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Competition between α-decay and β-decay for heavy and superheavy nuclei*

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Abstract: In this work, the β-stable region for $Z\geqslant 90$ is proposed based on a successful binding energy formula. The calculated β-stable nuclei in the β-stable region are in good agreement with the ones obtained by Möller et al. The half-lives of the nuclei close to the β-stable region are calculated and the competition between α-decay and β-decay is systematically investigated. The calculated half-lives and the suggested decay modes are well in line with the experimental results. The decay modes are mostly β^--decay above the β-stable region. Especially for $Z\leqslant 111$, all the decay modes are β^--decay. Regarding the nuclei above the β-stable region, α-decay and β^--decay ($\alpha+\beta^-$) can occur simultaneously when $Z\geqslant 112$. This is a very interesting phenomenon. The competition between α-decay and β-decay is very complex and drastic below the β-stable region. The predictions for half-lives and decay modes of the nuclei with $Z=107$–110 are presented in detail.

Key words: decay, β-stable region, half-life, superheavy nuclei

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1 Introduction

Nowadays, with the development of radioactive nuclear beams, many nuclei far from the β-stable line have been studied. Studying superheavy nuclei has been a hot topic in nuclear physics, and a large number of research results and publications have come out. Chinese physicists have made a great contribution to the study on superheavy nuclei [1–6]. Most superheavy nuclei are unstable and they can mainly be α-decay and β-decay. The two decay modes have been widely studied theoretically and experimentally [7–20].

In the early stage of the development of nuclear physics, scientists could only study the properties of the nuclei very close to the β-stable line. As a result, many nuclear phenomena, laws, formulae, methods, and models were based on the long-lived nuclei or stable nuclei close to the β-stable line. It is much easier to find and synthesize new nuclei close to the β-stable line. At present, the β-stable line for $Z<83$ has been well studied by physicists. For heavy and superheavy nuclei with $Z\geqslant 90$, most of them can be α-decay and β-decay simultaneously, and their half-lives are usually short. For this reason, it is more important to study the β-stable region than to study the β-stable line for these heavy and superheavy nuclei. In this article, we will investigate the boundary of the β-stable region based on a successful binding energy formula. The β-stable region for $Z\geqslant 90$ will be proposed. The half-lives of the nuclei close to the β-stable region will be calculated and the competition between α-decay and β-decay will be investigated. Then the decay modes can be suggested by the results of competition. The predictions are useful for quickly estimating the decay modes and half-lives in future superheavy experiments.

This article is organized in the following way. In Section 2, the β-stable region for $Z\geqslant 90$ is proposed. In Section 3, the half-lives of the nuclei close to the β-stable region are calculated and the competition between α-decay and β-decay is studied. A summary is given in Section 4.

2 The β-stable region for $Z\geqslant 90$

In this section, we will propose the β-stable region for $Z\geqslant 90$ based on a successful binding energy formula. To

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accurately measure and calculate the ground-state nuclear binding energies (or masses) is an important goal of nuclear physicists. The binding energy plays a crucial role for the nuclear stability on β-decay, α-decay and spontaneous fission of the heavy-mass region with $Z\geqslant 90$. In Ref. [21], Dong and Ren proposed a binding energy formula for heavy and superheavy nuclei. One can accurately reproduce the binding energies for the known heavy and superheavy nuclei with this formula. This formula is useful for accurately estimating the binding energies of unknown superheavy nuclei. Its form is the following:

$$\begin{aligned} B(Z,A) = {} & a_{\rm v}A-a_{\rm s}A^{2/3}-a_{\rm c}Z^2A^{-1/3}-a_{\rm a}\left(\frac{A}{2}-Z\right)^2A^{-1} \\ & +a_{\rm p}A^{-1/2}+\frac{a_6|A-252|}{A}-\frac{a_7|N-152|}{N} \\ & +\frac{a_8|N-Z-50|}{A}. \end{aligned} \tag{1}$$

The best fit parameters are

$$\begin{cases} a_{\rm v}= 15.8032\ \text{MeV}, \\ a_{\rm s}= 17.8147\ \text{MeV}, \\ a_{\rm c}= 0.71478\ \text{MeV}, \\ a_{\rm a}= 97.6619\ \text{MeV}, \\ a_6= 5.33\ \text{MeV}, \\ a_7= 21.0\ \text{MeV}, \\ a_8= -15.25\ \text{MeV}. \end{cases} \tag{2}$$

The coefficients of the pairing energy are

$$a_{\rm p}=\begin{cases} 12.26\ \text{MeV}, & \text{even-even nuclei}, \\ 3.0\ \text{MeV}, & \text{even-odd nuclei}, \\ 0\ \text{MeV}, & \text{odd-even nuclei}, \\ -8.0\ \text{MeV}, & \text{odd-odd nuclei}. \end{cases} \tag{3}$$

The mass formula has the form:

$$\begin{aligned} M(Z,A) &= ZM_{\rm H}+NM_{\rm n}-B(Z,A) \\ &= AM_{\rm n}+Z(M_{\rm H}-M_{\rm n})-B(Z,A), \end{aligned} \tag{4}$$

where $(M_{\rm H}-M_{\rm n})=-0.782$ MeV.

The decay energies of β^--decay and β^+-decay can be written as:

$$E_{\rm d}(\beta^-) = M(Z,A)-M(Z+1,A), \tag{5}$$

$$E_{\rm d}(\beta^+) = M(Z,A)-M(Z-1,A)-2m_{\rm e}, \tag{6}$$

where $2m_{\rm e}=1.022$ MeV.

