1\leqslant i\leqslant c, \tag{1}$$

where each $\beta_i\in\Omega_i\subset\boldsymbol{R}^k$, and each ε_i is a random vector with mean vector $\mu_i=0\in\boldsymbol{R}^t$ and covariance Σ_i. The switching regression is employed to find c linear regressions

$$y_{j,i}=\beta_{i0}+\beta_{i1}x_{j1}+\cdots+\beta_{ip}x_{jp},\quad i=1,\cdots,c, \tag{2}$$

that best fit the data structure.

Hathaway and Bezdek [16] first combined switching regressions with fuzzy c-means and referred to them as the fuzzy c-regressions models algorithm. The FCRM algorithm is to minimize the objective function $E_m(U,\{\beta_i\})$ defined in

$$E_m(U,\{\beta_i\})=\sum_{k=1}^{n}\sum_{i=1}^{c}u_{ik}^m E_{ik}(\beta_i), \tag{3}$$

where $m>1$ is fixed, and the $E_{ik}(\beta_i)$ is the measure of the error in $f_i(\boldsymbol{X}_k;\beta_i)$ as an approximation to $\boldsymbol{y}_k$, $1\leqslant i\leqslant c$ and $1\leqslant k\leqslant n$. The most common example for such a measure is the squared vector norm $E_{ik}(\beta_i)=\|f_i(\boldsymbol{X}_k;\beta_i)-\boldsymbol{y}_k\|^2$. U is the matrix of membership degree with $U=(u_{ik})$, and $0\leqslant u_{ik}\leqslant 1$, $\sum\limits_{i=1}^{c}u_{ik}=1$ stands for the probability that data (x_k,y_k) came from class i. The update equations for the minimization are

$$U_{ik}=\frac{1}{\sum\limits_{j=1}^{c}\left(\dfrac{E_{ik}}{E_{jk}}\right)^{\frac{1}{m-1}}} \tag{4}$$

and

$$\beta_i=[\boldsymbol{X}^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{X}]^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{Y}, \tag{5}$$

where $\boldsymbol{X}$ denotes the matrix in $\boldsymbol{R}^{n\times(p+1)}$ having $(1,x_j)$ as its jth row, $\boldsymbol{Y}$ denotes the vector in $\boldsymbol{R}^n$ having y_j as its jth component, and $\boldsymbol{D}_i$ denotes the diagonal matrix in $\boldsymbol{R}^{n\times n}$ having u_{ij}^m as its jth diagonal element. The calculation of u_{ik} uses the Lagrange multiplier and is given in the appendix.

$$\boldsymbol{X}=\begin{bmatrix}1 & x_1\\ 1 & x_2\\ \vdots & \vdots\\ 1 & x_j\end{bmatrix},\ \boldsymbol{Y}=\begin{bmatrix}y_1\\ y_2\\ \vdots\\ y_j\end{bmatrix},\ \boldsymbol{D}_i=\begin{bmatrix}u_{i1} & 0 & \dots & 0\\ 0 & u_{i2} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & u_{ij}\end{bmatrix}.$$

2.2 Noise resistance method

In the real situation, the data set always contain some noise signals, which affect the parameter fitting heavily. Here we suggest a noise resistance approach to avoid the effect of noise. In order to distinguish the noise from the effective data, $d_{ik}(x_k,y_k)$ is used to measure the distance between the data point (x_k,y_k) and the regression model $y=f(x,\beta_i)$. The distance is an abstract concept used to describe the similarity between the data point (x_k,y_k) and the regression model $y=f(x,\beta_i)$, commonly it comes with Euclidean distance. Then, setting a cut threshold w, if the distance $d_{ik}>w$ for $i=1,\cdots,c$, the data (x_k,y_k) will be considered as noise.

2.3 Initiation by the Hough transform

The FCRM strategy is to match the measurements (data points) to simple parameterized models (which in our case are lines or circles). However, the performance of FCRM depends heavily on initial values. Kuo-Lung Wu [17] shows that, without initiation, for two parallel lines there are about 53% trials that FCRM performs with correct results and 82% for two cross lines. In order to get a high quality solution avoiding the local minima, a method is needed that provides us with the approximate number of regression models and the approximate values of the parameters. Some existing methods can give us the initial values we need, such as k-mean clustering, the mountain method [18, 19] and the Hough transform. In this paper, for the track models which can be described as lines or circles, the Hough transform was used for this task.

