Optimization of single-step tapering amplitude and energy detuning for high-gain FELs*

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Abstract: We put forward a method to optimize the single-step tapering amplitude of undulator strength and initial energy tuning of electron beam to maximize the saturation power of high gain free-electron lasers (FELs), based on the physics of longitudinal electron beam phase space. Using the FEL simulation code GENESIS, we numerically demonstrate the accuracy of the estimations for parameters corresponding to the linac coherent light source and the Tesla test facility.

Key words: step-tapering, energy detuning, high-gain FELs

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1 Introduction

High-gain free-electron lasers (FELs), such as self-amplified spontaneous emission (SASE) and seeded harmonic generation (e.g. HGHG, EEHG), are capable of generating extremely high-brightness radiation in the ultraviolet and X-ray wavelengths. However, for a uniform-parameter undulator, the FEL efficiency at saturation is roughly given by the FEL scaling parameter ρ [1], which is typically on the order of 10^{-3}. Therefore, variable parameter undulators are broadly used in FEL operation to enhance performance [2, 3].

We have investigated the existing tapering strategies. One uses the “standard” KMR formulation [4] in which a synchronous electron with ponderomotive phase equal to the synchronous phase $\psi_{\rm r}$ maintains its resonant energy $\gamma_{\rm r}mc^2$ throughout the tapered undulator. As shown by KMR, maximizing the product of the area of ponderomotive well occurs at $\psi_{\rm r}\approx0.45$. Thus, the bucket decelerates together with the trapped electrons, yielding more energy in the form of radiation. A self-designed taper algorithm based upon the KMR formalism has been implemented in the GINGER simulation code [5]. Another approach is presented in Ref. [6], which empirically optimizes $K(z)$ and maximizes the output power at a fixed total undulator length without necessarily trying to keep the trapped particle fraction large at the undulator exit.

These tapering strategies are very useful for long undulators and can increase the radiation power by several times, even by a few orders of magnitude. In practice, the tapering of the long undulator is implemented through multiple step-tapering. However, FEL facilities usually construct undulators with a length equal to the nominal saturation length, or a little longer. On this condition, the FEL power enhancement of single-step tapering can be considerable and not much worse than other tapering schemes.

In this paper, we put forward a new method to optimize the single-step tapering amplitude of undulator strength for high gain FELs based on the physics of longitudinal electron beam phase space. We then numerically investigate the energy detuning of the electron beam in the same way and develop an empirical formula to optimize the energy detuning based on simulations.

2 Optimization of single-step tapering

In a free electron laser, the electron beam and radiation wave interact continually under the resonant condition. For a uniform undulator with undulator period $\lambda_{\rm u}$ and undulator strength parameter K, the fundamental resonant wavelength is

$$\lambda_{\rm s}=\frac{\lambda_{\rm u}}{2\gamma^2}\left(1+\frac{K^2}{2}\right), \quad (1)$$

where γ is the Lorentz factor of the electron. In the exponential gain region, the optical wave extracts energy from electrons and grows exponentially, meanwhile the average electron energy decreases. The saturation commences when electrons become trapped in the pondero-

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motive wave and the number of trapped electrons losing energy to the wave is balanced by the electrons gaining energy from the wave. At saturation the so-called sychrotron oscillations are responsible for the development of the sidebands of the radiation spectrum [7].

2.1 Theoretical estimation

Here, we consider the FEL process in the sight of longitudinal electron beam phase space (ϕ'-ϕ, where ϕ is the electrons' longitudinal phase). In the exponential gain region, the electrons trapped in the phase space bucket move down from the upper to the bottom to give energy to the radiation wave while the height of the bucket increases with the growth of the radiation power. When saturation occurs, the height of the bucket almost becomes stable as the radiation power tends to balance. For a planar undulator, the bucket height of the longitudinal phase space can be written as

$$H=2\Omega=\frac{2}{\gamma}\sqrt{2k_{\rm s}k_{\rm u}a_{\rm s}a_{\rm u}JJ}. \tag{2}$$

Where $k_{\rm s}$, $k_{\rm u}$ are the wave numbers of the radiation and undulator field while $a_{\rm s}$, $a_{\rm u}$ are the ponderomotive potential of the radiation and undulator field, and $JJ=J_0(\xi)-J_1(\xi)$ with $\xi=a_{\rm u}^2/(2+2a_{\rm u}^2)$. Here, Ω also means the frequency of sychrotron oscillation of the electron phase.