From the Eqs. (1), (4), (5) and (6), we get:

$$\begin{aligned} E_{\rm d}(\beta^-) = {} & 0.782-a_{\rm c}(2Z+1)A^{-1/3}+\frac{a_{\rm a}(A-2Z-1)}{A} \\ & +a_7\left(\frac{|A-Z-152|}{A-Z}-\frac{|A-Z-153|}{A-Z-1}\right) \\ & +\frac{a_8}{A}(|A-2Z-52|-|A-2Z-50|), \end{aligned} \tag{7}$$

$$\begin{aligned} E_{\rm d}(\beta^+) = {} & -1.804-a_{\rm c}(-2Z+1)A^{-1/3}-\frac{a_{\rm a}(A-2Z+1)}{A} \\ & +a_7\left(\frac{|A-Z-152|}{A-Z}-\frac{|A-Z-151|}{A-Z+1}\right) \\ & +\frac{a_8}{A}(|A-2Z-48|-|A-2Z-50|). \end{aligned} \tag{8}$$

If the values of $E_{\rm d}(\beta^-)$ and $E_{\rm d}(\beta^+)$ are set to zero, one can get the limits of β^--decay and β^+-decay for each isotopic chain. For each fixed proton number Z, one can get two different mass numbers for the limits of β^--decay and β^+-decay, respectively. For all the proton numbers from Z=90 to Z=126, two sets of mass numbers for the limits of β^--decay and β^+-decay can be obtained, respectively. Connecting two sets of mass numbers for the limits of β^--decay and β^+-decay in the coordinate space (Z, A), the boundary of the limits of β^--decay and β^+-decay can be obtained. The calculated results are plotted in Fig. 1.

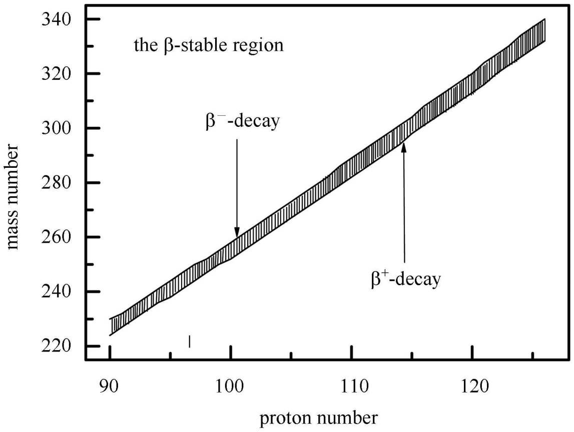

Fig. 1. The calculated β-stable region for $Z\geqslant 90$. The two curves denote the limits of β^--decay and β^+-decay, respectively.

In Fig. 1, the two curves denote the limits of β^--decay and β^+-decay, respectively. They are almost parallel. The shadow region is the calculated β-stable region. It is a long and narrow region between the two curves. According to the calculations, the nuclei above the calculated β-stable region can have β^--decay and the nuclei below the region can have β$^+$-decay. The nuclei in the β-stable region are the possible β-stable nuclei. Because there are few experimental data, we compare our calculated results with the results given by Möller et al. [22]. A comparison between our calculated results and Möller's results is shown in Table 1.

Table 1. The possible β-stable nuclei in the calculated β-stable region for $Z \geqslant 90$. The corresponding β-stable nuclei calculated by Möller et al. [22] are listed for comparison.

Z	mass number A of β-stable nuclei	
	Cal.	Möller
90	224−230	224, 226−230, 232
91	227−232	231
92	230−235	230, 232−236, 238
93	233−238	237
94	236−241	236, 238−242, 244
95	238−244	241#, 243#
96	241−247	240, 242−246, 248
97	244−250	247
98	247−252	246, 248−252, 254
99	250−255	253
100	252−258	252, 254−258, 260, 262
101	255−261	259
102	258−264	258, 260−264, 266
103	261−267	265
104	264−270	264, 266−268, 270, 272, 274
105	267−273	269, 271
106	270−276	268, 270, 272−276, 278, 280
107	273−279	277
108	276−282	274, 276, 278−284, 286
109	279−286	283#, 285#
110	282−289	282, 284−288, 290
111	285−292	289
112	288−295	288, 290−294, 296
113	291−298	293#, 295#
114	294−301	292, 294−298, 300, 302, 304
115	298−304	299, 301, 303
116	301−308	300, 302, 304−306, 308, 310−312, 314
117	304−311	307, 309
118	307−314	304, 306, 308, 310−314, 318
119	310−317	315
120	313−320	312, 316−318, 320, 322, 324
121	316−324	319
122	320−327	318, 320, 321, 323−326, 328, 330
123	324−330	322, 327
124	327−334	317, 323, 324, 326, 328−332, 334, 336, 338
125	329−337	325, 327, 333
126	332−340	326, 330, 332, 334−338

The mass numbers with # denote that the nuclei with these mass numbers are β-stable nuclei, estimated from systematic trends in neighboring nuclei. Our calculated results show that there are several (from six to nine) continuous β-stable nuclei in each isotopic chain. Even for Z, the two results are almost the same. On the whole, the range by Möller et al. is slightly larger than our calculated results. The calculated β-stable nuclei by Möller et al. are not continuous in their isotopic chains. Some nuclei are β-stable nuclei in our calculations, but they are not β-stable nuclei in Möller's results, and vice versa. For odd Z, Möller's results show that there are only one or two β-stable nuclei in their isotopic chains except for Z=115, 125, which is different from our results. But it can be seen that the β-stable nuclei from Möller's results are all in the middle of our calculated β-stable region except for Z=123, 125. From the above discussions, it can be said that the calculated β-stable region is in good agreement with Möller's results.

3 Competition between α-decay and β-decay of the nuclei close to the β-stable region

In the previous section, the β-stable region for $Z \geqslant 90$ has been proposed. Most nuclei with $Z \geqslant 90$ can have α-decay, β-decay and spontaneous fission simultaneously. In this section, we will calculate the half-lives of the nuclei close to the calculated β-stable region, and study the competition between α-decay and β-decay of them. There are plenty of experimental half-lives and the decay modes of many nuclei are very explicit in this region. The calculated results can be compared with these experimental data to test the reliability of the calculated results. On the other hand, the predictions are useful for quickly estimating the decay modes and half-lives in future superheavy experiments. Before calculating the half-lives, we firstly introduce several successful formulae for calculations.

