# Covariance analysis of an isospin-dependent probe*

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**Abstract:** Based on a modified quantum molecular dynamics model, we calculate the neutron-proton ratio and the nuclear stopping of reaction systems with different symmetry potentials and collision cross sections. We perform correlations of several probes using the covariance data processing method. It is shown that the correlation between the nuclear stopping and the isospin-dependent nucleon-nucleon cross sections is strong, but the nuclear stopping and symmetry potentials have a weak correlation. The correlation between neutron-proton ratio and symmetry potentials in the case of low energy is stronger. The correlation between neutron-proton ratio and isospin-dependent collision cross sections is enhanced with the increase of energy, but remains weak. In addition, the correlations of the emission numbers of the deuteron with the symmetry potentials and collision cross sections at different beam energies are not obvious compared to two prior physical quantities. In this paper, we define a parameter to quantitatively describe the sensitivity of isospin-dependent probes. By analyzing this parameter, one can extract more information about the isospin effects of the physical quantity.

**Key words:** isospin-dependent probe, correlation, covariance

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## 1 Introduction

When studying the properties of a nucleus, we should distinguish between protons and neutrons, namely, the isospin effect in a nucleus should be taken into account. This is important for studying the nuclear stopping properties, nucleon-nucleon cross sections and the nuclear equation of state, etc [1–3]. The symmetry potential is another very important physical quantity, which is important for the study of nuclear physics and astrophysics [4, 5], such as supernova explosions, the giant coupling effect of finite nuclei, and neutron star cooling.

When calculating nuclear data, the model used should consider the isospin effect of the nucleus and the symmetry potentials. The quantum molecular dynamics (QMD) model is one such model [6–8]. The QMD model has strong advantages in the study of the formation of fragments, dynamical fluctuations of heavy ion collisions and nuclear multi-fragmentation, etc [9, 10]. The QMD model is based on the classical molecular dynamics (CMD) method and can predict the formation of fragments by considering the uncertainty principle and Pauli blocking. In addition, it solves the shortcomings of the Boltzmann–Uehling–Uhlenbeck (BUU) [11, 12] equation, which has large fluctuations in the construction of the density-dependent mean field. The QMD model can not only get a more relaxed mean field, but is also able to describe nuclear multi-fragmentation, which is very important to study nuclear stopping, the neutron-proton ratio, the symmetry potentials and the collision cross sections [13, 14]. Li Qing-Feng et al calculated the nuclear stopping, neutron-proton ratio, isospin-dependent symmetry potentials and the momentum-dependent two-body collision cross sections. In intermediate energy heavy ion collisions, based on the isospin-dependent QMD model, they obtained qualitative relationships for some physical quantities, which are in good agreement with the experimental values [15–17]. Zhang Yin-Xun et al studied the liquid-gas phase transformation mechanism near the nucleus critical point within the framework of the QMD model. They also calculated the mass distribution of nuclear fragments, the density fluctuation and the maximum Lyapunov exponent for $^{124}$Sn and $^{208}$Pb nuclei. They found that the largest Lyapunov exponent and density fluctuation at the same temperature are maximized in heavy nuclear systems, and in this state the mass distribution of fragments is found to be in the largest liquid-gas coexistence region [18, 19].

After obtaining the desired data by a suitable physical model, a clearer and more accurate result can be

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obtained through appropriate data processing methods. Here, what we want to introduce is the covariance data processing method. Fattoyev et al dealt with theory prediction uncertainty in a relativistic mean field model: a large amount of information can be obtained through the strength of correlation according to the fluctuation near the lowest point of the $\chi^2$, and this information is difficult to extract with other methods [20]. Chen Zhen-Peng et al studied the features of covariance propagation in an $R$-matrix model by fitting the $^7$Li, $^{11}$B and $^{17}$O systems. They revealed partial regularity of covariance propagation, and found that the medium-energy-range component (MERC) of systematic error plays a very important role in propagation of covariance [21].

