Chinese Physics C Vol. 37, No. 8 (2013) 087001

Multi-pass, multi-bunch beam breakup for 9-cell Tesla cavities in the ERL *

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Abstract: Generally, the Energy Recovery Linac (ERL) needs specially designed high current superconducting RF cavities. In this paper, the threshold current of beam breakup for compact ERL facilities with 9-cell Tesla type cavities are investigated. The results show that it is feasible to adopt the 9-cell Tesla cavity for compact ERL test facilities with just a few cavities and beam current around 10 mA.

Key words: energy recovery linac, 9-cell Tesla cavity, beam breakup

PACS: 29.20.Ej **DOI:** 10.1088/1674-1137/37/8/087001

1 Introduction

Energy recovery linacs (ERLs) based on superconducting RF technology are suitable for running high current and low emittance electron beams with lower RF power supply than traditional linacs. Its merits indicate a broad prospect of applying ERLs to the next generation light source, high average power FEL, THz radiation and Compton back-scattering facilities [1, 2].

At Peking University, an ERL test facility, which will operate at 30 MeV and about several milliampere, is under construction. One of the key issues of the ERL is the multi-pass, multi-bunch beam breakup (BBU) caused by a higher order modes (HOMs) electromagnetic field in the RF cavities. It is the main limitation to the available beam current of ERLs. In order to suppress HOMs more efficiently, various types of superconducting cavities have been designed, such as the 5-cell cavity at BNL, the 7-cell cavity at Cornell University, the 9-cell ERL cavity at KEK/JAEA, etc. Compared with those cavities, the modules of 9-cell Tesla cavities are relatively mature after years of development and some facilities like the International Linear Collider(ILC) and European X-ray FEL have decided to adopt a 9-cell Tesla cavity in their main linacs. Although former studies show that 9-cell Tesla cavities may not be applicable for an ERL synchrotron light source which will operate with a current over 100 mA [3], they have the potential to be used in some compact ERLs with just a few cavities and an average current around 10 mA, such as the PKU-ERL test facility. In this paper, we discuss the HOMs and BBU threshold current when 9-cell Tesla cavities are placed in those compact ERLs.

2 Multi-pass, multi-bunch beam breakup

Because of the high quality factor of a superconducting cavity, HOMs excited by electron bunches may not be sufficiently suppressed. When an electron bunch enters a cavity with an excited HOM, it experiences a transverse kick and returns to the cavity with a transverse offset after traveling through the recirculating loop. This offset leads to an energy exchange between the HOM and the bunch. If the energy gain from bunches is beyond the suppression ability of the HOM coupler, HOM energy will grow and larger transverse kicks will be experienced by subsequent bunches, which will in turn lead to further growth of HOM energy. Then, a feedback loop is established and beam breakup occurs finally.

For the case of a single HOM in a single cavity, a theoretical equation of BBU threshold current can be expressed as [4]

$$I_{\mathrm{th}}=-\frac{2pc^2}{e\omega\left(\dfrac{R}{Q}\right)Q_{\mathrm{e}}M_{12}^{*}\sin(\omega T_{\mathrm{r}})}, \tag{1}$$

Received 28 August 2012

* Supported by Major State Basic Research Development Program of China (2011CB808303, 2011CB808304)

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where $\dfrac{R}{Q}$ is the shunt impedance of the HOM; Q_e is the HOM's external quality factor; ω is the HOM frequency and M_{12}^{*} is the transport line parameter:

$$M_{12}^{*}=T_{12}\cos^2\theta+\frac{1}{2}(T_{14}+T_{23})\sin 2\theta+T_{34}\sin^2\theta, \qquad (2)$$

where T_{ij} is the transport matrix element of the whole transport line; θ is the polarization angle of the HOM. Eq. (1) is only available for a single HOM in a single cavity. For ERLs with more cavities and more HOMs, computer simulations should be adopted. Some codes to calculate the threshold current of BBU have been developed: TDBBU, MATBBU and GBBU by Jefferson Lab; bi by Cornell University, etc. It has been proved that all these codes can agree well with the experimental results [5]. Here, the code bi [6] is used to calculate the threshold current of different cases when 9-cell Tesla cavities are launched in the main linac and the Elegant [7] program is used for particle tracking.