In Ref. [23], Ni et al. proposed a unified formula of half-lives for α-decay and cluster radioactivity. For α-decay, it is written as:

$$\log_{10} T_{1/2} = 2a\sqrt{\mu}(Z-2)Q_{\alpha}^{-1/2} + b\sqrt{\mu}[2(Z-2)]^{1/2} + c, \quad (9)$$

where $\mu = 4(A-4)/A$, $T_{1/2}$ is the half-life of α-decay (in seconds), and Q_{α} is α-decay energy (in MeV). A and Z are the mass number and the proton number of the parent nuclei respectively. The values of the parameters are $a = 0.39961$, and $b = -1.31008$. Parameter c is determined to be $c_{\text{e−e}} = -17.00698$ (for even-even nuclei), $c_{\text{e−o}} = -16.26029$ (for even-odd nuclei), $c_{\text{o−e}} = -16.40484$ (for odd-even nuclei), and $c_{\text{o−o}} = -15.85337$ (for odd-odd nuclei).

β-decay is also a very important decay mode for rich-neutron or rich-proton nuclei. For β$^-$-decay, in Ref. [24], Zhang et al. proposed a reliable formula to calculate the β$^-$-decay half-lives. It is written as:

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4 + \text{shell}(Z, N), \quad (10)$$

where

$$\begin{aligned}\mathrm{shell}(Z,N) = {} & c_5[\mathrm{e}^{-(N-29)^2/15}+\mathrm{e}^{-(N-50)^2/37} \\ & +\mathrm{e}^{-(N-85)^2/9}+\mathrm{e}^{-(N-131)^2/3}] \\ & +c_6\mathrm{e}^{-[(Z-51.5)^2+(N-80.5)^2]/1.9} \qquad (11)\end{aligned}$$

is the shell correction term. Z and N are the proton number and neutron number of the parent nuclei. $T_{1/2}$ is the half-life of β$^-$-decay (in seconds). The parameters are $c_1=3.37\times10^{-4}$, $c_2=-0.2558$, $c_3=0.4028$, $c_4=-1.0100$, $c_5=0.9039$, and $c_6=7.7139$.

For β$^+$-decay, in Ref. [25], Zhang et al. proposed a similar formula to Eq. (10). It is written as:

$$\log_{10}T_{1/2}=(c_1Z+c_2)N+c_3Z+c_4. \qquad (12)$$

For different order (the allowed β$^+$-transition, the first and the second forbidden β$^+$-transition), the parameters are different. The even-odd effect has been taken into account in the above equation. The best fit parameters are displayed in Table 2.

For a given proton number Z, we select ten continuous isotopes nearest to the top and bottom of the β-stable region, respectively. Thus there are 20 nuclei for each isotopic chain. Because only the half-lives of the allowed β$^+$-transition, the first and the second forbidden β$^+$-transitions can be calculated by Eq. (12), the nuclei with higher forbidden β$^+$-transition are not included. Because formula (9) can only calculate the half-lives of the nuclei with $Z\geqslant84$ and $N\geqslant128$, the nuclei with $N<128$ are not included also. So the number of the calculated nuclei of each isotopic chain may be less than 20. We calculate the half-lives of the nuclei from Z=90 to Z=126, and predict the decay modes of them. Because the calculated data are too many, we firstly compare the calculated results with the available experimental data [26]. The selected region for comparison is from Z=90 to Z=103, because there are many experimental data in this region. The results are listed in Table 3.

Table 2. The parameters of Eq. (12). The word "order" in the first column denotes the order of the β$^+$-decay from ground state to ground state. The even-odd effect has been included.

order	c_1	c_2	c_3	c_4		
				e-o, o-e	o-o	e-e
allowed	−0.00179	0.4233	−0.3405	−0.6443	−1.7089	−0.2132
first	−0.00127	0.3992	−0.4183	3.8215	3.7969	4.0364
second	−0.00162	0.3980	−0.3286	−0.1618	−0.4854	0.0267

Table 3. The comparison of the half-lives and decay modes between the calculated results and the experimental data by Audi et al. [26] from Z=90 to Z=103. Here $C=\log_{10}(T^{\mathrm{cal}}_{1/2}/T^{\mathrm{expt}}_{1/2})$

Z	A	Cal. $T^{\mathrm{cal}}_{\alpha 1/2}$/s	Cal. $T^{\mathrm{cal}}_{\beta 1/2}$/s	calculated decay modes and intensities (%)	experimental decay modes and intensities (%)	$T^{\mathrm{expt}}_{1/2}$/s	C
90	218	9.14×10^{-8}	718.8	α=100	α=100	1.17×10^{-7}	−0.11
90	219	3.11×10^{-6}	844.5	α=100	α=100	1.05×10^{-6}	0.47
90	221	5.67×10^{-4}	3136.2	α=100	α=100	1.68×10^{-3}	−0.47
90	222	2.80×10^{-3}	9912.9	α=100	α=100	2.05×10^{-3}	0.14
90	223	0.93	1.16×10^{4}	α=100	α=100	0.6	0.19
90	231	1.63×10^{17}	2823.1	β^-=100	β^-=100	9.19×10^{4}	−1.51
90	233	2.13×10^{21}	999.5	β^-=100	β^-=100	1309	−0.12
90	234	2.02×10^{21}	594.7	β^-=100	β^-=100	2.08×10^{6}	−3.54
90	235	3.78×10^{24}	353.9	β^-=100	β^-=100	432	−0.09
90	236	2.44×10^{24}	210.6	β^-=100	β^-=100	2238	−1.03
90	237	4.61×10^{29}	125.3	β^-=100	β^-=100	288	−0.36
90	238	2.87×10^{29}	74.6	β^-=100	β^-=100	564	−0.88
91	219	2.27×10^{-7}	115.0	α=100	α=100	5.30×10^{-8}	0.63
91	220	2.94×10^{-6}	208.9	α=100	α=100	7.80×10^{-7}	0.58
91	221	2.02×10^{-5}	424.7	α=100	α=100	5.90×10^{-6}	0.53
91	223	6.30×10^{-3}	1568.1	α=100	α=100, $\beta^+<0.001$#	5.10×10^{-3}	0.09
91	225	6.5	5789.2	α=100	α=100	1.7	0.58
91	226	694.0	1.05×10^{4}	α=94, β^+=6	α=74, β^+=26	108	0.78
91	233	7.47×10^{16}	4741.5	β^-=100	β^-=100	2.33×10^{6}	−2.69
91	234	1.15×10^{19}	2823.4	β^-=100	β^-=100	2.41×10^{4}	−0.93
91	235	3.53×10^{19}	1681.3	β^-=100	β^-=100	1466	0.06
91	236	1.35×10^{22}	1001.2	β^-=100	β^-=100	546	0.26

Table 3. Continued.