In intermediate energy heavy ion collisions, there are two very important isospin effects, i.e., isospin-dependent cross sections and symmetry potentials. We need to find a physical quantity, which is sensitive to one, but insensitive to the other. In this paper, we introduce several different reaction cross sections and symmetry potentials by modifying some parameters in the QMD model. We get the data of the neutron-proton ratio and nuclear stopping with different symmetry potentials and reaction cross sections. We then perform quantitative analysis of the correlations of several nuclear probes by using the covariance data processing method.

## 2 Model

The QMD model [22, 23] used for studying dynamics processes of intermediate energy heavy ion collisions consists of three main factors: the density-dependent mean field, the in-medium nucleon-nucleon cross sections and Pauli blocking. The density-dependent mean field contains both isospin-dependent symmetry potentials and Coulomb potential. Here we obtain different symmetry potentials by modifying some parameters. The interaction potential and parameters are [24]:

$$U(\rho)=U^{\rm Sky}+U^{\rm Coul}+U^{\rm Yuk}+U^{\rm Pauli}+U^{\rm MDI}+U^{\rm sym}, \quad (1)$$

where $U^{\rm sym}$ is a term for isospin-dependent symmetry potentials. There are many types of $U^{\rm sym}$, the most popular one being used in this paper. The empirical formula for the symmetry potentials $U^{\rm sym}$ has the following form

$$U^{\rm sym}=cu\delta\tau_z, \quad (2)$$

where $\tau_z = 1$ (neutron) and $\tau_z = -1$ (proton), $c$=32 MeV/u represents the symmetry potential strength, $u\equiv\rho/\rho_0$ represents the relative density of nuclear matter, $\delta=(\rho_{\rm n}-\rho_{\rm p})/(\rho_{\rm n}+\rho_{\rm p})$ denotes the relative neutron excess, and $\rho$, $\rho_0$, $\rho_{\rm n}$, $\rho_{\rm p}$ represent respectively nuclear density, saturation density, neutron density and proton density. In previous studies, the parameter $c$ was treated as a constant. Here $c$ is treated as a variable: we let it be 28, 30, 32, 34, 36 (MeV/u). Thus we can obtain different symmetry potentials.

In the isospin-dependent quantum molecular dynamics model (IQMD), we use the nucleon-nucleon cross sections in free-space of isospin-dependent parameterization extracted from experiment, and its specific form is as follows:

$$\sigma_{\rm np}^{\rm free} = \begin{cases} -\dfrac{5067.4}{E^2}+\dfrac{9069.2}{E}+6.9466\ ({\rm mb}), & E\leqslant 40\ ({\rm MeV}); \\ \dfrac{239380}{E^2}+\dfrac{1802.0}{E}+27.147\ ({\rm mb}), & 40<E\leqslant 400\ ({\rm MeV}); \\ 34.5\ ({\rm mb}), & 400<E\leqslant 800\ ({\rm MeV}). \end{cases} \quad (3)$$

$$\sigma_{\rm nn}^{\rm free}(\sigma_{\rm pp}^{\rm free}) = \begin{cases} -\dfrac{1174.8}{E^2}+\dfrac{3088.5}{E}+5.3107\ ({\rm mb}), & E\leqslant 40\ ({\rm MeV}); \\ \dfrac{93074}{E^2}-\dfrac{11.148}{E}+22.429\ ({\rm mb}), & 40<E\leqslant 310\ ({\rm MeV}); \\ \dfrac{887.37}{E}+0.05331E+3.5475\ ({\rm mb}), & 310<E\leqslant 800\ ({\rm MeV}). \end{cases} \quad (4)$$

Nuclear reactions generally occur in a medium, so one should use the in-medium nucleon-nucleon cross sections in the reaction dynamics. In addition, the reduced property of the in-medium nucleon-nucleon cross sections has been found by studying the collective flow in intermediate energy heavy ion collisions. Within the framework of the BUU model, Klakow et al found that the calculated balance energy can be in qualitative agreement with the results of the experiments when the in-medium nucleon-nucleon cross sections are equal to 80% of the free nucleon-nucleon cross sections [25]. We present an empirical formula for in-medium nucleon-nucleon cross sections:

$$\sigma_{\rm NN}^{\rm med}=\left(1-\gamma\frac{\rho}{\rho_0}\right)\sigma_{\rm NN}^{\rm free}(\gamma=0.2). \quad (5)$$

Similarly, here we let $\gamma$ respectively have values of

0.18, 0.19, 0.20, 0.21 and 0.22, thus giving different collision cross sections.