By using the Hough transform, data transformation should be processed first, i.e. transform the original c-regression data set into a parameter space. For those straight tracks, any two points can form a line, for example, any two points can have a regression equation $y=b_0+b_1x$ where the parameters of this equation is denoted by $b=(b_0,b_1)$. If we pair all points of the data set to form the regression lines, we will have a set of parameters $b_l=(b_{l0},b_{l1})$, $l=1,\cdots,C_n^2$ where C_n^2 denotes the number of combinations. Then the histogram of the parameters (b_{l0},b_{l1}) will form local maximum peaks in the parameter space. The number of the peaks indicate the regression models and the location of the peaks indicate the approximate values of the parameters. However, for those tracks that pass through the origin point, $b_{l0}\equiv 0$, the parameter of b_{l1} will form a new histogram with cleaner signal-to-background separation. Fig. 1 shows the original signals corresponding to non-perfect straight tracks with noise and the distribution of θ, here $\theta=\text{arctg}b_{l1}$, $l=1,\cdots,C_n^2$. It is obvious that the Hough transform can give us the parameters, both the number of models and the approximate values of (b_{l0},b_{l1}), we need in the FCRM, even though the tracks are not perfectly straight and are noisy.

For those circle tracks, if the circles going through the origin of a two-dimension x-y coordinate system (in fact most track models from TPC satisfy this demand), they will map onto straight lines in a u-v coordinate system by the transformation

$$u=\frac{x}{x^2+y^2},\quad v=\frac{y}{x^2+y^2},\tag{6}$$

where the circles are defined by the circle equation $(x-a)^2+(y-b)^2=r^2=a^2+b^2$. The straight lines in the $u-v$ plane are then given by

$$v=\frac{1}{2b}-u\frac{a}{b}.\tag{7}$$

Obviously, the problem finding tracks of circle models through the origin point will change to a problem finding straight tracks in the u-v space by the Hough transform, and then all the processes of the Hough transform used for finding straight tracks can be used naturally. Since the transformed straight lines from circle tracks do not always pass through the origin of the u-v coordinate system, a histogram of parameters should be carried out in a 2D parameter space. Fig. 2 shows the signals corresponding to non-perfect circle tracks with noise, the mapping straight lines on the u-v coordinate system and the 2D histogram of angles of slopes $\theta\left(\theta=\text{arctg}\left(-\frac{a}{b}\right)\right)$ and intercepts $\left(\frac{1}{2b}\right)$. The results shows that, based on the transformation of (6), the Hough transform is still an effective method for our initiation of FCRM.

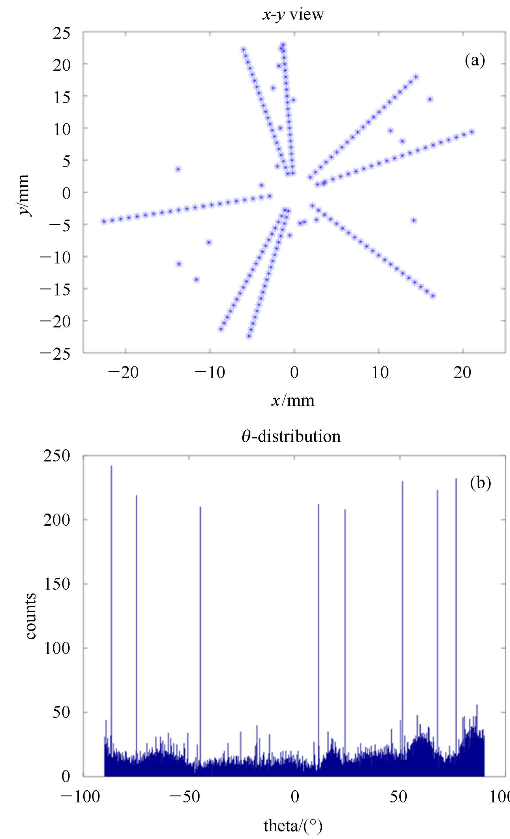

Fig. 1. (a) Original signals corresponding to non-perfect straight tracks with noise and (b) resulting θ-distribution.

