Properties of bottomonium in a semi-relativistic model

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Abstract: Using a semi-relativistic potential model we investigate the spectra and decays of the bottomonium ($b\bar{b}$) system. The Hamiltonian of our model consists of a relativistic kinetic energy term, a vector Coulomb-like potential and a scalar confining potential. Using this Hamiltonian, we obtain a spinless wave equation, which is then reduced to the form of a single particle Schrodinger equation. The spin dependent potentials are introduced as a perturbation. The three-dimensional harmonic oscillator wave function is employed as a trial wave function and the $b\bar{b}$ mass spectrum is obtained by the variational method. The model parameters and the wave function that reproduce the the $b\bar{b}$ spectrum are then used to investigate some of their decay properties. The results obtained are then compared with the experimental data and with the predictions of other theoretical models.

Key words: bottomonium, semi-relativistic model, decay rates

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1 Introduction

The bottomonium system has come into the news recently with the ATLAS detector at the Large Hadron Collider (LHC) discovering the previously unobserved $\chi_b(3P)$ state [1]. Bottomonium consists of a bottom quark and its antiquark ($b\bar{b}$). The study of heavy quarkonium systems has played an important role in the development of quantum chromodynamics (QCD). Heavy quarkonium decays may provide useful information on understanding the nature of inter-quark forces and decay mechanisms. Since the hadron spectrum cannot be obtained directly from QCD, one has to use other methods like potential model calculations [2–6], lattice gauge theory [7–11], effective field theory [12–15], etc., to investigate the hadron spectrum and its decays. Phenomenological potential models are still one of the important tools for studying the hadron spectrum and its decays. These models are either relativistic [16–27] or nonrelativistic [28–33].

Various nonrelativistic potential models have been proposed to understand the quarkonium spectra. The nonrelativistic Hamiltonian of these quark models usually contains a kinetic energy term, a linear confinement potential and a hyperfine interaction term, which has often been taken as an effective one gluon exchange potential (OGEP). Inclusion of relativistic effects is very important for the correct description of quarkonium spectra and its decays [20, 21]. It is also suggested [28, 34] that the conventional potential models with linear confinement overestimate the masses of quarkonia in the energy region above the open-flavor thresholds. With these points in mind, in the present work, we investigate the bottomonium spectrum and decays in a semi-relativistic quark model [35]. Using the relativistic kinetic energy for the quarks, we derive a wave equation for the mesons. The potentials are then introduced by the minimal coupling scheme [36], $E\rightarrow E-V$ and $m\rightarrow m+U$. Here V and U are vector and scalar potentials respectively. This relativistic wave equation is then reduced to an effective one-body Schrodinger type equation, which is then solved by the variational technique. We take V to be the short range Coulomb type potential. U is the scalar confining potential, which is taken to be nonlinear. In our analysis we use a power law confining potential, $U=Br^{\beta}$. Similar power law potentials [5, 37–40] have been used extensively to describe the hadron spectrum within the nonrelativistic formalism.

This paper is organised as follows: the details of our model are presented in Section 2. In Section 3 we discuss some of the decay properties of S and P wave bottomonium states. In Section 4 we discuss some of the theoretical and experimental uncertainties that are involved in our calculations. Results and discussions of our present analysis are given in Section 5.

2 Theoretical model

2.1 Spin-independent formalism

For a spinless two particle relativistic system, the free

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Hamiltonian is,

$$H=\sqrt{p_1^2+m_1^2}+\sqrt{p_2^2+m_2^2}, \tag{1}$$

and the corresponding relativistic wave equation can be written as

$$H\Psi=\left(\sqrt{p_1^2+m_1^2}+\sqrt{p_2^2+m_2^2}\right)\Psi. \tag{2}$$

If Ψ is an eigenstate of H with an eigenvalue E, Eq. (2) becomes,

$$E\Psi=\left(\sqrt{p_1^2+m_1^2}+\sqrt{p_2^2+m_2^2}\right)\Psi. \tag{3}$$

In the center-of-mass frame, for particles of equal mass, the above equation reduces to,

