# Simulation of effects of incident beam condition in p－p elastic scattering＊ 

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#### Abstract

The simulation is performed for the monitors of beam direction and beam position for p－p elastic scattering．We set several variables to simulate the monitors of incident beam condition changes：beam positions at the quadrupole magnet and target in beam line polarimeter（BLP2），distance between quadrupole magnet and target， size of plastic scintillators，distance between the target in BLP2 and the centers of plastic scintillators，and beam polarization．Through the rotation of the coordinate system，the distributions of scattered and recoiled protons in the laboratory system were obtained．By analyzing the count yields in plastic scintillators at different beam positions， we found that the beam incident angular change（ $0.35^{\circ}$ ）could be detected when the asymmetry of geometries of left and right scintillators in BLP2 was changed by $6 \%$ ．Therefore，the scattering angle measured in the experiment can be tracked by these monitors．


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## 1 Introduction

The high energy proton elastic scattering experiment plays an important role in exploring the inner structure of a nucleus．The angular distribution of elastic scatter－ ing is important for scattering experiments．It is well known that the differential cross section changes very rapidly with scattering angle in a high energy proton elastic scattering．The yield might change by more than $20 \%$ if the incident beam direction is changed by $0.1^{\circ}$ ． During experiment the beam direction at the target may change a little due to the instability of the cyclotron and beam transport condition，which could cause an unreli－ able measurement of scattering angle．Hence in order to monitor and correct the measured scattering angle，it is essential to monitor the beam incident condition on the target during the experiment．

This research was performed at the Research Center for Nuclear Physics（RCNP），Osaka University，Japan． Considering the present situation of the beam line in RCNP，there is no special facility for a beam condition monitor，except that two sets of beam line polarimeters （BLPs）are placed between the ring cyclotron and scat－
tering chamber for measurement of beam polarization and intensity［1］．Simulation is done by utilizing left－ right asymmetries of BLP2 depending on beam position and direction change．The Monte Carlo method is al－ lowed to simulate the behaviors of incident and scattered protons by kinematical coincidence．By using the simu－ lation results，we try to correct the measured angles in the E366 experiment．

## 2 Monitor of incident beam condition

## 2．1 Experimental setup

The overview of the WS beam line of RCNP is de－ scribed in Ref．［2］．The beam line polarimeters BLP1 and BLP2 are positioned at double－focus locations of the beam line，which are illustrated in Fig． 1 ［1，3］．It con－ sists of four（left，right，up and down）pairs of plastic scintillation counters to measure p－p scattering from the target．The targets in BLP1 and BLP2 are polyethylene foil and aramid foil，respectively．

When the proton beam is scattered from the target， the scattered and recoiled protons are simultaneously de－

[^0]tected by a pair of scintillation counters. Each pair of counters (forward and backward) is located at the scattering and recoil angles of $17.0^{\circ}$ and $70.5^{\circ}$ resulting in the large effective analyzing power $A_{y}=0.40$. Forward detectors in BLP2 have a smaller dimension than those in BLP1. The $y$-component of polarization is deduced by the asymmetrical counts of the left and right pairs of counters. The up and down pairs of counters are designed to obtain the beam intensity. During measurement the target is always inserted into the beam position. The polarization direction of the proton beam is reversed every second to reduce the systematic asymmetries [1].


Fig. 1. Setup of the BLPs on the WS beam line of RCNP (top view).

### 2.2 Principle of monitoring incident beam angular change

When a proton is scattered from a nucleus, the interaction between its spin and orbit angular momentum determines the probability of which direction the scattered proton will appear. The scattering direction of the incident proton with spin-up is opposite to that of the incident proton with spin-down. For the unpolarized beam, the particle counts of two sides along with the beam line should be the same due to the random directions of the beam spin. While the beam is polarized, the particle counts of two sides will be asymmetric, which can be used to measure the value of polarization.

In this beam line, as the incident condition of the polarized beam varies, the angular acceptances of four pairs of detectors in respectively BLP1 and BLP2 are different, which will also cause the asymmetry of the left and right counts along with the beam line. BLP1 will monitor the fluctuation of beam polarization and BLP2 can be employed as a monitor of the incident beam direction. When the angular change of the beam is observed
at BLP2, we could infer how the beam is changed at the target position during the beam transport process.