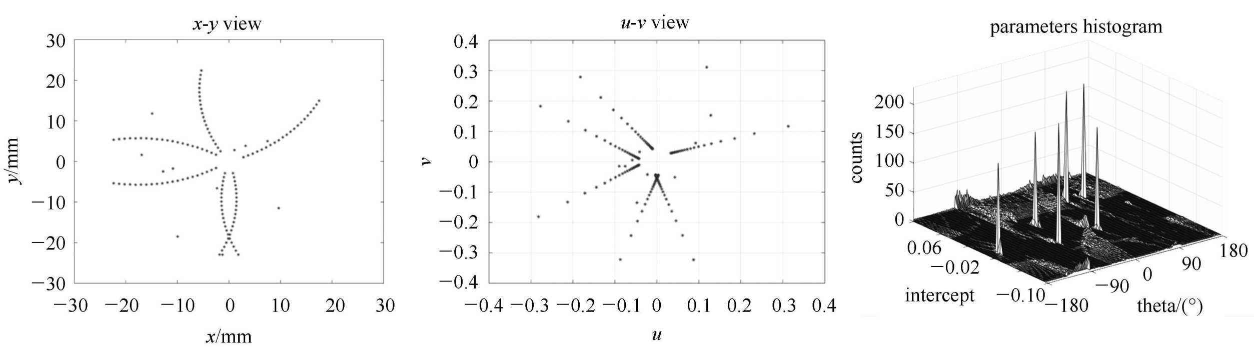

Fig. 2. (left) Original signals corresponding to non-perfect circle tracks with noise; (middle) straight lines transform $x-y$ space to $u-v$ space and (right) the 2D histogram of the parameters.

3 Simulations and results

In this section, some simulations have been done to examine the performance of the FCRM method. In the future the International Linear Collider (ILC) experiment, with a high spatial resolution (∼100 μm) is demanded for its track detector. Several prototypes TPC for ILC have been constructed and tested in the past few years. The spatial resolution of these prototypes is about 100 μm, a minimal spatial resolution of (53±5) μm was measured by Kaminski [20] and the result given by Oda [21] is 79 μm. Another prototype TPC was tested by Yulan LI [22] and a 100 μm spatial resolution was achieved. Based on the experimental results, in the following simulation the simulated data are spread by a normal distribution error with μ=0 and σ=100 μm. Here we assumed that the tracks had 21 measurement layers with a layer-to-layer distance of 5 mm, an inner radius of 30 mm, and outer radius of 130 mm. The detection efficiency, which was set to be 90%, was also considered in the simulation. Some outlier data generated in random were used to examine the noise resistance of FCRM. In fact, the real particle trajectories from ILC are complex and the track reconstruction was a systemic process with lots of factors to consider, such as the reconstruction efficiency, computer consumption, noise immunity and track multiplicity. The simple examples given in this paper are only a principle proof of the FCRM.

