Electron bunching in a Penning trap and accelerating process for CO_2 gas mixture active medium *

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Abstract: In PASER (particle acceleration by stimulated emission of radiation), in the presence of an active medium incorporated in a Penning trap, moving electrons can become bunched, and as they get enough energy, they escape the trap forming an optical injector. These bunched electrons can enter the next PASER section filled with the same active medium to be accelerated. In this paper, electron dynamics in the presence of a gas mixture active medium incorporated in a Penning trap is analyzed by developing an idealized 1D model. We evaluate the energy exchange occurring as the train of electrons traverses into the next PASER section. The results show that the oscillating electrons can be bunched at the resonant frequency of the active medium. The influence of the trapped time and population inversion are analyzed, showing that the longer the electrons are trapped, the more energy from the medium the accelerated electrons get, and with the increase of population inversion, the decelerated electrons are virtually unchanged but the accelerated electrons more than double their peak energy values. The simulation results show that the gas active medium needs a lower population inversion to bunch the electrons compared to a solid active medium, so the experimental conditions can easily be achieved.

Key words: PASER, Penning trap, active gas mixture medium, electron bunching

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1 Introduction

In particle acceleration by stimulated emission of radiation (PASER)[1–3], the energy stored in molecules or atoms is directly used to accelerate electrons. PASER does not need phase-matching or compensation for phase slippage, nor does it require a high power laser, a beam driver, or an electron gun. Recently, calculated results show that the wake generated by the trigger bunch of electrons in the active medium can achieve 1 GV/m [4–6].

In traditional accelerator structure, phase-matching is necessary to ensure that particles only experience the field pointing in the same direction as the particle motion, thereby allowing them to gain net energy. In PASER, however, the electrons interact with the active medium via a virtual photon. This photon is emitted by an excited molecule, so when the electron absorbs this photon, the electron gains energy equal to the energy emitted when the electron of the excited molecule returns from the upper to the lower energy state. The electric field of this virtual photon is not critical for this absorption process to occur. Hence, phase-matching is not needed.

In PASER, the electron passing near an excited molecule stimulates the molecule to emit a photon, which is absorbed directly by the electron. If the electrons are bunched together with a spacing equal to the wavelength of the emitted photons, i.e., at the resonance of the excited state, then there is a coherent effect that further enhances the energy exchange process. This is why it is preferable to have the electrons bunched at the resonance wavelength during the PASER process.

In the proof-of-principle PASER experiment [3] which gave the first experimental PASER result, the bunch equipment consisted of an existing accelerator, wiggler, and high power laser, which are very complex and expensive. In order to replace these three components, Schachter suggested a novel paradigm [7, 8] which relies on the possibility that in the presence of a solid active medium (Nd:YAG), the non-relativistic electrons oscillating in a Penning trap may get bunched at the resonance frequency of the active medium. During multiple round trips in the trap, the bunched electrons get enough

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kinetic energy, and can escape the trap forming a low energy optical injector. Thus, an electron gun is not needed.

When the bunched electrons escape the trap, they can enter the next PASER section to reach higher energies.

In Ref. [9], we performed some theoretical analysis and simulations of PASER. In this paper, electron dynamics in the presence of a CO_2 gas mixture active medium incorporated in a Penning trap is analyzed. Fig. 1 shows the schematics of a Penning trap. Compared to a solid active medium, the CO_2 gas mixture active medium is less costly and easier to get. In the following, based on the 1D solid active medium model [8], we develop an idealized 1D model in a gas mixture active medium to analyze the bunching process in the Penning trap. Further calculations are made with MATLAB to investigate the influence of the oscillating time in the trap and the population inversion. Furthermore, a calculated simulation of kinetic energy gain of the bunched electrons in the next PASER section is studied.

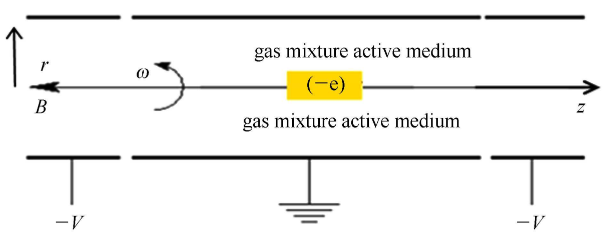

Fig. 1. (color online) Schematics of a Penning trap. The trap uses coils to generate a uniform axial magnetic field to provide radial confinement and applied end potentials provide axial confinement.