### 2.3 Simulation process

Our simulation is performed on the basis of the Monte Carlo method using Data Desk software. The simulation process for the monitor of beam incident condition is divided into three steps. Firstly, depending on the angular distribution of the differential cross section in p-p scattering, two million samples of scattering angle are generated, and then the behavior of every scattered proton can be simulated by kinematical coincidence; Secondly, several variables are set to indicate the beam incident condition, and distributions of scattered protons in the laboratory are obtained by the rotation of the coordinate system; Thirdly, the numbers of particles captured by the detectors are computed and the yields of detectors are simulated.

### 2.3.1 p-p elastic scattering process

We select beam energy $E_{\mathrm{p}}=295 \mathrm{MeV}$ and the vertical polarization $P_{y}=0.60$. In the simulation we consider the relativistic relations, the conservation principle of the total momentum and energy as well as the angles where the detectors in BLP2 are located. The data of the angular distribution of the cross section from $65^{\circ}$ to $75^{\circ}$ in p-p scattering at $E_{\mathrm{p}}=295 \mathrm{MeV}$ are listed in Table 1 [4].

Data of Table 1 give the fine agreement with the probability distribution of scattering angle in the Monte Carlo method. Therefore, the scattering cross section as a function of $\theta$ is generated, and random numbers of $\theta$ whose probability distribution satisfies $\sigma(\theta)$ are obtained.

Table 1. Angular distribution of cross section in p-p scattering at $E_{\mathrm{p}}=295 \mathrm{MeV}$ [4].

|  | $\mathrm{d} \sigma$ <br> $\theta_{\text {lab }} /\left(^{\circ}\right)$ | $\mathrm{d} \Omega$ <br> $(\mathrm{mb} / \mathrm{sr})$ | $\theta_{\mathrm{lab}} /\left(^{\circ}\right)$ | $\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega} /$ <br> $(\mathrm{mb} / \mathrm{sr})$ | $\theta_{\mathrm{lab}} /\left(^{\circ}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | | $\frac{\mathrm{d} \sigma}{\mathrm{d} \Omega} /$ |
| :---: |
| $(\mathrm{mb} / \mathrm{sr})$ |

From kinematical coincidence, the scattering angle of a scattered proton can be derived, and also the total energies, total momentums of scattered and recoiled protons can be obtained. In order to simulate the position vectors of every proton, an incident coordinate system whose original point is the center of the target in BLP2 is needed. $y$ and $z$ axes are the directions of polarization and incident beam. For the unpolarized proton beam, the differential cross section $\sigma$ is independent of $\phi$, and only dependent on scattering angle $\theta$. Now the value of polarization is selected as 0.60 , it will lead to the asymmetry of distributions of scattered protons around $z$-axis, i.e. the probability distribution of $\phi$ is not uniform. In fact, the cross section is calculated as follows to include the polarization,

$$
\begin{equation*}
\sigma=\sigma_{0}\left(1+P_{\text {beam }} A_{y}(\theta) \cos \phi\right) \tag{1}
\end{equation*}
$$

where $\sigma_{0}$ and $A_{y}(\theta)$ are the unpolarized cross section and analyzing power for p-p scattering, respectively. $P_{\text {beam }}$ represents the beam polarization. In our simulation, we assume the left and right detectors are respectively located at $\phi=180^{\circ}$ and $\phi=0^{\circ}$ (in the coordinate system in Fig. 1) due to the small solid angle of detectors. In addition, $A_{y}(\theta)$ can be considered as a constant $A_{y}=0.40$. Therefore, the cross sections for the left and right detectors can be expressed as

$$
\begin{align*}
& \sigma_{\mathrm{L}}=\sigma_{0}\left(1-P_{\text {beam }} A_{y}(\theta)\right),  \tag{2}\\
& \sigma_{\mathrm{R}}=\sigma_{0}\left(1+P_{\text {beam }} A_{y}(\theta)\right)
\end{align*}
$$

Then we consider the $\phi$ distribution of beam polarization $\left(\phi \in\left[-45^{\circ}, 315^{\circ}\right]\right)$. So the position vectors of every proton in the incident coordinate system are available. The distributions of recoiled and scattered protons can be obtained in the incident coordinate system.

### 2.3.2 Rotation of the coordinate system

In the simulation we set several variables to indicate the beam incident position and direction at quadrupole magnet QM6D and the target in BLP2. The distributions of scattered protons are obtained in the incident system $\left(Z_{\text {in }}\right)$, so we need to rotate it to the laboratory system $\left(Z_{\text {lab }}\right)$ to give the distributions of scattered protons.

