# Impact parameter dependence of the scaling of anisotropic flows in intermediate energy heavy-ion collisions

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**Abstract:** The scaling behaviors of anisotropic flows of light charged particles are studied for 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions at different impact parameters by the isospin-dependent quantum molecular dynamics model. The nucleon-number scaling of elliptic flow exists and the scaling of the ratios of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  is applicable to collisions at almost all impact parameters except for peripheral collisions.

Key words: impact parameter, anisotropic flow, scaling behavior

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#### 1 Introduction

Anisotropic flows are interesting subjects in theoretical and experimental investigations on nuclear reaction dynamics in both intermediate and high energy heavy-ion collisions. Many studies of the first and second anisotropic flows (directed flow and elliptic flow, respectively) dependence on beam energies, mass number or quark number, isospin and impact parameter have been done and revealed much interesting physics about the properties and origin of the collective flow [1– 10]. <sup>197</sup>Au+<sup>197</sup>Au collision experiments at RHIC energy demonstrated the number of constituent-quark (NCQ) scaling for the transverse momentum dependent elliptic flow for different mesons and baryons [11, 12], and a popular interpretation assumes that the mesons and baryons are formed by the coalescence or recombination of the constituent quarks. Our work [13] found the similar elliptic scaling for light particles in intermediate energy heavy ion collisions and it may also be the outcome of coalescence mechanism but at nucleonic level. As we know, the flow value strongly depends on the impact parameter, so the elliptic scaling is tested for light particles at different impact parameters of <sup>40</sup>Ca+<sup>40</sup>Ca collisions in intermediate energy in this paper, and  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  scalings predicted in RHIC energy are also investigated.

Anisotropic flows are defined as different nth harmonic coefficients  $v_n$  of the Fourier expansion for the particle invariant azimuthal distribution,

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi),\tag{1}$$

where  $\phi$  is the azimuthal angle between the transverse momentum of the particle and the reaction plane. The anisotropic flows  $v_n$  can further be expressed in terms of single-particle averages,

$$v_1 = \langle \cos \phi \rangle = \left\langle \frac{p_x}{p_t} \right\rangle,$$
 (2)

$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle, \tag{3}$$

$$v_3 = \langle \cos(3\phi) \rangle = \left\langle \frac{p_x^3 - 3p_x p_y^2}{p_t^3} \right\rangle, \tag{4}$$

$$v_4 = \langle \cos(4\phi) \rangle = \left\langle \frac{p_x^4 - 6p_x^2 p_y^2 + p_y^4}{p_t^4} \right\rangle,$$
 (5)

where  $p_x$  and  $p_y$  are, respectively, the projections of particle transverse momentum parallel and perpendicular to the reaction plane, and  $p_t$  is the transverse momentum  $(p_t = \sqrt{p_x^2 + p_y^2})$ .

### 2 Theoretical framework

Intermediate energy heavy-ion collision dynamics are complex since both mean field and nucleon-nucleon collisions play competing roles. Furthermore, the isospin-dependent role should be also incorporated for asymmetric reaction systems. Isospin-dependent quantum molecular dynamics model (IQMD) has been affiliated with isospin degrees of freedom with mean field and nucleon-nucleon collisions [14–20]. The IQMD model can explicitly represent the many-body state of the system and principally contains correlation effects to all orders and

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all fluctuations, and can well describe the time evolution of the colliding system. When the spatial distance  $\Delta r$  is smaller than 3.5 fm and the momentum difference  $\Delta p$  between two nucleons is smaller than 300 MeV/c, two nucleons can be coalesced into a cluster [14]. With this simple coalescence mechanism, which has been extensively applied in transport theory, different sized clusters can be recognized.