Using the IQMD model to simulate nuclear collisions, we calculated several physical quantities and discussed the correlation of these physical quantities by the covariance data analysis method. For the calculated physical quantities (nuclear stopping and neutron-proton ratio, etc.), we analyze these data and identify their covariance relationships, so gain correlations of several physical quantities. This is also very important for the relevant theoretical and experimental studies.

Calculating the correlation between two quantities, we use the covariance data analysis method. A concept of fundamental importance to correlation analysis is the covariance between two observables $A$ and $B$, denoted by cov $(A, B)$ [26]. By assuming that $(x(1), \cdots, x(M))$ represent $M$ points (or models) in the neighborhood of the optimal model $x(0)$=0, the covariance between $A$ and $B$ is defined as

$$\begin{aligned}\mathrm{cov}(A,B) &= \frac{1}{M}\sum_{m=1}^{M}[(A^{(m)}-\langle A\rangle)(B^{(m)}-\langle B\rangle)] \\ &= \langle AB\rangle-\langle A\rangle\langle B\rangle,\end{aligned} \tag{6}$$

where $A^{(m)} \equiv A(x^{(m)})$. From the above definition the correlation coefficient—often called the Pearson product-moment correlation coefficient—now follows

$$\rho(A,B)=\frac{\mathrm{cov}(A,B)}{\sqrt{\mathrm{var}(A)\mathrm{var}(B)}}, \tag{7}$$

where the variance of $A$ is simply given by $\mathrm{var}(A) = \mathrm{cov}(A,A)$. Note that two observables are considered to be fully correlated if $\rho(A,B)=1$, fully anti correlated if $\rho(A,B)=-1$ and uncorrelated if $\rho(A,B)=0$.

## 3 Results and discussion

Within the framework of an IQMD model, the neutron-proton ratio and nuclear stopping in reactions of $^{112}$Sn+$^{112}$Sn, $^{124}$Sn+$^{124}$Sn and $^{132}$Sn+$^{132}$Sn are investigated.

Figure 1 shows that the neutron-proton ratio is sensitive to the change of symmetry potentials, with strong correlation between them, but the correlation between the collision cross sections and the neutron-proton ratio is relatively weak. The energy values used in this figure are 50, 100, 150 and 200 MeV/u, among which the plots corresponding to 50, 100 and 150 MeV/u are all roughly horizontal stripes. This phenomenon shows the n-p ratio depends strongly on the symmetry potentials but weakly on the N-N cross sections ($\gamma$ factor). For neutron-rich reaction systems, the symmetry potential of the neutron is positive (repulsive force), and the symmetry potential of the proton is negative (attractive force). If the symmetry potential strength coefficient (c) is larger, a neutron-rich system will produce a stronger repulsive force for neutrons and attractive force for protons, so more neutrons will be repelled. Thus the gas phase neutron-proton ratio becomes larger. So we can see in Fig. 1(a) that the larger the symmetry potential strength coefficient is, the larger the neutron-proton ratio of the fragments will be. This phenomenon is not obvious in Fig. 1(d), because the symmetry potentials can only play a dominant role at lower reaction energies ($E_{\mathrm{beam}}$=50, 100 MeV/u). When the energy is increased to 150 MeV/u, or higher, the effect of the symmetry potentials becomes weak. The in-medium cross sections are higher while the isospin effect is not obvious at the lower energies. The $\gamma$ parameter represents the relative variation of in-medium cross sections. The cross section is smaller with higher $\gamma$, and the probability of N-N collisions is also small. In the lighter fragments, the number of neutrons and protons decreases concurrently, but their ratio remains almost unchanged, so the neutron-proton ratio depends weakly on the isospin effect of the in-medium nucleon-nucleon cross sections.