$$E\Psi=2\ \sqrt{p^2+m^2}\ \Psi. \tag{4}$$

The vector potential (V) and the scalar potential (U) are introduced in Eq. (4) through the transformation: $E\rightarrow E-V$ and $m\rightarrow m+U$. We obtain,

$$(E-V)\ \Psi=2\ \sqrt{p^2+(m+U)^2}\ \Psi, \tag{5}$$

or,

$$(E-V)^2\ \Psi=4\ (p^2+(m+U)^2)\ \Psi. \tag{6}$$

With the replacement $p\rightarrow -\mathrm{i}\nabla$ and

$$\Psi=R_{nl}(r)Y_{lm}(\theta,\phi)=\frac{u(r)}{r}Y_{lm}(\theta,\phi)$$

and after simplification Eq. (6) reduces to,

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{l(l+1)}{r^2}+V_{\mathrm{eff}}\right]\ u(r)=E_{\mathrm{eff}}\ u(r), \tag{7}$$

where,

$$V_{\mathrm{eff}}\ =\ \frac{4U^2+8mU+2EV-V^2}{4}, \tag{8}$$

$$E_{\mathrm{eff}}\ =\ \frac{E^2-4m^2}{4}. \tag{9}$$

Eq. (7) is analogous to the single particle Schrodinger equation, which represents a particle moving in an effective potential V_{eff} with energy E_{eff}.

In the present work, we have considered the potentials of the form,

$$V(r)\ =\ -\frac{A}{r}, \tag{10}$$

$$U(r)\ =\ B\ r^{\beta}+V_0. \tag{11}$$

In order to obtain the spin averaged masses of the $\mathrm{b\bar{b}}$ system, we have solved Eq. (7) by the variational method,

$$E_{\mathrm{eff}}(\Psi)=\frac{\langle\Psi|H_{\mathrm{eff}}|\Psi\rangle}{\langle\Psi|\Psi\rangle}=\langle H_{\mathrm{eff}}\rangle, \tag{12}$$

where

$$H_{\mathrm{eff}}=-\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{l(l+1)}{r^2}+V_{\mathrm{eff}}. \tag{13}$$

Once E_{eff} is obtained, the relativistic energy, E can be evaluated using Eq. (9).

In this work, the three dimensional harmonic oscillator wave function is used as the trial wave function for obtaining the $\mathrm{c\bar{c}}$ spectrum. In Refs. [18, 41], the harmonic oscillator wave function was successfully employed for the prediction of the meson spectrum and its decays. The harmonic oscillator wave function is given by:

$$\Psi_{nlm}(r,\theta,\phi)=N\left(\frac{r}{b}\right)^l L_n^{l+\frac{1}{2}}(r^2/b^2)\exp\left(-\frac{r^2}{b^2}\right)Y_{lm}(\theta,\phi), \tag{14}$$

where N is the normalization constant,

$$|N|^2=\frac{2\ n!}{b^3\pi^{\frac{1}{2}}}\frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)!, \tag{15}$$

and $L_n^{l+\frac{1}{2}}(x)$ are the associated Laguerre polynomials. In Eq. (14), b is treated as a variational parameter, which is determined for each state by minimising the expectation value of the Hamiltonian. The obtained b value is used in the harmonic oscillator wave function, Eq. (14), to find the energy and the value of the wave function and its derivatives at the origin. The parameters used in our investigation are listed in Table 1.

Table 1. Parameters used in our model.

parameter	value
A	0.4
B	0.395 $\mathrm{GeV}^{1.5}$
β	0.5
V_0	−0.3125 GeV
m_{q}	4.7 GeV
α_{s}	0.20

2.2 Spin-dependent interactions

The spin-dependent parts (hyperfine and fine structure terms) of the Hamiltonian providing the splitting of levels in quarkonium are sensitive to the Lorentz structure of the inter-quark potential.