In Fig. 2, the coordinates are described as

$$
\begin{array}{r}
x_{\mathrm{q}}=x_{\mathrm{q} 0}+x_{\mathrm{qr}}=x_{\mathrm{q} 0}+w_{\mathrm{q}} \times n o r m 1, \\
y_{\mathrm{q}}=y_{\mathrm{q} 0}+y_{\mathrm{qr}}=y_{\mathrm{q} 0}+w_{\mathrm{q}} \times \operatorname{norm} 1,  \tag{3}\\
x_{\mathrm{t}}=x_{\mathrm{t} 0}+x_{\mathrm{tr}}=x_{\mathrm{t} 0}+w_{\mathrm{t}} \times \operatorname{norm} 2, \\
y_{\mathrm{t}}=y_{\mathrm{t} 0}+y_{\mathrm{tr}}=y_{\mathrm{t} 0}+w_{\mathrm{t}} \times \mathrm{norm} 2,
\end{array}
$$

where $\left(x_{\mathrm{q}}, y_{\mathrm{q}}\right)$ and $\left(x_{\mathrm{t}}, y_{\mathrm{t}}\right)$ are the beam positions at the exit of quadrupole magnet QM6D and the target in BLP2, respectively. $x_{\mathrm{q} 0}, y_{\mathrm{q} 0}, x_{\mathrm{t} 0}, y_{\mathrm{t} 0}$ are the mean values of positions, $w_{\mathrm{q}}, w_{\mathrm{t}}$ are the standard deviations of
positions, and Norm1, Norm2 are the random numbers of standard normal distributions.

So we can derive the beam incident vectors $\left(x_{\mathrm{t}}-x_{\mathrm{q}}\right.$, $y_{\mathrm{t}}-y_{\mathrm{q}}, D_{\mathrm{q}}$ ), and also its polar angle $\theta_{\mathrm{R}}$ and azimuthal angle $\phi_{\mathrm{R}}$. The rotation process is followed as: rotate $\phi_{\mathrm{R}}$ about $z$-axis, then $\theta_{\mathrm{R}}$ about $x^{\prime}$-axis and $-\phi_{\mathrm{R}}$ about $z^{\prime \prime}$ axis. After the first two rotations, protons formerly located at the positions of detectors are messed up. Thus, finally we relocate them to the original places.


Fig. 2. A schematic of rotation of the coordinate system.

### 2.3.3 Simulation for the yields of detectors

After rotation, the scattering vectors $\left(x_{\mathrm{s}}, y_{\mathrm{s}}, z_{\mathrm{s}}\right)$ of every proton in the laboratory system can be deduced. Whether a particle is detected or not, depends on the relation between its vector and the plane of the detector. By calculating them in the corresponding field, the yield of every detector is simulated. Coincidence of the numbers of each pair of counters can be made to extract the p-p scattering events. The dependence of coincident events of detectors on incident beam condition can be realized.

### 2.4 Calculation for the monitoring variable

During our scattering experiment, a monitoring variable needs to be selected. First of all, yields on the two pairs of scintillators $\left(N_{\mathrm{L}}, N_{\mathrm{R}}\right)$ with spin-up $(\uparrow)$ and spindown $(\downarrow)$ modes are given as follows[3].

$$
\begin{align*}
& N_{\mathrm{L}}^{\uparrow}=N_{\mathrm{L}}^{\uparrow \text { pro }}-N_{\mathrm{L}}^{\uparrow \text { acc }}=\sigma_{0} N_{\mathrm{t}} N_{\mathrm{b}}^{\uparrow} \epsilon_{\mathrm{L}} \Delta \Omega_{\mathrm{L}}\left(1+A_{y}^{\text {eff }} P_{y}^{\uparrow}\right), \\
& N_{\mathrm{R}}^{\uparrow}=N_{\mathrm{R}}^{\uparrow \text { pro }}-N_{\mathrm{R}}^{\uparrow \text { acc }}=\sigma_{0} N_{\mathrm{t}} N_{\mathrm{b}}^{\uparrow} \epsilon_{\mathrm{R}} \Delta \Omega_{\mathrm{R}}\left(1-A_{y}^{\text {eff }} P_{y}^{\uparrow}\right), \\
& N_{\mathrm{L}}^{\downarrow}=N_{\mathrm{L}}^{\downarrow \text { pro }}-N_{\mathrm{L}}^{\downarrow \mathrm{acc}}=\sigma_{0} N_{\mathrm{t}} N_{\mathrm{b}}^{\downarrow} \epsilon_{\mathrm{L}} \Delta \Omega_{\mathrm{L}}\left(1+A_{y}^{\text {eff }} P_{y}^{\downarrow}\right),  \tag{4}\\
& N_{\mathrm{R}}^{\downarrow}=N_{\mathrm{R}}^{\downarrow \text { pro }}-N_{\mathrm{R}}^{\downarrow \mathrm{acc}}=\sigma_{0} N_{\mathrm{t}} N_{\mathrm{b}}^{\downarrow} \epsilon_{\mathrm{R}} \Delta \Omega_{\mathrm{R}}\left(1-A_{y}^{\text {eff }} P_{y}^{\downarrow}\right),
\end{align*}
$$