3 BBU simulation

3.1 HOMs in 9-cell Tesla cavity

According to Eq. (1), the most threatening HOMs to BBU should be the dipole modes with larger $(R/Q)Q_\mathrm{e}$. Typical simulation results for the 100 mA high-current cavity calculated by Cornell University show that the dipole HOMs should meet the demands of Eq. (3) [8]

$$(R/Q)Q_\mathrm{e}/f<1.4\times10^5\ \Omega/\mathrm{cm}^2/\mathrm{GHz}. \qquad (3)$$

In the 9-cell Tesla cavity, there are several HOMs with $(R/Q)Q_\mathrm{e}/f>1.4\times10^5$ and they are presented in Table 1.

Table 1. The 4 most threatening HOMs in a Tesla cavity.

mode No.	f/GHz	Q_e	(R/Q)/ (Ω/cm^2)	$((R/Q)Q_\mathrm{e}/f)$/ $(\Omega/\mathrm{cm}^2/\mathrm{GHz})$
1	1.7074	5×10^4	11.21	3.28×10^5
2	1.7343	2×10^4	15.51	1.79×10^5
3	1.8738	7×10^4	8.69	3.25×10^5
4	2.5751	5×10^4	23.80	4.62×10^5

3.2 Lattice configuration

The lattice configuration should be taken first of all. The transport matrix element T_{12} in Eq. (2) can be expressed in terms of β-function and phase advance $\Delta\Psi$:

$$T_{12}(\mathrm{i}\rightarrow\mathrm{f})=\gamma_\mathrm{i}\sqrt{\frac{\beta_\mathrm{i}\beta_\mathrm{f}}{\gamma_\mathrm{i}\gamma_\mathrm{f}}}\sin\Delta\Psi. \qquad (4)$$

We assume that all the cavities are fixed in a single cryomodule with no additional focusing between them. The recirculating optics was assumed to be symmetrical which means the recirculating loop (from the end of linac after acceleration to the beginning of linac before deceleration) has equal betatron phase advance in both horizontal and vertical planes and the β-function is the same both at the beginning and the end. The transport matrix of RF cavities can be described as [9]:

$$M_\mathrm{cav}=\begin{pmatrix}\cos\alpha-\sqrt{2}\sin\alpha & \sqrt{8}\dfrac{\gamma_\mathrm{i}}{\gamma'}\sin\alpha \\ -\dfrac{3}{\sqrt{8}}\dfrac{\gamma'}{\gamma_\mathrm{f}}\sin\alpha & \dfrac{\gamma_\mathrm{i}}{\gamma_\mathrm{f}}\left[\cos\alpha+\sqrt{2}\sin\alpha\right]\end{pmatrix}, \qquad (5)$$

where $\alpha=\dfrac{1}{\sqrt{8}}\ln\dfrac{\gamma_\mathrm{f}}{\gamma_\mathrm{i}}$, $\gamma_{\mathrm{i(f)}}$ is the initial (final) normalized energy of the particle. $\gamma'=qE_0\cos(\Delta\phi)/m_0c^2$ is the accelerating gradient of the RF cavity.

3.3 Simulation results

For an ERL with two 9-cell Tesla cavities, take the PKU-ERL test facility for example: 4 MeV injected beams will be accelerated to 30 MeV at the first pass. We scanned the betatron phase advance in 0–2π and calculated the BBU current. The results for such a scheme are presented in Fig. 1

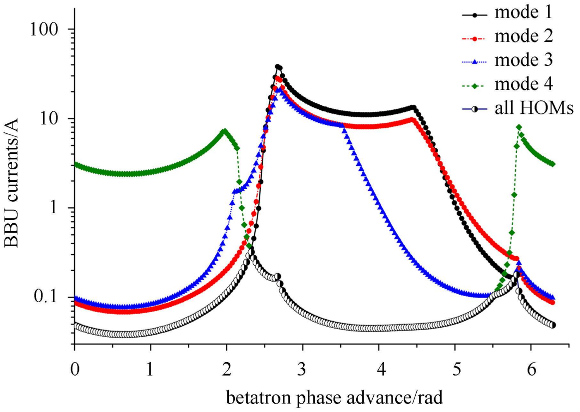

Fig. 1. The BBU current vs. the betatron phase advance of the recirculating loop. The 4 most threatening HOMs exist in each cavity.