Z	A	Cal. $T^{\mathrm{cal}}_{\alpha 1/2}$/s	Cal. $T^{\mathrm{cal}}_{\beta 1/2}$/s	calculated decay modes and intensities (%)	experimental decay modes and intensities (%)	$T^{\mathrm{expt}}_{1/2}$/s	C
91	237	1.17×10^{22}	596.2	$\beta^{-}=100$	$\beta^{-}=100$	522	0.06
91	238	7.10×10^{27}	355.0	$\beta^{-}=100$	$\beta^{-}=100$	136	0.42
91	239	8.19×10^{27}	211.4	$\beta^{-}=100$	$\beta^{-}=100$	6.48×10^{3}	-1.49
92	223	3.83×10^{-4}	212.4	$\alpha=100$	$\alpha\approx100$, $\beta^{+}=0.2$#	2.10×10^{-5}	1.26
92	225	0.18	779.4	$\alpha=100$	$\alpha=100$	0.061	0.47
92	226	0.34	2449.3	$\alpha=100$	$\alpha=100$	0.269	0.10
92	227	99.1	2860.9	$\alpha=97$, $\beta^{+}=3$	$\alpha=100$, $\beta^{+}<0.001$#	66	0.16
92	228	659.4	4508.8	$\alpha=87$, $\beta^{+}=13$	$\alpha>95$, $\varepsilon<5$	546	0.02
92	229	8.59×10^{4}	2888.3	$\alpha=3$, $\beta^{+}=97$	$\alpha\approx20$, $\beta^{+}\approx80$	3.48×10^{3}	-0.10
92	236	3.37×10^{15}	4753.1	$\beta^{-}=100$	$\alpha=100$	7.39×10^{14}	
92	237	1.62×10^{18}	2832.6	$\beta^{-}=100$	$\beta^{-}=100$	5.83×10^{5}	-2.30
92	239	1.62×10^{21}	1006.0	$\beta^{-}=100$	$\beta^{-}=100$	1.41×10^{3}	-0.15
92	240	8.57×10^{20}	599.5	$\beta^{-}=100$	$\beta^{-}=100$	5.08×10^{4}	-1.92
92	242	4.35×10^{23}	212.9	$\beta^{-}=100$	$\beta^{-}=100$	1.01×10^{3}	-0.68
93	227	1.34	385.2	$\alpha=100$	$\alpha\approx100$, $\beta^{+}=0.05$#	0.51	0.42
93	228	118.8	695.3	$\alpha=85$, $\beta^{+}=15$	$\alpha=40$, $\varepsilon=60$	61.4	0.22
93	229	1.03×10^{3}	1405.5	$\alpha=58$, $\beta^{+}=42$	$\alpha=68$, β^{+}?	240	0.39
93	230	3.00×10^{4}	64.6	$\beta^{+}=100$	$\beta^{+}\leqslant97$, $\alpha\geqslant3$	276	-0.63
93	231	4.85×10^{5}	1354.4	$\beta^{+}=100$	$\beta^{+}=98$, $\alpha=2$	2.93×10^{3}	-0.34
93	239	1.23×10^{16}	4779.5	$\beta^{-}=100$	$\beta^{-}=100$	2.04×10^{5}	-1.63
93	240	6.17×10^{16}	2850.5	$\beta^{-}=100$	$\beta^{-}=100$	3.71×10^{3}	-0.11
93	241	8.45×10^{18}	1700.1	$\beta^{-}=100$	$\beta^{-}=100$	834	0.31
93	242	2.05×10^{19}	1013.9	$\beta^{-}=100$	$\beta^{-}=100$	132	0.86
93	243	4.96×10^{19}	604.7	$\beta^{-}=100$	$\beta^{-}=100$	111	0.74
93	244	3.11×10^{21}	360.7	$\beta^{-}=100$	$\beta^{-}=100$	137	0.42
94	229	24.3	189.2	$\alpha=89$, $\beta^{+}=11$	$\alpha=50$, $\beta^{+}=50$	91	-0.63
94	231	1.78×10^{4}	194.6	$\alpha=1$, $\beta^{+}=99$	$\alpha=13$, $\beta^{+}=87$	516	-0.43
94	233	1.10×10^{6}	629.9	$\beta^{+}=100$	$\beta^{+}\approx100$, $\alpha=0.12$	1.25×10^{3}	-0.30
94	235	1.86×10^{8}	2038.6	$\beta^{+}=100$	$\beta^{+}\approx100$, $\alpha=0.003$	1.52×10^{3}	0.13
94	242	1.17×10^{13}	4821.1	$\beta^{-}=100$	$\alpha=100$	4.93×10^{11}	
94	243	2.30×10^{15}	2877.5	$\beta^{-}=100$	$\beta^{-}=100$	1.78×10^{4}	-0.79
94	244	1.85×10^{15}	1717.5	$\beta^{-}=100$	$\alpha\approx100$, SF $=0.12$	1.05×10^{14}	
94	245	2.50×10^{17}	1025.1	$\beta^{-}=100$	$\beta^{-}=100$	3.78×10^{4}	-1.57
94	246	4.72×10^{17}	611.9	$\beta^{-}=100$	$\beta^{-}=100$	9.36×10^{5}	-3.18
94	247	1.00×10^{20}	365.2	$\beta^{-}=100$	$\beta^{-}=100$	1.96×10^{5}	-2.73
95	235	3.10×10^{5}	1202.3	$\beta^{+}=100$	$\beta^{+}\approx100$, $\alpha=0.4$	618	0.29
95	237	2.29×10^{7}	932.6	$\beta^{+}=100$	$\beta^{+}\approx100$, $\alpha=0.025$	4.42×10^{3}	-0.67
95	245	5.58×10^{12}	4878.1	$\beta^{-}=100$	$\beta^{-}=100$	7.38×10^{3}	-0.18
95	246	4.73×10^{13}	2913.8	$\beta^{-}=100$	$\beta^{-}=100$	2.34×10^{3}	0.10
95	247	1.27×10^{15}	1740.5	$\beta^{-}=100$	$\beta^{-}=100$	1.38×10^{3}	0.10
96	239	1.58×10^{6}	927.3	$\beta^{+}=100$	$\beta^{+}\approx100$, $\alpha=0.0062$	1.04×10^{4}	-1.05
96	249	2.21×10^{13}	2959.7	$\beta^{-}=100$	$\beta^{-}=100$	3.69×10^{3}	-0.11
96	251	1.89×10^{13}	1057.7	$\beta^{-}=100$	$\beta^{-}=100$	1.01×10^{3}	0.02
97	242	5.02×10^{5}	1750.9	$\beta^{+}=100$	$\beta^{+}\approx100$	420	0.62
97	243	1.65×10^{5}	3498.3	$\alpha=2$, $\beta^{+}=98$	$\beta^{+}\approx100$, $\alpha\approx0.15$	1.62×10^{4}	-0.67
97	251	1.38×10^{11}	5040.9	$\beta^{-}=100$	$\beta^{-}=100$	3.34×10^{3}	0.18
98	241	522.5	130.6	$\alpha=20$, $\beta^{+}=80$	$\alpha\approx25$, β^{+}?	141	-0.13
98	243	8.78×10^{3}	462.8	$\alpha=5$, $\beta^{+}=95$	$\alpha\approx14$, $\beta^{+}\approx86$	642	-0.17
98	245	1.66×10^{4}	1640.1	$\alpha=9$, $\beta^{+}=91$	$\alpha=36$, β^{+}?	2.70×10^{3}	-0.26
98	246	1.22×10^{5}	5064.3	$\alpha=4$, $\beta^{+}=96$	$\alpha=100$	1.29×10^{5}	
98	253	1.69×10^{9}	8598.6	$\beta^{-}=100$	$\beta^{-}\approx100$, $\alpha\approx0.31$	1.54×10^{6}	-2.25
98	255	1.84×10^{11}	3082.3	$\beta^{-}=100$	$\beta^{-}=100$	5.10×10^{3}	-0.22
99	243	33.4	61.6	$\alpha=65$, $\beta^{+}=35$	$\alpha\geqslant61$, $\beta^{+}\leqslant39$	21.6	0.01
99	245	120.3	217.0	$\alpha=64$, $\beta^{+}=36$	$\alpha=40$, β^{+}?	66	0.07