The spin-spin hyperfine interaction is entirely due to the Lorentz vector and is given by [42, 43]

$$H_{\mathrm{SS}}=\frac{2}{3m_{\mathrm{q}}^2}\vec{S}_{\mathrm{Q}}\cdot\vec{S}_{\bar{\mathrm{Q}}}\ \Delta V_{\mathrm{v}}(r), \tag{16}$$

where m_{q} is the mass of the quark and Δ is the Laplacian operator. $\vec{S}_{\mathrm{Q}}$ and $\vec{S}_{\bar{\mathrm{Q}}}$ are the spin operators for the quark and antiquark respectively, with

$$\langle\vec{S}_{\mathrm{Q}}\cdot\vec{S}_{\bar{\mathrm{Q}}}\rangle=\begin{cases}\frac{1}{4} & \text{for } \vec{S}=1\\ -\frac{3}{4} & \text{for } \vec{S}=0\end{cases}. \tag{17}$$

The spin-orbit potential is given by [42, 43]:

$$H_{\mathrm{LS}}=\frac{1}{2m_{\mathrm{q}}^2 r}\vec{L}\cdot\vec{S}\ \left[3\frac{\mathrm{d}}{\mathrm{d}r}V_{\mathrm{v}}(r)-\frac{\mathrm{d}}{\mathrm{d}r}V_{\mathrm{s}}(r)\right], \tag{18}$$

where $\vec{S}=\vec{S}_Q+\vec{S}_{\bar{Q}}$ is the total spin of the bound state and $\vec{L}$ is the relative angular momentum of its constituents.

The tensor potential is given by [42, 43]:

$$H_{\rm T}=\frac{1}{12m_{\rm q}^2}S_{12}\left[\frac{1}{r}\frac{\rm d}{{\rm d}r}V_{\rm v}(r)-\frac{{\rm d}^2}{{\rm d}r^2}V_{\rm v}(r)\right], \qquad (19)$$

where the spin dependent factor,

$$S_{12}=2\left[3\frac{(\vec{S}\cdot\vec{r})^2}{r^2}-\vec{S}^2\right]. \qquad (20)$$

$H_{\rm LS}$ and $H_{\rm T}$ only affect the states with $L>0$. $H_{\rm SS}$ gives the spin singlet-triplet splittings, while $H_{\rm LS}$ and $H_{\rm T}$ give the fine structure of the states. The spin-dependent interactions are added perturbatively to the previously obtained spin-averaged spectrum.

3 Decay widths of b$\bar{\rm b}$ mesons

3.1 Leptonic decay widths

The vector ($^3S_1$) states of bottomonium can decay into a lepton pair through a virtual photon. This leptonic decay width is given by the Van Royen and Weisskopff relation [44, 45],

$$\bar{\Gamma}(nS) = \frac{4\alpha^2e_{\rm b}^2|R_{\rm nS}(0)|^2}{m_{\rm nS}^2}\left(1-\frac{16\alpha_{\rm s}}{3\pi}\right), \qquad (21)$$

where $\alpha(=1/137)$ is the electromagnetic fine structure constant, $\alpha_{\rm s}$ is the QCD coupling constant, $e_{\rm b}=-1/3$ is the b quark charge, $m_{\rm nS}$ is the mass of the n^3S_1 state, $|R_{\rm nS}(0)|$ is the value of the relative nS b$\bar{\rm b}$ wavefunction at the origin. The term in the parenthesis of Eq. (21) is the first order QCD correction factor.

The QCD coupling constant $\alpha_{\rm s}$ used in our analysis is given by [46]

$$\alpha_{\rm s}(Q^2)=\frac{12\pi}{(33-2n_{\rm f})\ln(Q^2/\Lambda^2)}. \qquad (22)$$

3.2 Two-photon and two-gluon decay widths

The C even states ($^1S_0$, $^3P_0$, $^3P_2$) of bottomonium can decay into two photons and also to two gluons. The two-photon and two-gluon decay widths are sensitive to the behavior of the b$\bar{\rm b}$ wave function and its derivatives near the origin. The decay widths for b$\bar{\rm b}$ states to decay into two photons including the first-order QCD correction factors are given by [45],