In this model the nuclear mean-field potential is parameterized as

$$U(\rho, \tau_z) = \alpha \left(\frac{\rho}{\rho_0}\right) + \beta \left(\frac{\rho}{\rho_0}\right)^{\gamma} + \frac{1}{2}(1 - \tau_z)V_{\rm C}$$
$$+ C_{\rm sym} \frac{(\rho_{\rm n} - \rho_{\rm p})}{\rho_0} \tau_z + U^{\rm Yuk}, \tag{6}$$

where  $\rho_0$  is the normal nuclear matter density (0.16 fm<sup>-3</sup>);  $\rho_n$ ,  $\rho_p$  and  $\rho$  are the neutron, proton and

total densities, respectively; and  $\tau_z$  is the zth component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively. The coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  are the parameters for the nuclear equation of state (EOS).  $C_{\rm sym}$  is the symmetry energy strength due to the density difference of neutrons and protons in the nuclear medium, which is important for asymmetric nuclear matter ( $C_{\rm sym} = 32$  MeV is used).  $V_{\rm C}$  is the Coulomb potential and  $U^{\rm Yuk}$  is the Yukawa (surface) potential. In the present work, we take  $\alpha = 124$  MeV,  $\beta = 70.5$  MeV and  $\gamma = 2$ , which corresponds to the so-called hard EOS with an incompressibility of K = 380 MeV.

## 3 Results and discussions

Now we move to the calculations. About 200000  $^{40}\mathrm{Ca}+^{40}\mathrm{Ca}$  collisions have been simulated with hard EOS

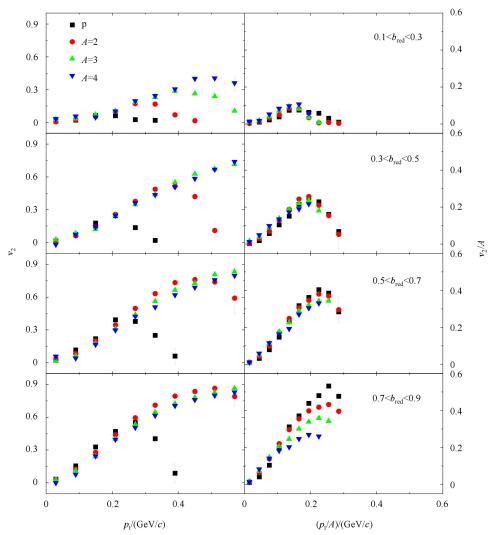


Fig. 1. (color online) The transverse momentum dependence of elliptic flow and the transverse momentum per nucleon dependence of elliptic flow per nucleon for light charged particles from 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions with different reduced impact parameters.

at 25 MeV/u. In this study, we extract the physical results at 200 fm/c for light charged particles when the system has been frozen out.

Figure 1 shows the transverse momentum dependence of elliptic flows (left column) and the transverse momentum per nucleon dependence of elliptic flow per nucleon (right column) for light charged particles from 25 MeV/u  $^{40}\mathrm{Ca}+^{40}\mathrm{Ca}$  collisions with different reduced impact parameters. The reduced impact parameter is defined as  $b_{\mathrm{red}}\!=\!b/b_{\mathrm{max}}$  and  $b_{\mathrm{max}}\!=\!R_{\mathrm{p}}\!+\!R_{\mathrm{t}},$  where  $R_{\mathrm{p}}$  and  $R_{\mathrm{t}}$  are the radii of projectile and target respectively. The four rows are for four different reduced impact parameter bins of  $0.1 < b_{\mathrm{red}} < 0.3,\ 0.3 < b_{\mathrm{red}} < 0.5,\ 0.5 < b_{\mathrm{red}} < 0.7$  and  $0.7\!<\!b_{\mathrm{red}}\!<\!0.9$ , respectively. The squares represent protons, the circles represent fragments of  $A\!=\!2$ , the up-

triangles represent A=3, and the down-triangles represent A=4. From the figures of the transverse momentum dependence of elliptic flows (left panel), the elliptic flow is positive and increases with increasing  $p_t$  up to a certain  $p_t$ , after which it begins to decrease with increasing  $p_t$ , the heavier fragment having a greater  $p_t$  at the inflection point. In the figures of the transverse momentum per nucleon dependence of elliptic flow per nucleon (right panel), however, the curves for different particles overlap with each other. This behavior is apparently similar to the number of constituent quarks scaling of elliptic flow versus transverse momentum per constituent quark  $(p_t/n)$  for mesons and baryons, which was observed at RHIC [9]. We call it the nucleon-number scaling of elliptic flow [13], which reflects that the formation of the