Table 3. Continued.

Z	A	Cal. $T^{\rm cal}_{\alpha 1/2}$/s	Cal. $T^{\rm cal}_{\beta 1/2}$/s	calculated decay modes and intensities (%)	experimental decay modes and intensities (%)	$T^{\rm expt}_{1/2}$/s	C
99	247	3.89×10^3	116.8	α=3, β^+=97	$\alpha\approx7$, $\beta^+\approx93$	273	−0.39
99	248	2.65×10^5	1355.9	β^+=100	$\beta^+\approx100$, $\alpha\approx0.25$	1.62×10^3	−0.08
99	249	5.97×10^5	2693.4	β^+=100	$\beta^+\approx100$, $\alpha\approx0.57$	6.13×10^3	−0.36
99	256	3.72×10^9	8802.2	β^-=100	β^-=100	1.52×10^3	0.76
99	257	8.50×10^9	5274.1	β^-=100	β^-=100	6.65×10^5	−2.10
100	247	37.2	16.5	α=31, β^+=69	$\alpha>50$, $\beta^+<50$	31	−0.49
100	248	34.3	310.5	α=90, β^+=10	α=93, β^+=7	36.1	−0.07
100	251	2.39×10^4	411.3	α=2, β^+=98	α=1.8, β^+=98.2	1.91×10^4	−1.67
101	250	106.6	82.6	α=44, β^+=56	α=7, β^+=93	52	−0.05
101	252	2.66×10^3	3.30	β^+=100	$\beta^+>50$, α?	138	
102	254	33.3	228.1	α=87, β^+=13	α=90, β^+=10	51.2	−0.25
102	256	2.33	789.7	α=100	$\alpha\approx100$	2.91	−0.10
103	253	1.68	1.30	α=44, β^+=56	α=90, β^+=1#	0.63	−0.01
103	254	16.8	17.3	α=51, β^+=49	α=72, β^+=28	17.1	−0.19
103	256	13.6	0.60	α=4, β^+=96	α=85, β^+=15	27	−1.68

When using formula (9) to calculate the half-lives of α-decay and β^+-decay, we need α-decay energies Q_α, spins and parities. Here, all data are taken from Ref. [26, 27]. If there are no experimental data, we use the calculated data obtained by Möller et al. [22]. The fifth column is the calculated decay modes and intensities (in %). The decay mode can be regard as a competition between α-decay and β-decay. Here we define a symbol R to denote the ratio of α-decay half-life and β-decay half-life. $R=T^{\rm cal}_{\alpha 1/2}/T^{\rm cal}_{\beta 1/2}$. If the α-decay half-life is shorter than the β-decay half-life by 100 times (i.e. $R<0.01$) in a nucleus, we can say the decay mode of this nucleus is α-decay. If the $R>100$ in a nucleus, we can say the decay mode of this nucleus is β-decay. If $0.01<R<100$, the decay mode can be regard as a coexistence state of both α-decay and β-decay, and we use the symbol α+β^+ to denote it. The data marked with # denote the values from systematic trends in neighboring nuclei. The symbol C in the last column is the ratio of calculated half-life and experimental one, and it is in the form of $C=\log_{10}(T^{\rm cal}_{1/2}/T^{\rm expt}_{1/2})$. The nuclei which do not have explicit experimental decay modes and intensities (in %) are not included in Table 3. There are altogether 91 nuclei. It can be seen that the predicted decay modes are in excellent agreement with the experimental ones. There are only five nuclei whose decay modes are not in line with the predicted decay modes and their proton and mass number are marked in bold italic type. The values of C are mostly between 1.0 and −1.0. This means that the most differences between calculated half-lives and experimental ones are less than ten times. Usually, if the differences between theoretical half-lives and experimental ones are less than 10^3 times, it can be said that the results are satisfactory. So we can say the calculated half-lives in this region are in good agreement with the experimental data.