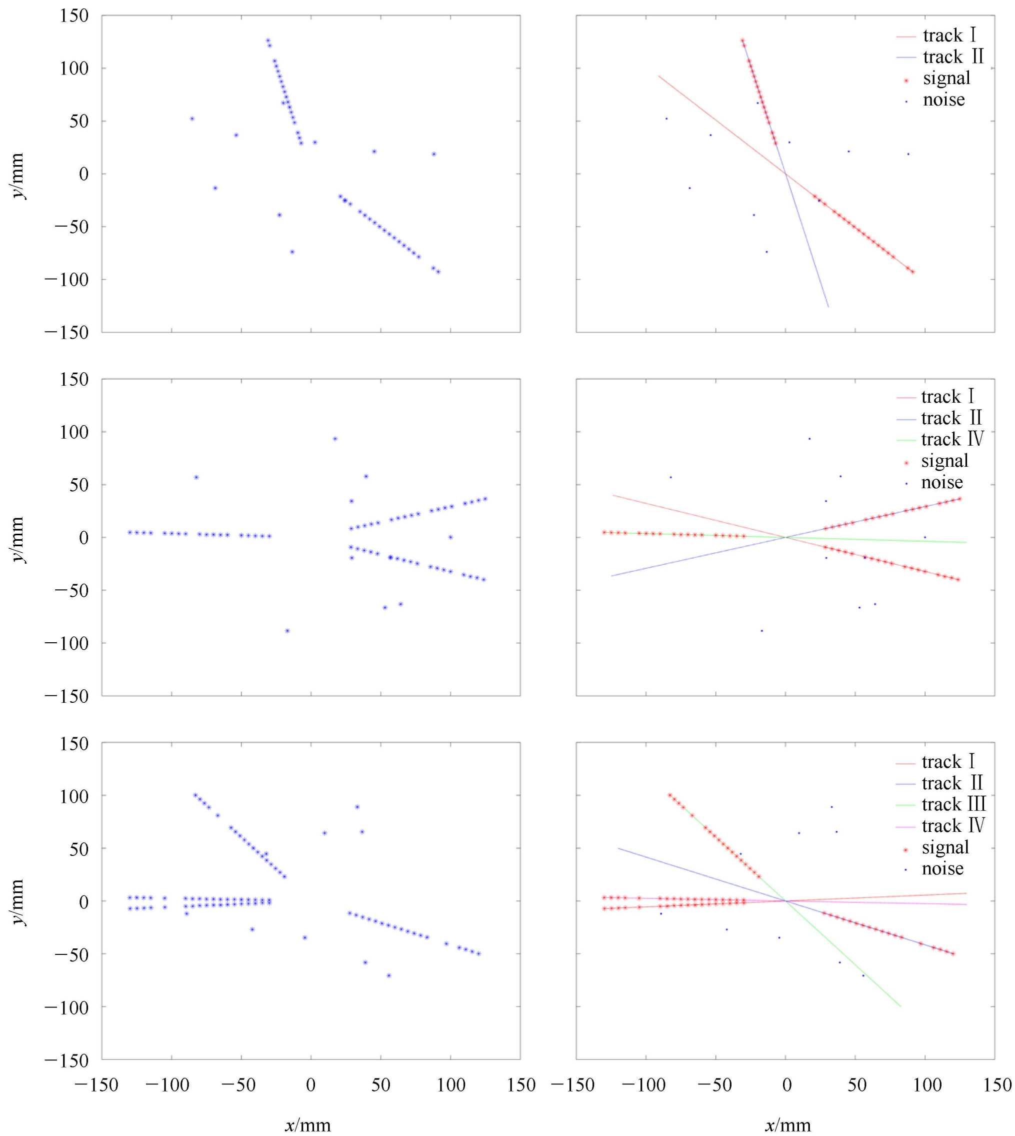

Fig. 3. (left) The original signals and (right) the reconstructed tracks by FCRM.

3.1 FCRM for line reconstruction

Figure 3 shows some results of the reconstructed straight tracks. The left of Fig. 3 shows the original signals obtained from the experiment which contain position uncertainties and noise and the right of that shows the reconstructed tracks from FCRM. Because the detection efficiency is 90%, some measurement points are lost in our simulation, as shown in the left of Fig. 3. The results indicate that it is easy to distinguish multitracks from each other using FCRM, and it is also efficient in distinguishing signals from noises.

In order to evaluate the performance of the FCRM algorithm, 10000 double tracks events are generated and the distribution of the fitting parameters are shown in Fig. 4. The true values of the parameters of the two tracks are $\beta_{11}=0.7265$, $\beta_{12}=0$, $\beta_{21}=1$, $\beta_{22}=0$, which means that the angle of slopes are 36° and 45° and both of them pass through the origin point. The values of the fitting parameters are $\hat{\beta}_{11}=0.7265\pm7.5445\times10^{-4}$, $\hat{\beta}_{12}=-4.5020\times10^{-4}\pm0.0527$, $\hat{\beta}_{21}=1.0000\pm7.6962\times10^{-4}$, $\hat{\beta}_{22}=5.4008\times10^{-4}\pm0.0469$. The results show that the values of the fitting agree well with the true values of the parameters. The error of the slope given by the least squares method can be calculated as [23]:

$$\sigma_m=\frac{\sigma_y}{x_N-x_0}\frac{1}{\sqrt{N+1}}\sqrt{\frac{12N}{N+2}}. \tag{8}$$