As shown in Fig. 1, the most threatening modes in the 9-cell Tesla cavity are mode 1 and mode 4. Both of them have larger $(R/Q)Q_\mathrm{e}/f$ than other HOMs and they determine the threshold current of the 9-cell Tesla cavity. The BBU current due to some HOMs is sensitive to the betatron phase advance so that a slight shift of betatron phase advance can lead to an obvious change of BBU current. The maximum value of BBU current that can be achieved by lattice adjustment for this case is about 300 mA and the minimum value is about 35 mA so that the threshold current for this case should be about 300 mA.

For an ERL with higher energy, more 9-cell Tesla cavities are required. Along with the increasing number of cavities, the electron beam will suffer more kicks and the offset after recirculating will be larger so that more energy exchange will occur between HOMs and the beam. We set ERLs with different number of cavities and calculate their BBU current. The simulation results are shown in Fig. 2.

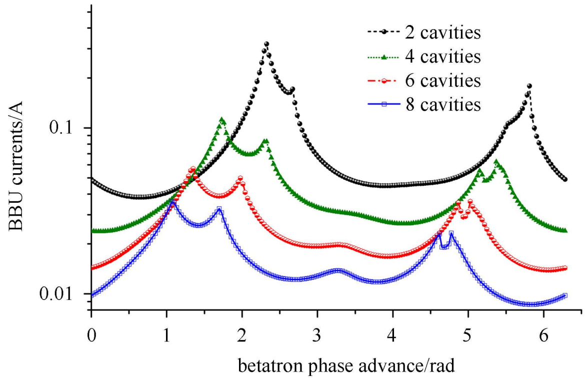

Fig. 2. The BBU current vs. the betatron phase advance of a recirculating loop for different cavity numbers. The injection energy is 4 MeV.

From Fig. 2 we can find for the case of 8 cavities (in an ILC cryomodule) with an accelerating injected beam from 4 MeV to 100 MeV (square line in Fig. 2), the BBU threshold current is just about 31 mA. That means if we want to apply a 9-cell Tesla cavity to such a scheme with average current higher than 31 mA, some additional methods should be considered.

4 Influence of inhomogeneous HOMs to BBU

During the fabrication of Tesla cavities, some errors and uncertainties are inevitable. These errors will make the dipole HOMs in real cavities slightly different from the same HOMs in ideal cavities. According to the former study [10], the frequency spread of the dipole HOMs due to the fabrication error is of the order of 10 MHz compared with the ideal cavity. For an ERL with several Tesla cavities, a frequency spread of the same HOMs between different cavities will be introduced. This frequency spread may interrupt the coupling of HOMs in different cavities and increase the BBU current. Fig. 3 and Fig. 4 show the BBU current vs. HOM frequency spread of mode1 in an 8-cavity scheme. The optics was chosen correspondingly to both the minimum and maximum current values from Fig. 2 and frequency has uniform distribution.

Clearly the worst scenario of BBU is the case that all cavities have the same HOM frequency. The HOM frequency spread between various cavities leads to a several times larger BBU current for $\sigma > 3.5$ MHz, reaching about 50 mA for this case. At the same time we can find that when $\sigma > 3.5$ MHz, the BBU current does not increase as fast as $\sigma < 3.5$ MHz. That means the ability of increasing BBU current by HOM frequency spread is limited.

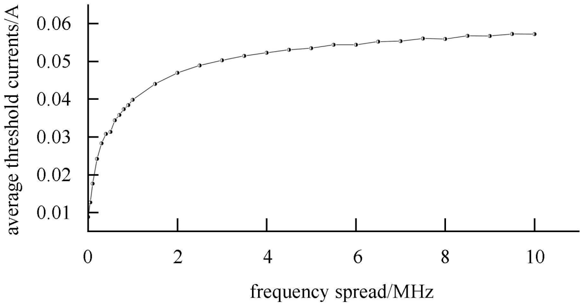

Fig. 3. The BBU current vs. the frequency spread. The lattice corresponds to the minimum value of BBU current $I_{\rm th}$=8.7 mA.