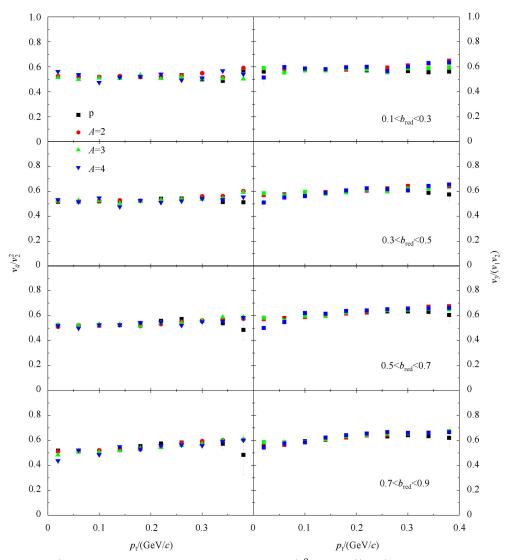


Fig. 2. (color online) Transverse momentum dependence of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  for light charged particles from 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions with different reduced impact parameters. The symbols are the same as that of Fig. 1.

fragments during the reaction obeys the coalescence mechanism. Fig. 1 also shows that the elliptic flow increases with impact parameter, which indicates that the fragments prefer more to be emitted in the reaction plane with greater eccentricity at larger impact parameter. But the nucleon-number scaling is broken for fragments with great  $p_{\rm t}/A$  at large impact parameters of  $0.7 < b_{\rm red} < 0.9$ , which may indicate that the collective effects on lighter fragments are much stronger than on heavier fragments, i.e., lighter fragments are emitted at a higher thermal pressure in the overlap zone.

The RHIC experimental data also demonstrated a scaling relationship between  $v_4$  and  $(v_2^2)$  [21]. It has been shown that such a scaling relation follows from a naive quark coalescence model [12, 22, 23] that only allows quarks with equal momentum to form a hadron. Denot-

ing the meson anisotropic flows by  $v_{n,\mathrm{M}}(p_{\mathrm{t}})$  and baryon anisotropic flows by  $v_{n,\mathrm{B}}(p_{\mathrm{t}})$ , Kolb et al. [22] found that if quarks have no higher-order anisotropic flows than the fourth term, one can show that  $\frac{v_{4,\mathrm{M}}}{v_{2,\mathrm{M}}^2} \approx \frac{1}{4} + \frac{1}{2} \frac{v_{4,\mathrm{q}}}{v_{2,\mathrm{q}}^2}$  and

 $\frac{v_{4,\mathrm{B}}}{v_{2,\mathrm{B}}^2} \approx \frac{1}{3} + \frac{1}{3} \frac{v_{4,\mathrm{q}}}{v_{2,\mathrm{q}}^2}, \text{ where } v_{n,\mathrm{q}} \text{ denotes the quark anisotropic}$ 

flows. The meson and baryon anisotropic flows thus satisfy the scaling relations if the quark anisotropic flows also satisfy such relations, and this ratio is experimentally determined to be 1.2 [24]. In view of the above behaviors of the flows at RHIC energies, we display the anisotropic flows at intermediate energy. The left panel of Fig. 2 shows the transverse momentum dependence of  $v_4/v_2^2$  for light particles from 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions with different reduced impact parameters. It