After having compared the calculated results with the experimental data from Z=90 to Z=103, we predict some half-lives and decay modes for some heavier nuclei. Next, we select the region from Z=107 to Z=110 to make some predictions. There are many researches in this region [28–37]. We hope that our predictions will be useful for future experiments on heavy and superheavy nuclei. Because there are almost no explicit spins and parities for the nuclei with $Z\geqslant107$ except even-even nuclei, it is difficult for us to judge the orders of β^+-decay. When using the formula (12) to calculate half-lives of β^+-decay, we suppose all the orders of β^+-decay are the first β^+-transition for simplifying the calculations. The calculated results are listed in Table 4.

Table 4. The calculated half-lives and the predicted decay modes of the nuclei from Z=107 to Z=110. Some available experimental half-lives of α-decay [28, 29, 32, 37] are listed for comparison. Here $D=T^{\rm cal}_{\alpha 1/2}/T^{\rm expt}_{\alpha 1/2}$.

Z	A	Cal. $T^{\rm cal}_{\alpha 1/2}$/s	Cal. $T^{\rm cal}_{\beta 1/2}$/s	calculated decay modes and intensities(%)	$T^{\rm expt}_{\alpha 1/2}$/s	D
107	264	0.16	2.40	α=94, β^+=6	0.9 [37]	0.18
107	265	1.99	4.64	α=70, β^+=30	0.94 [28]	2.12
107	266	20.9	8.04	α=28, β^+=72	5 [26]	4.18
107	267	36.3	15.6	α=30, β^+=70	17 [32]	2.14
107	268	55.3	27	α=33, β^+=67		
107	269	86.5	52.4	α=38, β^+=62		
107	270	63.7	90.9	α=59, β^+=41	61 [29]	1.04
107	271	0.91	176.3	α=100		

Table 4. Continued.

Z	A	Cal. $T^{\text{cal}}_{\alpha 1/2}$/s	Cal. $T^{\text{cal}}_{\beta 1/2}$/s	calculated decay modes and intensities(%)	$T^{\text{expt}}_{\alpha 1/2}$/s	D
107	272	34.1	305.5	α=90, β^{+}=10	9.8 [29]	3.48
107	273	56.2	β-stable	α=100		
107	274	3.99×10^{3}	β-stable	α=100		
107	275	108.5	β-stable	α=100		
107	276	4.00×10^{4}	β-stable	α=100		
107	277	4.58×10^{6}	β-stable	α=100		
107	278	9.12×10^{5}	β-stable	α=100		
107	279	1.07×10^{7}	β-stable	α=100		
107	280	2.49×10^{10}	1.19×10^{4}	β^{-}=100		
107	281	3.24×10^{11}	7.16×10^{3}	β^{-}=100		
107	282	3.42×10^{13}	4.31×10^{3}	β^{-}=100		
107	283	1.52×10^{12}	2.60×10^{3}	β^{-}=100		
107	284	1.16×10^{12}	1.57×10^{3}	β^{-}=100		
107	285	9.45×10^{11}	945.6	β^{-}=100		
107	286	4.41×10^{15}	570.1	β^{-}=100		
107	287	5.48×10^{14}	343.7	β^{-}=100		
107	288	7.69×10^{15}	207.2	β^{-}=100		
107	289	1.43×10^{15}	124.9	β^{-}=100		
108	266	2.72×10^{-3}	1.83	α=100	2.3×10^{-3} [32]	1.18
108	267	0.129	2.04	α=94, β^{+}=6	0.058 [32]	2.22
108	268	0.0376	6.12	α=100		
108	269	9.44	6.82	α=42, β^{+}=58	9.7 [32]	0.97
108	270	1.88	20.4	α=92, β^{+}=8	3.6 [32]	0.52
108	271	0.21	22.8	α=100		
108	272	0.011	68.3	α=100		
108	273	0.21	76.2	α=100		
108	274	0.49	228.4	α=100		
108	275	0.31	254.6	α=98, β^{+}=2	0.19 [29]	1.63
108	276	66.3	β-stable	α=100		
108	277	8.00×10^{3}	β-stable	α=100		
108	278	516.2	β-stable	α=100		
108	279	9.31×10^{4}	β-stable	α=100		
108	280	1.98×10^{6}	β-stable	α=100		
108	281	4.57×10^{10}	β-stable	α=100		
108	282	1.60×10^{10}	β-stable	α=100		
108	283	3.50×10^{11}	1.25×10^{4}	β^{-}=100		
108	284	1.60×10^{10}	7.54×10^{3}	β^{-}=100		
108	285	1.71×10^{10}	4.55×10^{3}	β^{-}=100		
108	286	8.25×10^{9}	2.48×10^{3}	β^{-}=100		
108	287	9.52×10^{13}	1.66×10^{3}	β^{-}=100		
108	288	4.23×10^{12}	999.3	β^{-}=100		
108	289	1.09×10^{14}	602.9	β^{-}=100		
108	290	2.22×10^{13}	363.8	β^{-}=100		
108	291	1.09×10^{14}	219.5	β^{-}=100		
108	292	4.83×10^{12}	132.5	β^{-}=100		
109	269	7.33×10^{-3}	0.891	α=100		
109	270	3.62×10^{-4}	1.54	α=88, β^{+}=12	5×10^{-3} [32]	0.07
109	271	0.073	2.96	α=98, β^{+}=2		
109	272	0.018	5.1	α=100		
109	273	1.45×10^{-3}	9.84	α=100		
109	274	1.14	17	α=94, β^{+}=6	0.445 [29]	2.56
109	275	9.82×10^{-3}	32.7	α=100	9.7×10^{-3} [29]	1.01
109	276	1.57	56.3	α=97, β^{+}=3	0.72 [29]	2.18
109	277	4.28	108.9	α=96, β^{+}=4		
109	278	240.6	187.2	α=44, β^{+}=56		
109	279	1.29×10^{3}	β-stable	α=100		
109	280	3.21×10^{4}	β-stable	α=100		
109	281	1.90×10^{5}	β-stable	α=100		
109	282	1.94×10^{9}	β-stable	α=100		
109	283	2.06×10^{9}	β-stable	α=100		
109	284	1.70×10^{10}	β-stable	α=100		