At saturation, since the radiation power $P_{\rm sat.}\approx\rho P_{\rm e}$, where $P_{\rm e}$ is the power of the electron beam, we then have

$$a_{\rm s,sat.}=\frac{2(1+a_{\rm u}^2)}{a_{\rm u}JJ}\rho^2. \tag{3}$$

By inserting Eq. (3) into Eq. (2), the bucket height at saturation can be estimated as

$$H_{\rm sat.}=4\sqrt{2}k_{\rm u}\rho. \tag{4}$$

Through investigating the longitudinal phase space of the electron beam, we consider using a new undulator with a strength parameter of K–ΔK to shift the bucket down by the amplitude of $\Delta\phi'=H_{\rm sat.}$ from the position a little before saturation in order to make the radiation wave keep extracting energy from the electron beam, which is equivalent to locating most of the electrons in the upper of a new bucket, as shown in Fig. 1. Obviously the resonant electron energy decreases and its variation can be given as

$$\frac{\Delta\gamma_{\rm r}}{\gamma_{\rm r}}=\frac{\Delta\phi'}{2k_{\rm u}}=2\sqrt{2}\rho. \tag{5}$$

By applying Eq. (1), we obtain the variation of the undulator strength

$$\frac{\Delta K}{K}=\left(1+\frac{2}{K^2}\right)\frac{\Delta\gamma_{\rm r}}{\gamma_{\rm r}}=2\sqrt{2}\rho\left(1+\frac{2}{K^2}\right). \tag{6}$$

However, since the sychrotron oscillation at this time is very fast, the radiation will reach a new saturation soon.

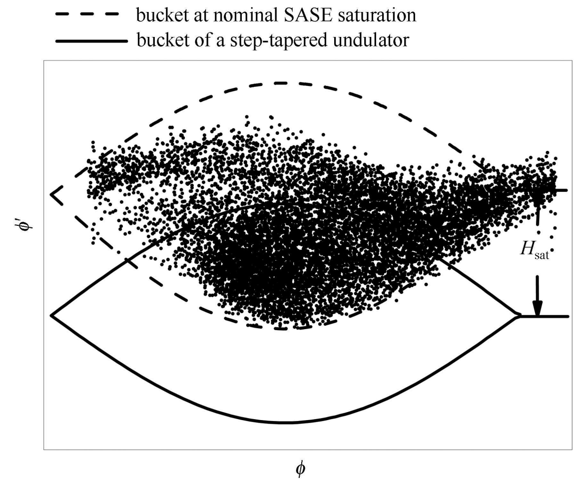

Fig. 1. Schematic view of shifting down the phase space bucket by the amplitude of $H_{\rm sat}$. with a step-tapered undulator.

2.2 Simulation examples

We use the FEL code GENESIS [8] to simulate SASE FELs with a single-step tapered undulator based on the linac coherent light source (LCLS) [9] and the Tesla test facility (TTF) [10] like parameters, as shown in Table 1. The start point of the step-tapered undulator is the last undulator gap before saturation. We have scanned the step-tapered undulator strength with single-frequency simulations. The results are shown in Fig. 2 and Fig. 3, corresponding to the LCLS and TTF like parameters, respectively, which clearly show that single-step tapering increases the radiation power and the radiation resaturates in a short undulator length.

Table 1. GENESIS simulation parameters.

parameter	LCLS	TTF
electron energy/GeV	4.3	1.0
slice energy spread	0.025%	0.02%
peak current/kA	2.0	2.5
normalized emittance/(mm·mrad)	1.2	2.0
undulator period/cm	3.0	2.73
undulator strength K	3.4995	1.2671
radiation wavelength/nm	1.5095	6.44578
pierce parameter	1.22×10^{-3}	1.85×10^{-3}

For LCLS like parameters, the saturation length for normal SASE is 34.08 m, which happens in the undulator gap and so the step-tapering starts from the next undulator segment. From Fig. 1, the radiation has the highest saturation power and the shortest saturation length while $\Delta K=0.375\%$. According to the theory above, the optimal ΔK is calculated to be 0.4%. Similarly, for

TTF like parameters, the optimized ΔK from simulation is 1.05% while it is 1.17% for theoretical estimation. Obviously, the theoretical estimation agrees with the simulation results very well. Furthermore, it is worth mentioning that the radiation power has a good tolerance on the step-tapering amplitude.

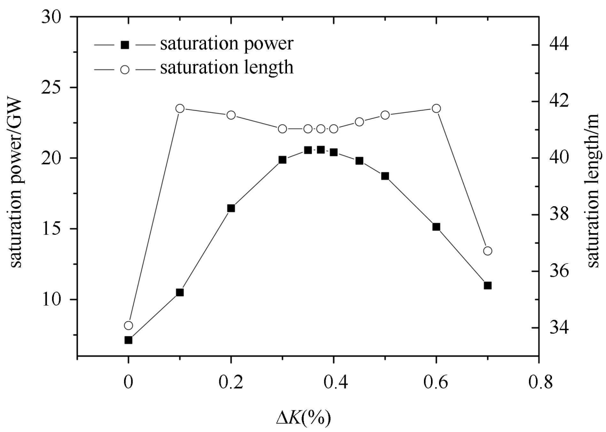

Fig. 2. The simulation results of saturation power and length with a single-step tapered undulator based on LCLS like parameters. The optimisation for theoretical estimation is 0.4%.