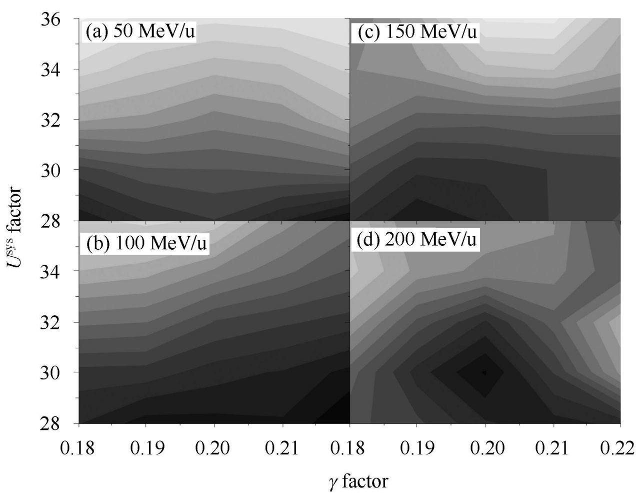


Fig. 1. The isospin effect of neutron-proton ratio in the $^{124}$Sn+$^{124}$Sn system.

Figure 2 shows that the nuclear stopping varies with the symmetry potentials and the $\gamma$ value. It can clearly be seen from the figure that the nuclear stopping changes obviously as $\gamma$ increases, but changes only slightly with the increase of the symmetry potentials ($U^{\mathrm{sym}}$). Thus in reactions of $^{124}$Sn+$^{124}$Sn the correlation between nuclear stopping and collision cross sections is stronger, and it is weaker between nuclear stopping and symmetry potentials. In heavy ion reactions, the initial direction of motion of the incident nucleus is defined as the $z$-axis. If the $z$-axis signifies the horizontal direction, then the $x$-$y$ plane perpendicular to the $z$-axis will represent the vertical direction. Nuclear stopping is the ratio of the momentum component of the reaction products in the vertical direction and the horizontal direction. Before reaction, the nucleon momentum is almost purely in the horizontal direction. After collision, the nuclear motion can be in any direction, and the vertical component of the momentum is increased, while the nuclear reaction cross sections has a greater impact on the collision, so the nuclear stopping is very sensitive to collision cross sections. Smaller $\gamma$ values correspond to bigger cross sections, as can be seen clearly in Fig. 3, especially at higher reaction energies ($E \geqslant 100$ MeV/u). This is because the collision cross sections play a dominant role under such conditions, whereas the role of the symmetry potentials is to exclude neutrons, to make neutrons move faster (protons being slower or bound), so the symmetry potentials almost have no effect on the nuclear stopping.

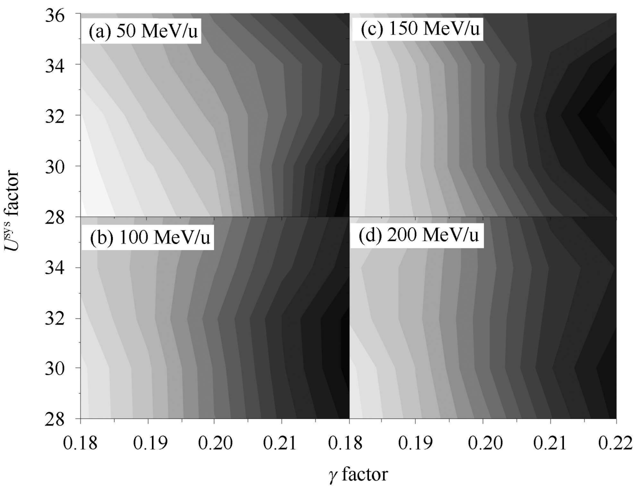


Fig. 2. The isospin effect of nuclear stopping in the $^{124}$Sn+$^{124}$Sn system.