where $N^{\text {pro }}$ and $N^{\text {acc }}$ are the numbers of prompt and accidental coincident events, respectively. $\sigma_{0}$ and $A_{y}$ are the unpolarized cross section and analyzing power, respectively. $N_{\mathrm{t}}$ and $N_{\mathrm{b}}$ are the numbers of the target and beam particles, respectively. $P$ is the beam-polarization vector $\left(P^{\downarrow}<0\right.$ and $\left.P^{\uparrow}>0\right) . \epsilon$ and $\Delta \Omega$ are the efficiency and the solid angle of each scintillation detector, respectively. The accidental coincidence is estimated using the number of forward counter $L(R)$ event coincident with the event of backward counter $L^{\prime}\left(R^{\prime}\right)$ of the next beam
bunch [3]. Theoretically, due to the smaller dimension of detectors, the value of polarization measured by BLP2 should be sensitive to the angular change of beam condition. It can be obtained as follows

$$
\begin{align*}
k_{\mathrm{L}} & =\frac{N_{\mathrm{L}}^{\uparrow}}{N_{\mathrm{L}}^{\downarrow}}, \quad k_{\mathrm{R}}=\frac{N_{\mathrm{R}}^{\uparrow}}{N_{\mathrm{R}}^{\downarrow}} \\
P_{y}^{\uparrow} & =\frac{1}{A_{y}} \frac{\left(k_{\mathrm{L}}+k_{\mathrm{R}}\right)-2 k_{\mathrm{L}} k_{\mathrm{R}}}{k_{\mathrm{L}}-k_{\mathrm{R}}}  \tag{5}\\
P_{y}^{\downarrow} & =\frac{1}{A_{y}} \frac{2-\left(k_{\mathrm{L}}+k_{\mathrm{R}}\right)}{k_{\mathrm{L}}-k_{\mathrm{R}}}
\end{align*}
$$

Figure 3 shows the simulation result of polarization using BLP2 data when changing the beam position $x_{\mathrm{q} 0}$ at the exit of QM6D from -3 cm to $3 \mathrm{~cm}\left(P_{\mathrm{u}}=P_{y}^{\uparrow}\right.$, $\left.P_{\mathrm{d}}=\left|P_{y}^{\downarrow}\right|\right)$. It fluctuates in a random way and changes only about $0.7 \%$. It is concluded that the measured value of polarization is free from incident beam condition.


Fig. 3. Simulation result of polarization using BLP2 data when changing beam position $x_{\mathrm{q} 0}$.

Finally, the asymmetry of the geometries of the left and right scintillators is adopted, namely $\epsilon_{\mathrm{L}} \Delta \Omega_{\mathrm{L}} / \epsilon_{\mathrm{R}} \Delta \Omega_{\mathrm{R}}$. It can be derived from Eq. (5) as follows

$$
\begin{align*}
\epsilon_{\mathrm{L}} \Delta \Omega_{\mathrm{L}} & =\frac{N_{\mathrm{R}}^{\downarrow} N_{\mathrm{L}}^{\uparrow}-N_{\mathrm{R}}^{\uparrow} N_{\mathrm{L}}^{\downarrow}}{2 \sigma_{0}\left(N_{\mathrm{R}}^{\downarrow}-N_{\mathrm{R}}^{\uparrow}\right)} \\
\epsilon_{\mathrm{R}} \Delta \Omega_{\mathrm{R}} & =\frac{N_{\mathrm{R}}^{\uparrow} N_{\mathrm{L}}^{\downarrow}-N_{\mathrm{R}}^{\downarrow} N_{\mathrm{L}}^{\uparrow}}{2 \sigma_{0}\left(N_{\mathrm{L}}^{\downarrow}-N_{\mathrm{L}}^{\uparrow}\right)}  \tag{6}\\
\frac{L}{R} & =\frac{\epsilon_{\mathrm{L}} \Delta \Omega_{\mathrm{L}}}{\epsilon_{\mathrm{R}} \Delta \Omega_{\mathrm{R}}}=\frac{N_{\mathrm{L}}^{\uparrow}-N_{\mathrm{L}}^{\downarrow}}{N_{\mathrm{R}}^{\downarrow}-N_{\mathrm{R}}^{\uparrow}} .
\end{align*}
$$

In the derivation process, the effects of polarization and analyzing power on the counts of detectors are eliminated, and the value of geometry $L / R$ is a reflection of the change of angular acceptance of the left and right detectors, which only depends on the incident beam position. In our simulation all the variables are exactly consistent with the experimental setup, $w_{\mathrm{q}}=0.6282 \mathrm{~cm}$ and $w_{\mathrm{t}}=0.2569 \mathrm{~cm}$ at the exit of QM6D and the target in BLP2, respectively.