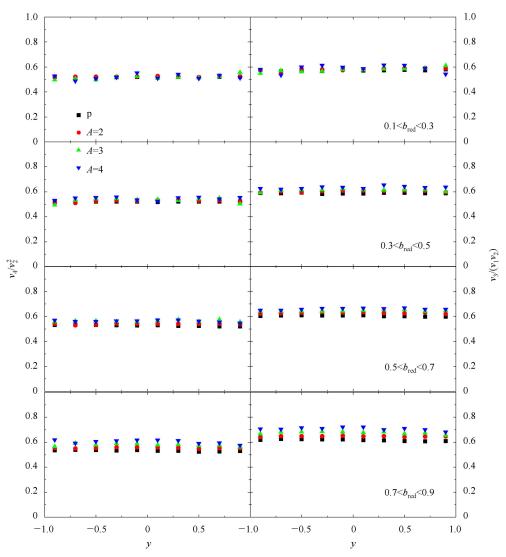


Fig. 3. (color online) Rapidity dependence of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  for light charged particles from 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions with different reduced impact parameters. The symbols are the same as that of Fig. 1.

shows that the ratios of  $v_4/v_2^2$  for different fragments are a nearly constant value of 0.5 at all the impact parameter bins. If we assume the scaling laws of mesons and baryons are also valid for A=2 and 3 nuclear clusters, respectively, then  $v_4/v_2^2$  for A=2 and 3 clusters will indeed give the same value of 1/2 as nucleons(protons). Coincidentally the predicted value of the ratio of  $v_4/v_2^2$ for hadrons is also 1/2 if the matter produced in ultrarelativistic heavy ion collisions reaches thermal equilibrium and its subsequent evolution follows the laws of ideal fluid dynamics [25]. It is interesting to note the same ratio was predicted in two different models at very different energies, which is of course worth further investigation in the near future. In addition, Kolb et al. suggested another scaling relationship between  $v_3$  and  $v_1v_2$  as also insinuated by the coalescence model. The right panel of Fig. 2 displays the  $p_t$  dependence of the ratio  $v_3/(v_1v_2)$ . It shows that the ratios of  $v_3/(v_1v_2)$  for different LCPs are also scaled and have a nearly constant value 0.6. This may be another parameter that can reflect the thermalization of the matter in the overlap zone. It is also worthy of further study theoretically and experimentally at both RHIC energy and intermediate energy.

Figure 3 shows the rapidity dependence of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  for light charged particles from 25 MeV/u  $^{40}$ Ca+ $^{40}$ Ca collisions with different reduced impact parameters, which are integrated for fragments with all possible  $p_{\rm t}$ . So the ratios of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  are almost the same as the expected constant of 0.5 and 0.6 at differ-

ent rapidities, respectively. For the peripheral collisions of  $0.7 < b_{\rm red} < 0.9$ , however, the heavier fragments have a slightly greater ratio value than the lighter ones. That may because of the flow difference of the light particles at high  $p_{\rm t}/A$ , as shown in Fig. 1. It may reflect that the overlap zone is approaching thermal equilibrium at the moment when the lighter fragments are emitted for the peripheral collisions.

## 4 Summary

We have investigated the scaling behaviors of elliptic flows and the ratios of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$  for light charged particles at different collision parameters for the simulations of <sup>40</sup>Ca+<sup>40</sup>Ca at 25 MeV/u with the IQMD model. It is shown that nucleon-number scaling of elliptic flow exists at broad reduced impact parameter, except for some deviations at the peripheral collisions  $(0.7 < b_{\rm red} < 0.9)$ . The ratios of  $v_4/v_2^2$  and  $v_3/(v_1v_2)$ , which may reflect the degree of thermalization of the matter produced in these heavy ion collisions, are almost constant at about 0.5 and 0.6 respectively, for all light fragments at different transverse momenta, rapidities and impact parameters except for some deviations in the peripheral collisions. It may reflect that during peripheral collisions different fragments may be emitted at different thermalization extent of the overlap zone because of the large eccentricity. Experimental investigations at intermediate energy are expected to be made.

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