From Figs. 1 and 2, we show that there are strong correlations between the neutron-proton ratio and symmetry potentials, and between the nuclear stopping and collision cross sections. This conclusion is the same as previous studies. In addition, we also calculated correlations of the emission numbers of the deuteron with the symmetry potentials and collision cross sections at different beam energies in the $^{124}$Sn+$^{124}$Sn system; the results show that the correlation rule is not obvious compared to the two prior physical quantities.

Using the covariance formula, we calculate the correlation between the nuclear stopping at different energies and symmetry potentials, and also the correlation between the nuclear stopping and isospin-dependent cross sections. Also, the correlation of the n-p ratio can also be known. As we can see in Fig. 3, in the $^{112}$Sn, $^{124}$Sn and $^{132}$Sn systems, no matter how high the energy is, the nuclear stopping always depends sensitively on the isospin-dependent cross sections (solid lines) and weakly on the symmetry potentials (dotted lines). The solid line is always higher because the cross section decides the probability of the nucleon-nucleon collision and with the change of the isospin-dependent cross sections, as the probability of N-N collision changes, the proportion of the collided particles to total particles becomes larger, so the nuclear stopping is larger and the correlation is stronger. The dotted lines are lower, which shows that the nuclear stopping depends weakly on the symmetry potentials. In the process of reactions, the density of neutrons and protons is different, and the density of the neutron is always greater than that of the proton; neutrons will be affected by the isotropic rejection and protons will be affected by the isotropic attraction. The ratio of their momenta in all directions is almost unaffected; even if it is affected weakly, it will be offset by a rejection and an attraction. The nuclear stopping does not take into account the neutrons and protons, but just considers components of the momentum, while the symmetry potentials considers the types of nucleon, so the correlation between them is very weak.

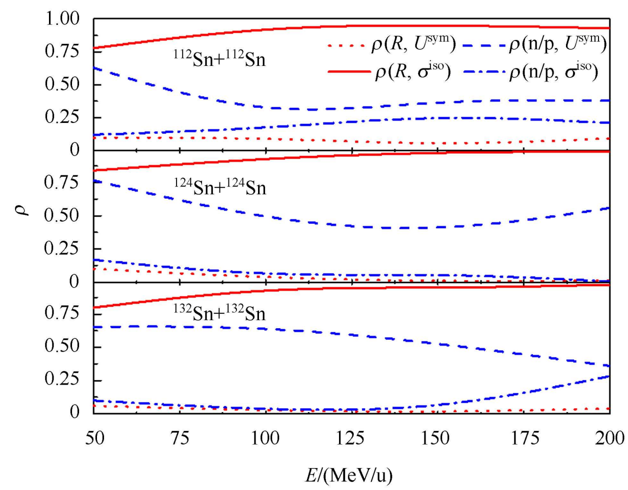


Fig. 3. (color online) The isospin-dependent factor evolution with beam energy.

The correlation between the n-p ratio and the symmetry potentials (dashed lines) is stronger in the case of low-energy, and weak at high energies. The correlation between the n-p ratio and isospin-dependent cross sections (dashed-dotted lines) increases with the increase of energy, but the correlation is consistently weak. When the incident energy is low, the symmetry potential is obvious, so the correlation of the dashed line on the left is relatively stronger. When the incident energy is increased to 100–200 MeV/u, N-N collisions play a leading role, and the probability of collision is determined by the cross sections, while the symmetry potentials intensity factor 32 MeV/u is very small with respect to the incident energy. The dashed-dotted lines, at the bottom of the plots, shows that the correlation between the n-p ratio and isospin-dependent cross sections, is weak, and the reason is that considering various cross sections will affect the probability of collision; after a few more collisions there will be more particles to collide, and the number of both protons and neutrons involved in collisions will change, but their ratio changes little. Therefore the correlation between the cross sections and the n-p ratio is relatively weak. With the increase of energy, cross sections play a leading role at energies from 100 MeV/u to 400 MeV/u, and the symmetry potentials play a leading role at energies below 100 MeV/u, so the right-hand side of the solid lines are a little higher than the left.