Table 4. Continued.

Z	A	Cal. $T^{\text{cal}}_{\alpha 1/2}$/s	Cal. $T^{\text{cal}}_{\beta 1/2}$/s	calculated decay modes and intensities(%)	$T^{\text{expt}}_{\alpha 1/2}$/s	D
109	285	8.22×10^{8}	β-stable	α=100		
109	286	4.41×10^{8}	β-stable	α=100		
109	287	1.69×10^{9}	7.97×10^{3}	β^-=100		
109	288	1.40×10^{12}	4.81×10^{3}	β^-=100		
109	289	2.02×10^{12}	2.90×10^{3}	β^-=100		
109	290	1.47×10^{13}	1.75×10^{3}	β^-=100		
109	291	7.61×10^{12}	1.06×10^{3}	β^-=100		
109	292	1.88×10^{13}	639.6	β^-=100		
109	293	1.00×10^{12}	386.3	β^-=100		
109	294	1.62×10^{9}	233.2	β^-=100		
109	295	9.23×10^{8}	140.8	β^-=100		
109	296	1.40×10^{10}	85.1	β^-=100		
110	272	1.02×10^{-4}	1.15	α=100		
110	273	2.16×10^{-4}	1.28	α=100	1.7×10^{-4} [32]	1.27
110	274	3.32×10^{-5}	3.81	α=100		
110	275	8.90×10^{-4}	4.23	α=100		
110	276	2.53×10^{-3}	12.6	α=100		
110	277	0.081	14.0	α=100		
110	278	0.091	41.6	α=100		
110	279	1.40	46.1	α=97, β^+=3	2 [29]	0.7
110	280	9.18	137.5	α=94, β^+=6		
110	281	584.3	152.4	α=21, β^+=79	240 [26]	2.43
110	282	1.94×10^{9}	β-stable	α=100		
110	283	3.97×10^{9}	β-stable	α=100		
110	284	1.14×10^{9}	β-stable	α=100		
110	285	1.13×10^{9}	β-stable	α=100		
110	286	8.62×10^{9}	β-stable	α=100		
110	287	5.29×10^{9}	β-stable	α=100		
110	288	8.76×10^{9}	β-stable	α=100		
110	289	3.36×10^{9}	β-stable	α=100		
110	290	3.12×10^{10}	8.45×10^{3}	β^-=100		
110	291	1.74×10^{11}	5.10×10^{3}	β^-=100		
110	292	2.24×10^{10}	3.08×10^{3}	β^-=100		
110	293	6.49×10^{10}	1.86×10^{3}	β^-=100		
110	294	6.07×10^{9}	1.13×10^{3}	β^-=100		
110	295	7.67×10^{6}	680.7	β^-=100		
110	296	6.59×10^{6}	411.4	β^-=100		
110	297	7.84×10^{7}	248.6	β^-=100		
110	298	2.38×10^{6}	150.2	β^-=100		
110	299	2.26×10^{8}	90.8	β^-=100		

The symbol D in the last column is the ratio of calculated half-life of α-decay and experimental one, and it is in the form of $D=T^{\text{cal}}_{\alpha 1/2}/T^{\text{expt}}_{\alpha 1/2}$.

For β^--decay, the values of half-life vary from 10^1 s to 10^5 s for all Z. The nearer the nuclei are close to the β-stable region, the longer their half-lives are. For β^+-decay, on the whole, it is similar to the case of β^--decay. For α-decay, the half-lives of α-decay approximately vary from 10^{-4} s to 10^{16} s in this region. The half-life of α-decay of a nucleus above the β-stable region is much longer than its half-life of β^--decay on the whole. Thus the nuclei above the β-stable region have β^--decay mainly. However, for most nuclei below the β-stable region, their half-lives of α-decay are slightly less than the ones of β-decay. So the decay modes of these nuclei are mainly α or α+β^+-decay. There are 18 experimental half-lives of α-decay in this region. It can be seen that the calculated half-lives of α-decay are in agreement with the experimental ones. Except for ^{270}Mt (D=0.07), the values of D vary from 0.18 to 4.18. It is a good approximation. It must be pointed out that the half-life is very sensitive to the α-decay energy. A small change in α-decay energy will lead to a very large difference in half-life. There are few experimental α-decay energies in this region, and most α-decay energies used for calculation are the estimated data [27] or the calculated results [22].

To clearly understand the competition between α-decay and β-decay of the nuclei close to the calculated β-stable region, we draw the predicted decay modes from

Z=90 to Z=126 in Fig. 2.

In Fig. 2, one can clearly see the decay modes of the nuclei close to the calculated β-stable region. Especially for $Z\leqslant111$, all the decay modes are β^--decay. The decay modes are very complex below the β-stable region. All the three cases of decay mode can occur from Z=90 to Z=126. It indicates that the competition between α-decay and β-decay is very complex and drastic below the β-stable region. It can be seen that the nuclei above the β-stable region can have α-decay and β^--decay ($\alpha+\beta^-$) simultaneously when $Z\geqslant112$. It is a very interesting phenomenon, because there is no decay mode of $\alpha+\beta^-$according to experimental results by Audi et al. for all Z.