2 Bunching process of electrons in the presence of CO_2 gas mixture active medium in a Penning trap

Based on the 1D solid active medium model [8], considering the difference in dielectric function ε and "plasma" frequency ω_p between the solid and gas active mediums, electron dynamics in the presence of a CO_2 gas mixture active medium incorporated in a Penning trap is investigated by deducing an idealized 1D model and simulations.

The dielectric function of gas active medium is expressed by

$$\varepsilon(\omega>0)\equiv 1+\frac{\omega_p^2}{\omega_0^2-\omega^2+2j\omega/T_2}, \quad (1)$$

which satisfies $\varepsilon(\omega<0)=\varepsilon^*(\omega>0)$. It is assumed that the medium has a single resonance frequency chosen to correspond to the macro-bunch modulation $\omega_0=2\pi c/\lambda_0$, where ω_p is the "plasma" frequency, $\omega_p^2\equiv e^2\Delta n/m\varepsilon_0$, with m being the rest mass of the electron and Δn representing the population density of the resonant atoms. For an excited medium, when the population density is inverted ($n<0$), the plasma frequency is negative ($\omega_p^2<0$); T_2 is the relaxation time.

We assume that in the Penning trap there are N_{mp} macro-particles, and each macro-particle contains N_{el} electrons. If the νth macro-particle's trajectory is represented by $(r_\nu(t),z_\nu(t))$, then the longitudinal current density can be expressed by:

$$J_z(r,z,t)=-eN_{el}\sum_\nu \dot{z}_\nu(t)\frac{1}{2\pi r}\delta[r-r_\nu(t)]\delta[z-z_\nu(t)]. \quad (2)$$

The magnetic potential associated with the above current density in the boundless case is given by

$$A_z(r,k,\omega)=\mu_0\int dr'r'G(\Lambda r,\Lambda r')J_z(r',k,\omega), \quad (3)$$

in which $J_z(r',k,\omega)$ is the spatial and temporal Fourier transform of the longitudinal current density in Eq. (2), and $\Lambda^2=k^2-\varepsilon(\omega)\omega^2/c^2$

$$G(\Lambda r,\Lambda r')=\begin{cases} I_0(\Lambda r)K_0(\Lambda r'), r<r' \\ K_0(\Lambda r)I_0(\Lambda r'), r>r'. \end{cases} \quad (4)$$

So we can get the longitudinal electric field generated by the ensemble of electrons in the trap as follows:

$$\begin{aligned} E_z(r,z,t) = & \frac{\mu_0 eN_{el}N_{mp}}{(2\pi)^3}\int_{-\infty}^{\infty} d\omega \exp(jwt) \\ & \times\int_{-\infty}^{\infty} dk\exp(-jkz)\frac{c^2\Lambda^2}{j\omega\varepsilon(\omega)} \\ & \times\int_{-\infty}^{\infty} dt'\exp(-jwt') \\ & \times\langle G[\Lambda r,\Lambda r_\nu(t')]\dot{z}_\nu(t')\exp[jkz_\nu(t')]\rangle_\nu. \quad (5) \end{aligned}$$

Therefore, the force on a single macro-particle (the νth macro-particle) is expressed by

$$\begin{aligned} F_\nu(t) = & -eN_{el}E_z[r_\nu(t),z_\nu(t),t] \\ = & -\frac{\mu_0 e^2N_{el}^2N_{mp}}{(2\pi)^3}\int_{-\infty}^{\infty} d\omega\exp(jwt) \\ & \times\int_{-\infty}^{\infty} dk\exp[-jkz_\nu(t)]\frac{c^2\Lambda^2}{j\omega\varepsilon(\omega)} \\ & \times\int_{-\infty}^{\infty} dt'\exp(-jwt') \\ & \times\langle\langle G[\Lambda r_\nu(t),\Lambda z_\mu(t')]\rangle\dot{z}_\mu(t')\exp[jkz_\mu(t')]\rangle_\mu. \quad (6) \end{aligned}$$