Where σ_m is the error of the slope and $\sigma_y=100$ μm is the spatial resolution, $N=21$ is the number of the measurement, $x_N=130$ mm, $x_0=30$ mm. The calculated value of σ_m is 7.0571×10^{-4}. The error of intercept is given as:

$$\sigma_{\mathrm{b}}=\frac{\sigma_y}{\sqrt{N+1}}Z(r,N), \tag{9}$$

where σ_b is the error of intercept and $r=\dfrac{x_N+x_0}{2(x_N-x_0)}$, $Z(r,N)=\sqrt{\dfrac{12r^2+1+2N^{-1}}{1+2N^{-1}}}$. So the calculated value of σ_b is 0.0603 mm. Obviously, the parameters' errors obtained from FCRM are consistent with that from the least squares method.

3.2 FCRM for circle reconstruction

Circle tracks always come from a projection of helices in a certain readout plane or Cherenkov rings. From section 3.2 we can transform the problem of circle track reconstruction into a problem of straight line reconstruction in the u-v space by Hough transform, and then the FCRM can be used naturally. However, we suggest a new approach for circle tracks finding and fitting here, which is based on new regression models.

Based on the initial parameters, such as the number of tracks and the approximate value of the parameters given by the Hough transform, we reconstruct the circle tracks by a new regression model. Chernov and Ososkov [24] gave four regression models for circle fitting and here we use one of them for our FCRM. The equation of the circle can be written as $x^2+y^2+Ax+By+C=0$, supposing that $z=x^2+y^2$ in the circle equation, the circle equation will be transformed into a linear regression equation

$$z=mx+ny+p, \tag{10}$$

where $m=2a$, $n=2b$, $p=R^2-a^2-b^2$ are treated as the new unknown parameters, the circle center defined as (a,b) and the radius equals R. Change our one-dimensional linear regression to multi variable linear regression models as

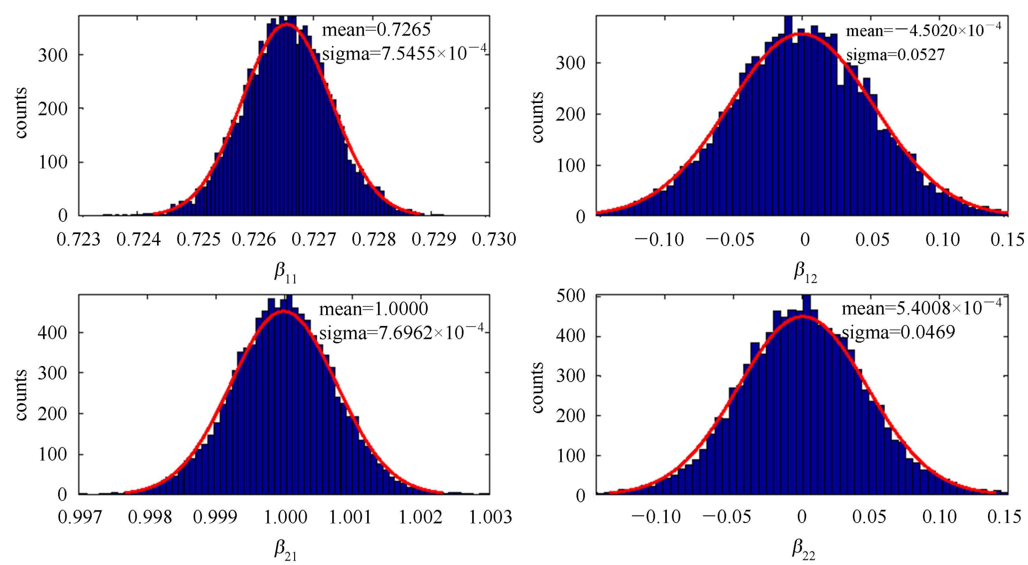

Fig. 4. The distribution of the fitting parameters.