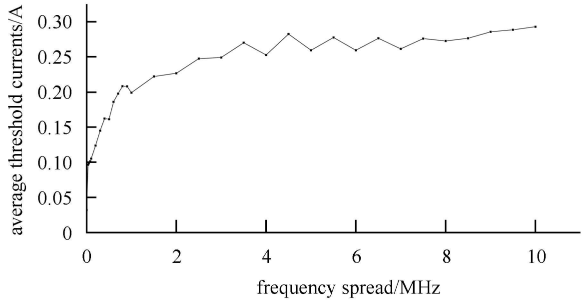

Fig. 4. The BBU current vs. the frequency spread. The lattice corresponds to the maximum value of BBU current $I_{\rm th}$=31 mA.

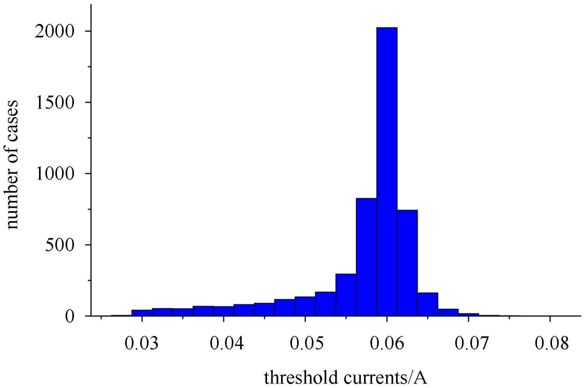

Fig. 5. Statistics of the BBU current for 10 MHz frequency spread.

The distribution of HOM frequency is random so that its effect on BBU current is also random. Fig. 5 shows the statistics of BBU current against different cases of frequency spread in the 8-cavity scheme. The HOM frequency spread behaves a uniform distribution with

$\sigma=10$ MHz around mode 1. The BBU current is calculated 5000 times and the average current of this case is about 60 mA, corresponding to the original value of BBU current $I_{\rm th}$=8.7 mA.

Apart from the frequency spread, cavity fabrication error will also introduce $Q_{\rm e}$ spread of HOMs. For example, a slight adjustment of the depth of the HOM coupler antenna insert into the cavity will cause an obvious change on $Q_{\rm e}$. The shift of $Q_{\rm e}$ due to cavity assembling uncertainties might be as large as one order of magnitude. Fig. 6 shows the statistics of the BBU current against the $Q_{\rm e}$ spread. The BBU currents were calculated by determining the thresholds in 1000 random seeds that have $Q_{\rm e}$ randomly distributed between 2.5×10^4 and 2.5×10^5. The betatron phase advance of the lattice was chosen, which corresponds to the lowest value of BBU current 8.7 mA. The average BBU current of the statistical result is about 3.1 mA.

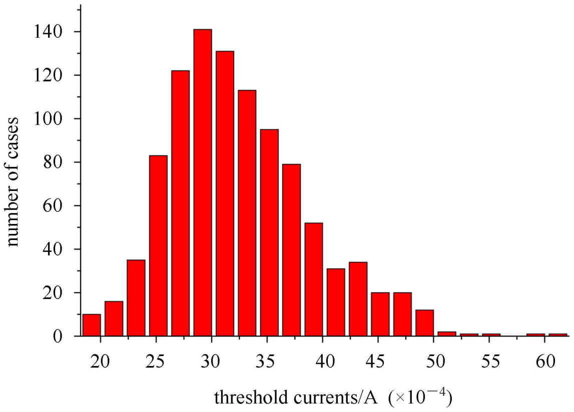

Fig. 6. Statistics of the BBU current for different cases of $Q_{\rm e}$ distribution. HOM parameters: $f\approx1.7074$ GHz, $R/Q=87.54$ Ω, $Q_{\rm e}\in(2.5\times10^4, 2.5\times10^5)$.