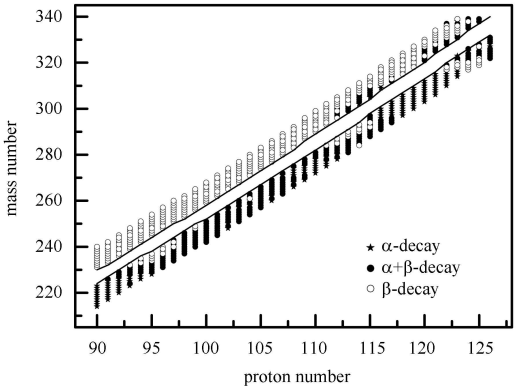

Fig. 2. The predicted decay modes of the nuclei close to the calculated β-stable region. The dark asterisks denote α-decay. The dark circles denote α + β-decay. The hollow circles denote β-decay.

4 Conclusions

In summary, we propose the β-stable region for $Z\geqslant90$. The predicted β-stable nuclei in the calculated β-stable region are in good agreement with the ones obtained by Möller et al. We calculate the half-lives of the nuclei close to the calculated β-stable region and systematically study the competition between α-decay and β-decay. The calculated half-lives and the suggested decay modes are in good agreement with the experimental results from Audi's Table. The predictions for half-lives and decay modes of the nuclei with Z=107–110 are presented. We draw the predicted decay modes from Z=90 to Z=126 in a figure. We find that the nuclei above the β-stable region can have α-decay and β^--decay ($\alpha+\beta^-$) simultaneously when $Z\geqslant112$. It is a very interesting phenomenon. The competition between α-decay and β-decay is very complex and drastic below the β-stable region. The calculated results on the half-lives and the decay modes of the nuclei close to the calculated β-stable region are useful for the future experiments on heavy and superheavy nuclei.

References

1 ZHENG S J, XU F R, YUAN C X, QI C. Chin. Phys. C (HEP & NP), 2009, **33**: 107
2 ZHAO W J, ZHANG Y Q, WANG H L et al. Chin. Phys. C (HEP & NP), 2010, **34**: 1609
3 XU H S, ZHOU X H, XIAO G Q et al. Nucl. Phys. Rev., 2003, **20**: 76
4 ZHANG H F, ZUO W, REN X Z, ZHOU X H, LI J Q. Nucl. Phys. Rev., 2004, **21**: 203
5 ZUO W, LI J Q, ZHAO E G. Nucl. Phys. Rev., 2006, **23**: 375
6 HUANG W X, WANG Y, ZHU Z C et al. Nucl. Phys. Rev., 2006, **23**: 383
7 Hofmann S, Münzenberg G. Rev. Mod. Phys., 2000, **72**: 733
8 Koonin S E. Nature, 1991, **354**: 468
9 Haxton W C, Johnson C. Phys. Rev. Lett., 1990, **65**: 1325
10 Wilk P A, Gregorich K E, Türler A et al. Phys. Rev. Lett., 2000, **85**: 2697
11 REN Z Z, XU G O. Phys. Rev. C, 1987, **36**: 456
12 ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
13 XU C, REN Z Z. Phys. Rev. C, 2006, **73**: 041301(R)
14 REN Y J, REN Z Z. Phys. Rev. C, 2012, **85**: 044608
15 PENG J S, LI L L, ZHOU S G, ZHAO E G. Chin. Phys. C (HEP & NP), 2008, **32**: 634
16 DONG J M, ZHANG H F, WANG Y Z et al. Chin. Phys. C (HEP & NP), 2009, **33**: 633
17 ZHANG G L, LE X Y. Chin. Phys. C (HEP & NP), 2009, **33**: 354
18 SU X L, ZHANG H F, ZUO W, LI J Q. Nucl. Phys. Rev., 2009, **26**: 177
19 XU C, REN Z Z. Nucl. Phys. Rev., 2013, **30**: 308
20 BAO X J, ZHANG H F, LI J Q, ZHANG H F. Nucl. Phys. Rev., 2013, **30**: 318
21 DONG T K, REN Z Z. Phys. Rev. C, 2008, **77**: 064310
22 Möller P, Nix J R, Kratz K -L. At. Data Nucl. Data Tables, 1997, **66**: 131
23 NI D D, REN Z Z, DONG T K et al. Phys. Rev. C, 2008, **78**: 044310
24 ZHANG X P, REN Z Z, ZHI Q J et al. J Phys. G: Nucl. Part. Phys., 2007, **34**: 2611
25 ZHANG X P, REN Z Z, ZHI Q J. Commun. Theor. Phys., 2007, **48**: 1072
26 Audi G, Kondev F G, Wang M et al. Chin. Phys. C (HEP & NP), 2012, **36**: 1157
27 WANG M , Audi G, Wapstra A H et al. Chin. Phys. C (HEP & NP), 2012, **36**: 1603
28 GAN Z G, GUO J S, WU X L et al. Eur. Phys. J. A, 2004, **20**: 385
29 Oganessian Yu Ts. J Phys. G: Nucl. Part. Phys., 2007, **34**: R165
30 Oganessian Yu Ts, Dmitriev S N, Yeremin A V et al. Phys. Rev. C, 2009, **79**: 024608
31 Düllmann Ch E et al. Nature, 2002, **418**: 859
32 Gupta M, Burrows T W. Nucl. Data Sheets, 2005, **106**: 251
33 Dragojević I, Gregorich K E, Düllmann Ch E et al. Phys. Rev. C, 2009, **79**: 011602(R)
34 PEI J C, XU F R, LIN Z J et al. Phys. Rev. C, 2007, **76**: 044326
35 Dvorak J, Brüchle W, Chelnokov M et al. Phys. Rev. Lett., 2008, **100**: 132503
36 ZHANG H F, ZUO W, LI J Q et al. Phys. Rev. C, 2006, **74**: 017304
37 Morita K, Morimoto K, Kaji D et al. J Phys. Soc. Jpn., 2004, **73**: 1738