The total energy exchange is given by

$$\begin{aligned} W_{\rm ex} &= -N_{\rm mp}\int_{-\infty}^{\infty}{\rm d}t\langle \dot{z}_\mu(t)F_\nu(t)\rangle_\nu \\ &= -\frac{\mu_0 e^2 N_{\rm el}^2 N_{\rm mp}^2}{(2\pi)^3}\int_{-\infty}^{\infty}{\rm d}t\int_{-\infty}^{\infty}{\rm d}\omega\exp({\rm j}wt) \\ &\quad\times\int_{-\infty}^{\infty}{\rm d}k\frac{c^2\Lambda^2}{{\rm j}\omega\varepsilon(\omega)}\times\int_{-\infty}^{\infty}{\rm d}t'\exp(-{\rm j}wt') \\ &\quad\times\langle\dot{z}_\nu(t')\exp[-{\rm j}kz_\nu(t)]G[\Lambda r_\nu(t),\Lambda r_\mu(t')] \\ &\quad\times\dot{z}_\mu(t')\exp[{\rm j}kz_\mu(t')]\rangle_{\nu,\mu}. \end{aligned} \tag{7}$$

Before proceeding, we make the following assumptions: (1) the transverse distribution is independent of the longitudinal distribution, and its contribution to the energy exchange is negligible; (2) in the radial direction, the electrons are uniformly distributed in the range $0<r<R_{\rm b}$, so the transverse filling factor $\langle G[\Lambda\rho_\nu,\ \Lambda\rho_\mu]\rangle_{\nu,\mu}$ can be replaced by the transverse form factor $F_\perp(\Lambda R_{\rm b})=2[1-2K_1(\Lambda R_{\rm b})I_1(\Lambda R_{\rm b})]/(\Lambda R_{\rm b})^2$. For a CO_2 gas mixture active medium, the transverse wavelength is much larger than the radius of the electron ensemble, so $\Lambda R_{\rm b}$ is very small, and we can make the approximation

$$[1-2K_1(\Lambda R_{\rm b})I_1(\Lambda R_{\rm b})]\approx(\pi x/2)^2.$$

In the Penning trap, the angular frequency Ω is determined by $\Omega=(2c/L)\sqrt{2eV_0/mc^2}$, where L is the length of the Penning trap, so ignoring the damping during one period of the oscillation, the trajectory of the νth macro-particle can be assumed to be given by

$$z_\nu(t)=\frac{L}{2}\{1+\cos[\Omega(t-t_\nu)]\}. \tag{8}$$

So the velocity can be expressed by

$$\dot{z}_\nu(t)=-\frac{L}{2}\Omega\sin[\Omega(t-t_\nu)]. \tag{9}$$

Therefore, after a series of derivations, the total energy exchange is as follows:

$$\begin{aligned} W_{\rm ex} &= \frac{-e^2N_{\rm el}^2N_{\rm mp}^2(2\pi)^3e^2\Delta n}{4\pi\varepsilon_0\Omega^2(2L)m\varepsilon_0}\times\Big\langle\cos[\omega_0(t_\nu-t_\mu)] \\ &\quad\times\exp\left[-\frac{1}{T_2}(t_\nu-t_\mu)\right]h(t_\nu-t_\mu)\Big\rangle_{\nu,\mu}. \end{aligned} \tag{10}$$

So the force on the νth macro-particle contributing directly to the energy exchange is

$$\begin{aligned} F_\nu(t) &= \frac{2e^2N_{\rm el}N_{\rm mp}(2\pi)^2e^2\Delta n}{4\pi\varepsilon_0\Omega^2L^2m\varepsilon_0}\times\Big\langle\cos[\omega_0(t_\nu-t_\mu)] \\ &\quad\times\exp\left[-\frac{1}{T_2}(t_\nu-t_\mu)\right]h(t_\nu-t_\mu)\Big\rangle_\mu\sin[\Omega(t-t_\nu)], \end{aligned} \tag{11}$$

and the amplitude of the force is

$$\begin{aligned} f_\nu(t) &= \frac{2e^2N_{\rm el}N_{\rm mp}(2\pi)^2e^2\Delta n}{4\pi\varepsilon_0\Omega^2L^2m\varepsilon_0} \\ &\quad\times\Big\langle\cos[\omega_0(t_\nu-t_\mu)] \\ &\quad\times\exp\left[-\frac{1}{T_2}(t_\nu-t_\mu)\right]h(t_\nu-t_\mu)\Big\rangle_\mu. \end{aligned} \tag{12}$$

Assuming the effective impact of the trap on the particles is represented by an ideal harmonic oscillator, then its force is $\Omega^2 z$, and the equation of motion of the νth macro-particle is

$$F_\nu(t)+F_{\rm scatt}+\Omega^2z_\nu=N_{\rm el}m\ddot{z}_\nu, \tag{13}$$

in which $F_{\rm scatt}$ is the effect of the elastic collisions of the electrons with the gas.