Typically the fabrication uncertainties of cavities will cause the $Q_{\rm e}$ of the HOM to be larger than a nominal value so that the BBU current will be smaller than that of the ideal cavities. From Eq. (1), it can be seen that cavities at lower energy section contribute more to the instability because bunches are easier to be deflected in those cavities. Therefore, it is beneficial to set the first and last cavities in ERL to be well damped $Q_{\rm e}$.

5 Methods to suppress BBU

According to Eq. (1), a smaller $(R/Q)Q_{\rm e}$ indicates a higher threshold current so that the best approach of suppressing beam breakup is to fabricate the cavities with sufficient HOM damping. For ERLs with 9-cell Tesla cavities, as shown above, several not-well-damped HOMs may increase the risk of beam breakup instability of ERLs. Some methods can be applied to reduce this risk, such as a random frequency distribution introduced to HOMs among cavities [11], or a dedicated section to adjust the betatron phase advance of recirculating loop [12]. For the former method, as shown in Fig. 3 and Fig. 4, the ability of increasing the threshold current is limited. For the latter method, we cannot ensure the value of $M^*_{12}\sin\omega T_r\approx0$ for each HOM in each cavity so that for ERLs with more cavities and HOMs the ability of suppressing BBU is also limited.

Except that, another two methods for beam optics control of BBU can also be applied, one is a reflection transport matrix which interchanges the horizontal and vertical planes betatron motion, while the other one is a rotation matrix which rotates the betatron phase plane by 90° [4]. These functions can be realized by a solenoid or a set of skew-quadruples inserted in the beamline. The coupled transport matrixes make the T_{12} and T_{34} of transport matrix zero so that $M^*_{12}=0$. The reflection matrix $M_{\rm ref}$ and rotation matrix $M_{\rm rot}$ can be expressed as

$$M_{\rm ref}=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\ M_{\rm rot}=\begin{pmatrix}0 & I\\ -I & 0\end{pmatrix}, \tag{6}$$

I is the 2×2 identity matrix. We insert a reflection section and a rotation section into the recirculating beamline of an 8-cavity scheme respectively and calculate their BBU current. The simulation results of these two configurations are shown in Fig. 7.

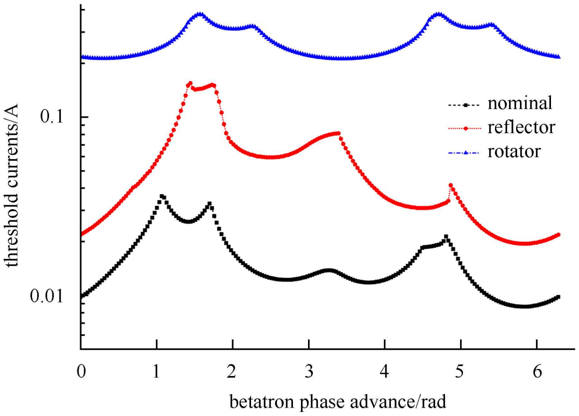

Fig. 7. (color online) The 8×9 cell Tesla cavities scheme with a reflector (red squares) or a rotator (green triangle) in the transport line.

As shown in Fig. 7, a reflector or rotator in a transport line would increase the threshold current obviously. For the reflection matrix, the BBU threshold is increased by a factor of about 5 and for the rotation matrix the factor is about 10. Theoretically these methods will lead to an infinite threshold current for a single HOM in a single cavity. However, for larger ERLs with more cavities and cryomodules, a more complicated situation of HOMs may lead to more destructive mode coupling and degrade the suppression performance. Furthermore, for ERLs of more than 2 turns, the coupling induced by these two methods will increase the difficulty of beam transportation.

6 Conclusion

The BBU threshold currents of compact ERLs with 9-cell Tesla cavities are investigated. The study shows that by adjusting the betatron phase advance of the recirculating lattice and introducing frequency spread between different cavities, a BBU threshold current of up to hundreds mA can be obtained for an ERL test facility with two 9-cell Tesla cavities, which is sufficient for the requirement of the PKU-ERL test facility. For an ERL test facility with 8×9-cell Tesla cavities, the BBU threshold current up to tens mA can be obtained, too. It is feasible to use a 9-cell Tesla cavity on some compact ERL test facilities with just a few cavities and beam current around tens mA.

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