$$y_{ji}=\beta_{i0}+\beta_{i1}x_{j1}+\beta_{i2}x_{j2},\quad i=1,\cdots,c, \tag{11}$$

where $x_{j1}=x_j$, $x_{j2}=y_j$ and $y_{ji}=z_j$. Then the fitting parameters can be obtained by

$$\beta_i=\left[\boldsymbol{X}^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{X}\right]^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{Y}, \tag{12}$$

where

$$\boldsymbol{X}=\begin{bmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ \vdots & \vdots & \vdots\\ 1 & x_j & y_j\end{bmatrix},\boldsymbol{Y}=\begin{bmatrix}(x_1^2+y_1^2)\\ (x_2^2+y_2^2)\\ \vdots\\ (x_j^2+y_j^2)\end{bmatrix},\boldsymbol{D}_i=\begin{bmatrix}u_{i1} & 0 & \cdots & 0\\ 0 & u_{i2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & u_{ij}\end{bmatrix}.$$

Then, the FCRM algorithm can be used and all the calculation steps are the same steps as mentioned above.

It is worth mentioning that the $E_{ik}(\beta_i)$, which stands for the error in $f_i(\boldsymbol{X}_k;\beta_i)$ as an approximation to $\boldsymbol{y}_k$ in function (3), should be changed as below

$$\begin{aligned}E_{ik}(\beta_i) &= E_{ik}(m_i,n_i,p_i)\\ &= |(x_k-a_i)^2+(y_k-b_i)^2-R_i^2|.\end{aligned} \tag{13}$$

Where $a_i=\dfrac{m_i}{2}$, $b_i=\dfrac{n_i}{2}$, $R_i^2=p_i^2+a_i^2+b_i^2$. Fig. 5 shows some simulation results of double circle tracks and triple circle track reconstruction by FCRM. Table 1 lists the comparison of the real values and the fitting values of the parameters from 10000 double circle track events, both the center of the circles (a,b) and the radius (R) contained. The results show that FCRM is effective for the reconstruction of circle tracks.

Table 1. Comparison of the true values and fitting values of the circle parameters.

parameters	circle I			circle II		
	a_1	b_1	R_1	a_2	b_2	R_2
true value	−70.71	70.71	100.00	80.90	58.78	100.00
fitting value	−70.67±0.57	70.71±0.07	99.98±0.50	80.81 ±0.48	58.78±0.12	100.00±0.46