Explicitly, the equation of motion can be expressed by

$$\frac{{\rm d}^2z_\nu}{{\rm d}t^2}+\left(\frac{2}{\tau_\nu^{\rm (csk)}}+\frac{2}{\tau_{\rm scatt}}\right)\frac{{\rm d}z_\nu}{{\rm d}t}+\Omega^2z_\nu=0, \tag{14}$$

in which the decay parameter $\tau_\nu^{\rm (csk)}$ is represented by

$$\begin{aligned} \frac{2}{\tau_\nu^{\rm (csk)}} &= -\frac{2\pi^2\Delta ne^2r_{\rm e}N_{\rm mp}N_{\rm el}}{\varepsilon_0mc(2eV_0/mc^2)^{3/2}}\times\Big\langle\cos[\omega_0(t_\nu-t_\mu)] \\ &\quad\times\exp\left[-\frac{1}{T_2}(t_\nu-t_\mu)\right]h(t_\nu-t_\mu)\Big\rangle_\mu. \end{aligned} \tag{15}$$

Assuming that, during one round trip in the trap, the decay parameter does not change significantly, then solving the equation of motion, we can get that after one round trip the νth macro-particle's phase space can be given by

$$z_\nu(T)=z_\nu(0)\exp(-T/\tau_\nu), \tag{16}$$

$$\dot{z}_\nu(T)=\dot{z}_\nu(0)\exp(-T/\tau_\nu). \tag{17}$$

Here, $(z_\nu(0),\ \dot{z}_\nu(0))$ is the initial phase space.

For the CO_2 gas mixture active medium, $\lambda_0=10.6$ μm, and we assume the population density is $\Delta n\approx10^{11}$ m^{-3}, the relaxation time T_2=2 ms, the trap is of length L=0.01 m, the voltage V_0 on the central anode is 400 V, $N_{\rm mp}=500$, $N_{\rm el}=10^6$, and the decay time $\tau_{\rm scatt}$ is 0.08 ms. Based on the parameters and the equations above, we make numerical simulations to trace a fraction of the ensembles that populates one optical period and further assume that there is no significant difference from one period to another. Figure 2 shows the amplitude of the force on the νth macro-particle in one optical period. We can see that during one optical period some electrons are accelerated while others are decelerated.

The variation of total energy exchange versus population inversion is illustrated in Fig. 3; Fig. 3(a) shows the lower population inversion region while Fig. 3(b) illustrates the higher region. The total energy exchange is increased with the increasing of the population inversion.

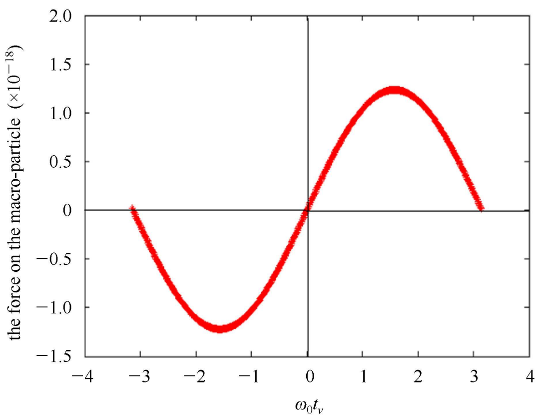

Fig. 2. (color online) Amplitude of the force on the νth macro-particle in one optical period.