In previous studies, we qualitatively discussed the isospin-dependent probe. However, its degree of sensitivity was not considered. In this article, we use the absolute value of the difference between the two correlations to represent degrees of sensitivity of these probes. It is defined as the sensitivity parameter of isospin-dependent probes, and its form is:

$$\Delta\rho(F)=||\rho(\sigma,F)|-|\rho(U^{\rm sym},F)||. \tag{8}$$

In intermediate-energy heavy ion collisions, there are two important isospin effects, and the two physical quantities are very uncertain, such as the incompressible coefficient in the symmetry potential changing from $-400$ MeV to 400 MeV. When $\Delta\rho$ tends to 1, the probe is sensitive to one of the isospin effects and quite insensitive to the other; when $\Delta\rho$ tends to 0, the difference of the correlation between the observable and the two physical quantities is small. The two isospin effects may simultaneously affect the observable with almost the same intensity, or both do not have an impact on the observable, so the observable is not a good probe. One can choose an observable with higher $\Delta\rho$ in experiment, and extract information from one of the two isospin effects.

In Fig. 4, the horizontal axis shows the beam energy, and the vertical axis is the absolute value of the difference between the correlations of a probe with the cross sections and that of symmetry potential, namely the sensitivity parameter of isospin-dependent probes. The sensitivity parameter ranges from 0 to 1, and its value is closer to 1, indicating that it is sensitive to one of the isospin effects and insensitive to the other. As can be seen from this figure, the solid line is high, which means that the nuclear stopping is always a sensitive physical quantity for probing the isospin effect at energies from 50 MeV/u to 200 MeV/u. The n-p ratio is a good probe only at lower energy, but the emission numbers of the deuteron is not a good probe within the energy we have studied. The sensitivity of isospin-dependent probes is clearly shown in Fig. 4, and through analysis of the sensitivity parameter of isospin-dependent probes, we conclude that nuclear stopping is a better isospin effect probe than the n-p ratio and the emission numbers of the deuteron.

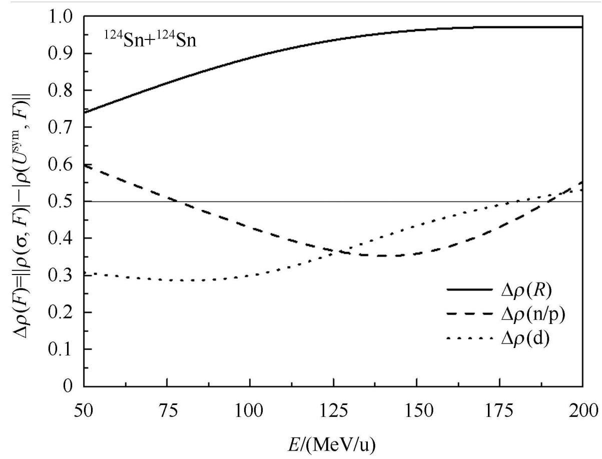


Fig. 4. The sensitivity parameter of isospin-dependent probe evolution with beam energy.

## 4 Conclusion

Within the framework of improved quantum molecular dynamics model, we studied the correlations of the n-p ratio, nuclear stopping and emission numbers of the deuteron with the symmetry potentials and collision cross sections at different beam energies. The results showed that with the increase of intensity coefficient of the symmetry potentials, the n-p ratio of the free nucleons produced in collision reactions becomes larger, and the n-p ratio is insensitive to the collision cross sections, the $\Delta\rho$ of the n-p ratio is large at lower energy. The nuclear stopping is more sensitive to the change of $\gamma$, and the $\Delta\rho$ of the nuclear stopping is always large, while it is small for the emission numbers of the deuteron at the energies we studied. According to the above results, considering the parameter ($\Delta\rho$) defined in this paper, we conclude that this parameter can quantitatively describe the isospin effect.

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