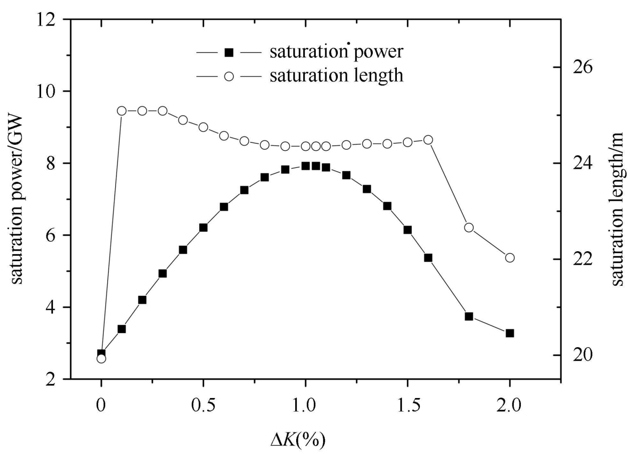

Fig. 3. The simulation results of saturation power and length with a single-step tapered undulator based on TTF like parameters. The optimisation for theoretical estimation is 1.17%.

In addition, we consider starting the single-step tapered undulator from an earlier point. Accordingly, the step-tapering amplitude should be scaled down with the bucket height. The simulation results suggest that the radiation does not have a stronger power than it did in the previous case.

3 Optimization of energy detuning

It is well known that using an electron beam with energy above resonance can enhance the radiation power [11]. As in Part 2, we investigate energy detuning with the longitudinal electron beam phase space. As we know, electrons move down in the bucket as they lose energy to the radiation wave, then the bucket height increases and more electrons are captured. According to the conservation of energy, for a single electron trapped in the bucket, the higher the electron energy is, the more the energy will be exchanged and the stronger the saturation power will be. However, for an electron beam with disorganized electron phases, there is a threshold value for the electron energy. When the electron energy exceeds this value and keeps increasing, more and more electrons will not be captured by the bucket. Therefore, the optimal energy detuning should be a balance between the energy loss of a single electron and the number of trapped electrons.

In SASE FELs, the radiation starts from a small signal (quadratic gain for seeded FELs) and then quickly grows exponentially as the bucket height varies. Therefore, it is difficult to analytically calculate the optimized energy detuning for the whole process. So we expect to develop empirically optimized energy detuning that maximizes the saturation power through numerical simulations.

We have done many simulations and the results imply that the optimized energy detuning has an approximate linear relationship with the bucket height at normal SASE saturation ($H_{\text{sat.}}$), as shown in Fig. 4. For LCLS like parameters, the electron energy variation corresponding to $H_{\text{sat.}}$ is $\Delta\gamma = H_{\text{sat.}}/(2k_{\text{u}}) = 0.345\%$ while the optimized energy detuning from simulation is $\delta\gamma_{\text{opt.}} \approx 0.156\% = 0.452\Delta\gamma$, where $\delta\gamma_{\text{opt.}} = (\gamma_{\text{opt.}} - \gamma_{\text{r}})/\gamma_{\text{r}}$. For the case of TTF like parameters, $\Delta\gamma$ is about 0.523% while $\delta\gamma_{\text{opt.}} \approx 0.235\% = 0.449\Delta\gamma$. Based on these, we have the empirical relationship

$$\delta\gamma_{\text{opt.}} \approx 0.45 H_{\text{sat.}}/(2k_{\text{u}}) = 0.9\sqrt{2}\rho. \quad (7)$$

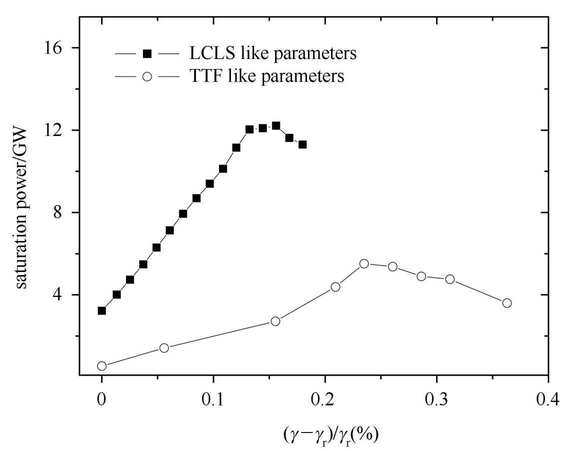

Fig. 4. The simulation results of saturation power varying with energy detuning based on LCLS and TTF like parameters.

We have checked this empirical formula for the parameters of high gain harmonic generation based on the Hefei soft X-ray proposal [12] ($\delta\gamma_{\rm opt.}$=0.437$\Delta\gamma$), the results also agree with Eq. (7) well.

4 Conclusions

In this paper we present a method to estimate the single-step tapering amplitude in the sight of longitudinal electron beam phase space. Through the simulations based on the LCLS and TTF like parameters, we have shown that this method can be an effective means to optimize the undulator parameters in high gain FELs. However, it is especially useful for the FEL facilities whose undulator is just slightly longer than the saturation length.

Furthermore, we have studied the energy detuning and found that the optimized energy detuning is proportionate to the bucket height at normal SASE saturation. An empirical formula has been developed through numerical simulations using the LCLS and TTF like parameters. It has then been checked with the other two FEL parameters settings and the results also agree with the formula well.

These conclusions are effective for high gain FELs, including SASE and seeded FELs.

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