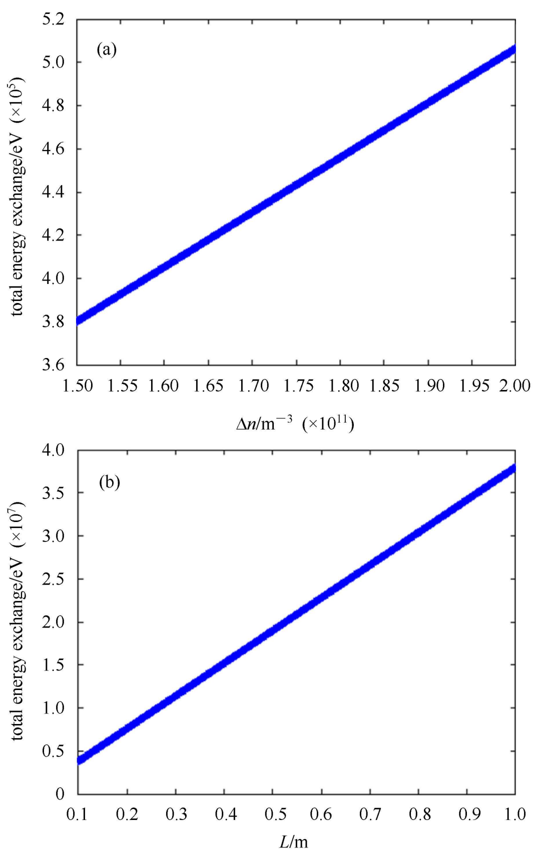

Fig. 3. (color online) Total energy exchange versus population inversion.

We considered the one round trip amplification factor $\exp(-T/\tau_\nu)$ with and without regular scattering in Fig. 4, which show that in the absence of normal scattering, a significant fraction of electrons can absorb energy from the medium; as the scattering effect increases, this fraction diminishes. In order to envision the impact of this effect, we show in Fig. 5 the relative changes in the total energy of the electrons with and without normal scattering after 500 round trips. We observe that in the absence of normal scattering, a significant fraction of electrons can absorb energy from the active medium, primarily those around phase $\pi/2$.

When the population inversion is $\Delta n \approx 2\times 10^{11}$ m^{-3}, the relative energy changes after 400, 500, and 600 roundtrips with normal scattering are shown in Fig. 6. This plot illustrates that the longer the electrons are trapped, the more energy the accelerated electrons get from the medium, but at the same time, the energy of the decelerated electrons varies slowly.

Figure 7 indicates the changes in relative energy with different population inversions after 500 roundtrips, and shows that with the increase of population inversion, the decelerated electrons are virtually unchanged but the accelerated electrons more than double their peak energy values.

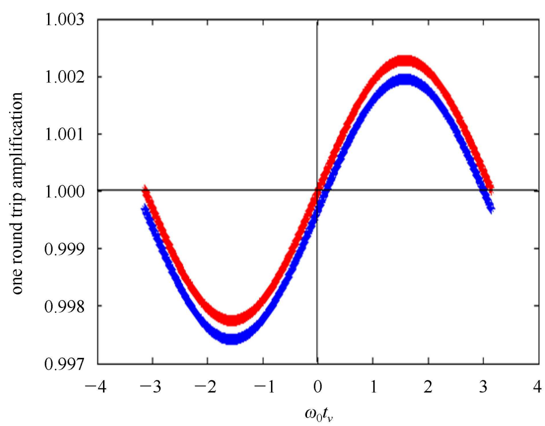

Fig. 4. (color online) Amplification factor $\exp(-T/\tau_\nu)$ with (blue line) and without (red line) scattering included.

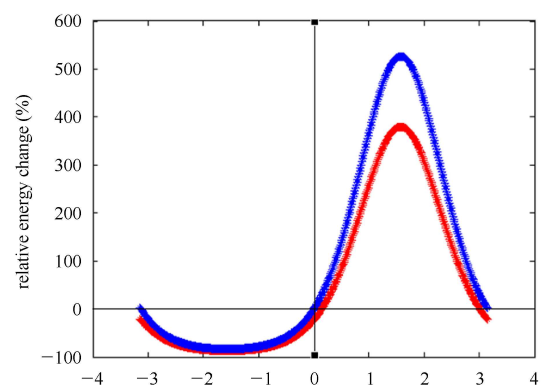

Fig. 5. (color online) Relative changes in the total energy of the electrons with (red line) and without (blue line) normal scattering after 500 round trips.