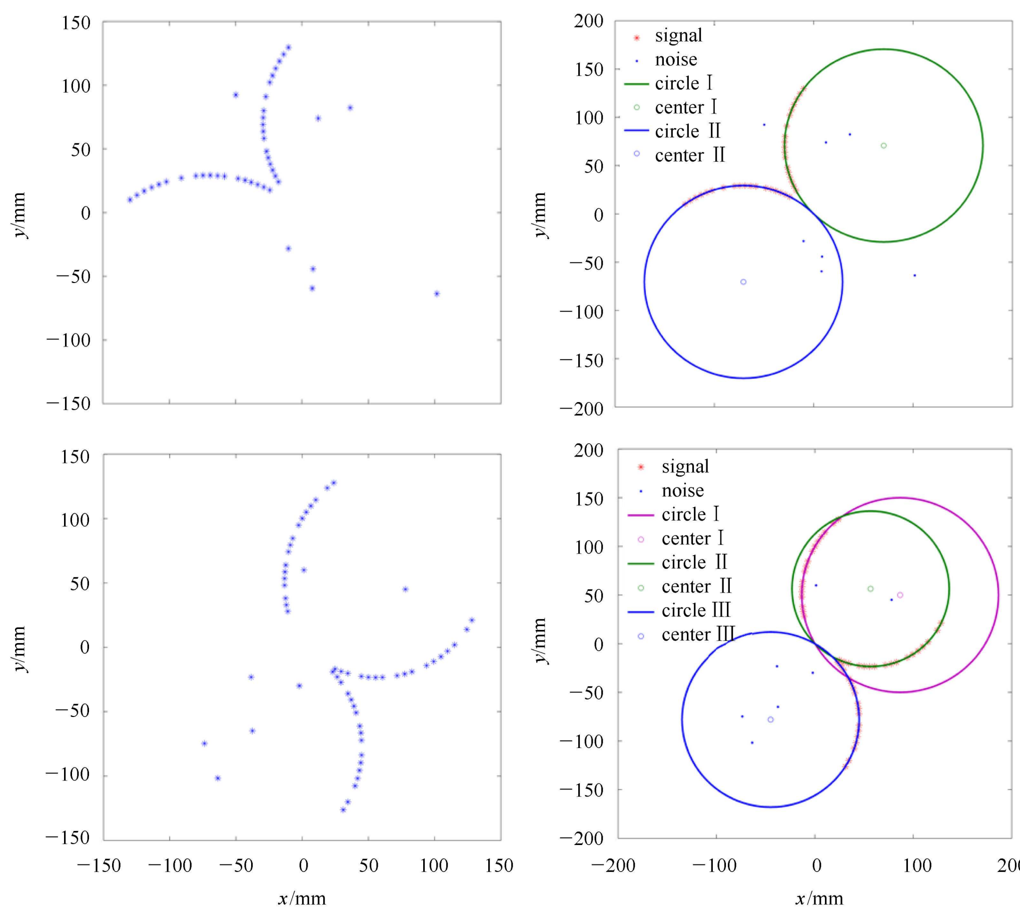

Fig. 5. (color online) (left) The original measured signals and (right) the reconstructed circle tracks by FCRM.

4 Conclusions

In this paper, a novel track reconstruction method using fuzzy c-regression models is proposed, which combines the finding and fitting problem into a single approach. For those straight and circle tracks, which can be described as regression function, the method of FCRM could be used effectively. By initializing a set of regression models with the Hough transform, both the multiple straight and circle tracks can be found and fitted in synchronously. The weighted least squares method is used to solve the regression functions and an iteration process is used to derive the minimum of the objective function, then the track parameters are obtained. From the simulation results one may say that the FCRM method is useful and provides an effective fitting.

Appendix A

The explicit formulas for the new iterates u_{ik} $(1\leqslant i\leqslant c)$ are based on the Lagrange multiplier method. Let

$$F(\lambda,u_{ik})=\sum_{i=1}^{c}(u_{ik})^m(E_{ik}(\beta_i))+\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right), \tag{A1}$$

then the minimum of $F(\lambda,u_{ik})$ can be obtained by solving Eq. (A2) and (A3):

$$\frac{\partial(\lambda,u_{1k},\cdots,u_{ck})}{\partial u_{ik}}=m(u_{ik})^{m-1}(E_{ik}(\beta_i))+\lambda=0, \tag{A2}$$

$$\frac{\partial(\lambda,u_{1k},\cdots,u_{ck})}{\partial\lambda}=\sum_{i=1}^{c}u_{ik}-1=0, \tag{A3}$$

from (A2), we can derive

$$u_{ik}=\left(\frac{-\lambda}{m(E_{ik}(\beta i))}\right)^{\frac{1}{m-1}}=\left(\frac{-\lambda}{m}\right)^{\frac{1}{m-1}}\left(\frac{-\lambda}{(E_{ik}(\beta_i))}\right)^{\frac{1}{m-1}}, \tag{A4}$$

put (A4) into (A3)

$$\begin{aligned}\sum_{i=1}^{c}u_{ik} &= \sum_{i=1}^{c}\left(\frac{-\lambda}{m}\right)^{\frac{1}{m-1}}\left(\frac{-\lambda}{E_{ik}(\beta_i)}\right)^{\frac{1}{m-1}}\\ &= \left(\frac{-\lambda}{m}\right)^{\frac{1}{m-1}}\sum_{i=1}^{c}\left(\frac{-\lambda}{E_{ik}(\beta_i)}\right)^{\frac{1}{m-1}},\end{aligned} \tag{A5}$$

then

$$\left(\frac{-\lambda}{m}\right)^{\frac{1}{m-1}}=\frac{1}{\sum_{i=1}^{c}\left(\frac{-\lambda}{E_{ik}(\beta_i)}\right)^{\frac{1}{m-1}}}, \tag{A6}$$

put (A6) into (A4)

$$u_{ik}=\frac{1}{\sum_{j=1}^{c}\left(\frac{E_{ik}}{E_{jk}}\right)^{\frac{1}{m-1}}}. \tag{A7}$$

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