Figure 4 shows the simulation result of the change of geometry $L / R$ depending on the beam condition. Geometry $L / R$ is changed by about $6 \%$ when the beam position $x_{\mathrm{q} 0}$ at QM6D is changed by 1 cm , i.e. the beam incident angular is changed by $0.35^{\circ}$. It is shown that the geometry $L / R$ is sensitive enough to monitor the change of beam condition.


Fig. 4. Simulation result of change of geometry $L / R$ depending on beam position.

## 3 BLP data processing of E366 and experimental correction

As the first step of data reduction of E366, we need to normalize the scaler data of BLP2 by beam intensity.

$$
\begin{align*}
& N_{\mathrm{L}(\mathrm{R})}^{\uparrow \mathrm{nor}}=\frac{N_{\mathrm{L}(\mathrm{R})}^{\dagger \text { pro }}-N_{\mathrm{L}(\mathrm{R})}^{\dagger \text { acc }}}{B \cdot I(\uparrow) B \cdot S}  \tag{7}\\
& N_{\mathrm{L}(\mathrm{R})}^{\downarrow \mathrm{nor}}=\frac{N_{L(R)}^{\downarrow \text { pro }}-N_{\mathrm{L}(\mathrm{R})}^{\downarrow \mathrm{acc}}}{B \cdot I(\downarrow) B \cdot S}
\end{align*}
$$

where $B \cdot I(\uparrow)$ and $B \cdot I(\downarrow)$ are the beam intensity with spin-up and spin-down modes, respectively, and B.S is the scale of beam intensity. Then geometry $L / R$ of every run can be calculated from Eq. (7), which is shown in Fig. 5.


Fig. 5. The results of geometry $L / R$ of 144 runs in E366.


Fig. 6. In the E366 experiment the change $\Delta \theta_{\text {BLP }}$ of beam incident angle. (a) at BLP2; (b) at target chamber.

From the simulation results in Section 2.4, the function of $x_{\mathrm{q} 0}$ with geometry $L / R$ can be obtained by the polynomial fitting method. Then the change of beam position $x_{\mathrm{q} 0}$ as well as the change $\Delta \theta_{\text {BLP }}$ of incident angle at BLP2 can be deduced from the function. Fig. 6(a)
describes the change $\Delta \theta_{\text {BLP }}$ of incident angle at BLP2 in E366.

To monitor the beam condition at the position of the target chamber, in the simulation we select the achromatic mode of the WS beam line [2] to transport the beam from BLP2 to the target chamber. In this part the horizontal magnification $M_{x}^{I I I-V}$ and the angular magnification $A_{M_{x}}^{\text {II-V }}$ are equal to ${ }^{x} 1.36698$ and 0.68523 , respectively. Therefore, the angular change $\Delta \theta_{\text {BLP }}$ should be transformed by multiplying the angular magnification from BLP2 to the target chamber, i.e. $\Delta \theta_{\mathrm{tgt}}=$ $\Delta \theta_{\text {BLP }} \times A_{M_{x}}^{\text {III-V }}$. The final result of $\Delta \theta_{\text {tgt }}$ is shown in Fig. 6(b). For the elastic scattering process, we can make a correction to the scattering angle measured in E366 on the basis of the angular change of the incident beam, i.e. $\theta_{\text {cor }}=\theta_{\text {mea }}+\Delta \theta_{\text {tgt }}$. Because the data reduction of E366 has not been finished yet, we will expect to show the corrected results in the later paper.

## 4 Summary

We simulate the behavior of incident and scattered protons by kinematic coincidence on the basis of the Monte Carlo method. From the simulation results of the asymmetry of geometries of the left and right scintillators in BLP2 depending on the incident beam condition, it is found that the beam angular change ( $0.35^{\circ}$ ) can be detected when the ratio of the left-right geometries is changed by $6 \%$, which is very helpful to obtain the more precise scattering angle. Furthermore, in our future work, we need to introduce the feedback system for this monitor to adjust the beam incident direction and position automatically.

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