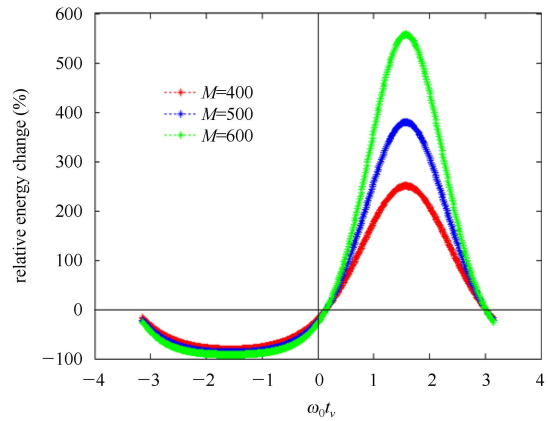

Fig. 6. (color online) Relative energy changes for 400, 500, and 600 roundtrips with normal scattering. The population inversion is $\Delta n \approx 2\times10^{11}$ m^{-3}.

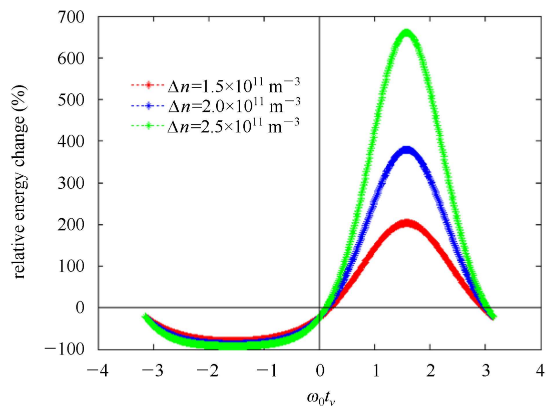

Fig. 7. (color online) Relative energy changes for 500 roundtrips with different population inversions.

It is important to note that the above theory simulation results show that the gas active medium needs a lower population inversion to bunch the electrons compared to the solid active medium, so the experimental condition is easy to achieve.

When the oscillating electrons get bunched and gain enough energy from the active medium, they escape from the trap, forming an electron bunch train. The train of the bunched electrons can traverse into the next acceleration unit, which is filled with the same gas mixture active medium, to be accelerated to higher energies [9].

3 Discussion and conclusions

From the numerical simulations above, we can see that it is possible to use an excited gas mixture inside a Penning trap to bunch oscillating electrons. These can then be accelerated to higher energies in the next acceleration unit, which is filled with the same gas mixture active medium.

From the simulation results above we know that a significant fraction of electrons can absorb energy from the medium, but as the scattering effect increases, this fraction diminishes. The total energy exchange is linearly proportional to the interaction length and is influenced by the initial bunch kinetic energy when the bunch energy is low and independent of γ for relativistic energies.

Because PASER is truly a direct photon-to-electron acceleration process, it does not rely on an intermediary medium, such as plasma, to accelerate the electrons. This is one of the ways it is fundamentally different from plasma accelerators. The advantage of PASER is that it is a simpler acceleration technique and is potentially more compact. PASER does not require a terawatt laser or high-energy electron beam as the driver for generating the plasma wave. In particular, PASER does not need phase matching or compensation for phase slippage, and the total energy exchange is linearly proportional to the interaction length. We can therefore extend the interaction length by bending the electrons' trajectory and re-circulating them in the same cell in order to get higher energies.

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References

1 Schachter L. Physics Letters A, 1995, **205**: 355–358
2 Banna S et al. Laser and Photon Rev., 2009, **3**(1–2): 97–122
3 Banna S, Berezovsky V, Schachter L. Phys. Rev. E, 2006, **74**: 046501
4 Schachter L. Phys. Rev. Lett., 1999, **83**: 92–95
5 Voin M, Kimura W D, Schächter L. Nuclear Instruments and Methods in Physics Research, A, 2014, **740**: 117–123
6 Voin M, Schächter L. Phys. Rev. Lett., 2014, **112**: 054801
7 Schachter L. Phys. Rev. Lett., 2009, **102**: 034801
8 Schachter L. J. Appl. Phys., 2011, **109**: 054907
9 TIAN X F, WU Cong-Feng. Chinese Journal of Quantum Electronics, 2014, **31**(1): 33–39