$$\begin{aligned}\bar{\Gamma}_{\gamma\gamma}(^1S_0) &= \frac{12\alpha^2e_{\rm b}^4|R_{\rm nS}(0)|^2}{m_{\eta}^2}\left(1-\frac{3.4\alpha_{\rm s}}{\pi}\right),\\ \bar{\Gamma}_{\gamma\gamma}(^3P_0) &= \frac{2^4 3^3\alpha^2e_{\rm b}^4|R'_{\rm nP}(0)|^2}{m_{\chi_{\rm b0}}^4}\left(1+\frac{0.54\alpha_{\rm s}}{3\pi}\right),\\ \bar{\Gamma}_{\gamma\gamma}(^3P_2) &= \frac{2^6 3^2\alpha^2e_{\rm b}^4|R'_{\rm nP}(0)|^2}{5\ m_{\chi_{\rm b2}}^4}\left(1-\frac{16\alpha_{\rm s}}{3\pi}\right).\end{aligned} \qquad (23)$$

The two-gluon decay widths are given by [45]

$$\begin{aligned}\bar{\Gamma}_{\rm gg}(^1S_0) &= \frac{8\alpha_{\rm s}^2|R_{\rm nS}(0)|^2}{3\ m_{\eta}^2}\left(1+\frac{4.4\alpha_{\rm s}}{\pi}\right),\\ \bar{\Gamma}_{\rm gg}(^3P_0) &= \frac{2^5 3\alpha_{\rm s}^2|R'_{\rm nP}(0)|^2}{m_{\chi_{\rm b0}}^4}\left(1+\frac{10.0\alpha_{\rm s}}{\pi}\right),\\ \bar{\Gamma}_{\rm gg}(^3P_2) &= \frac{2^7\alpha_{\rm s}^2|R'_{\rm nP}(0)|^2}{5\ m_{\chi_{\rm b2}}^4}\left(1-\frac{0.1\alpha_{\rm s}}{\pi}\right).\end{aligned} \qquad (24)$$

4 Theoretical and experimental uncertainties in the model

The basic aim of the present work is to obtain a reliable estimate of both the spectra and the decay widths with the same set of parameters and to understand the uncertainties in the calculation in the frame work of the model. The model that we have employed has many basic features of the QCD and has reasonably succeeded in predicting the spectra and decay widths of hadrons as it allows direct calculations of the relevant matrix elements for each hadron.

In our present work, the Hamiltonian consists of a relativistic kinetic energy term, a Lorentz vector Coulombic potential and a power law confinement potential. We have chosen harmonic oscillator wave function as the trial wave function which does not take into account the importance of the Coulombic potential for a heavy quarkonium system [47]. The standard way of estimating the uncertainties in any model is to vary different parameters in the model. In our work, we have chosen harmonic oscillator wave function as the trial wave function which is most reliable. But, as has been pointed out the harmonic oscillator wave function does not take into account the importance of the Coulombic potential for a heavy quarkonium system [47]. Also it is known that a larger harmonic oscillator basis increases the wave function at the origin since the higher order states mixing into the wave function probe the short distance of the potential [30, 48]. But, in our present work, we have not used a diagonalization procedure to obtain the bottomonium mass spectrum. Hence there is a slight theoretical uncertainty in the value of the oscillator size parameter (b) used as a variational parameter. Also, there is theoretical uncertainty in the form of the wave function $\psi(0)$ at the origin. The $\psi(0)$ is relatively flat for linear and harmonic oscillator potentials, but it rises sharply for the Columbic potential. Ultimately, the form of the wave function at the origin has to be settled by lattice QCD calculations which will be the most reliable results as they are calculations from the first principles.

Another source of theoretical uncertainty is the mass of the bottom quark (m_b). The spectra of the quarkonium are not very sensitive to the mass of the quark, but $|\psi(0)|^2$ is strongly dependent on m_b, which is required in the estimation of leptonic decay widths. The estimated value of $|\psi(0)|^2$ is different in different models. It is larger for the Cornell than the Martin potential. For a potential which varies as r^α, $|\psi(0)|^2$ is proportional to $m_q^{3/(2+\alpha)}$ [31]. Therefore, the actual behaviour of the potential and the mass of the quark are of importance. Since $|\psi(0)|^2$ depends on the potential, it is necessary to know the potential accurately and then solve the Schrodinger equation. But to find the potential, we have only clues from the QCD (linear at long distance and Coulombic at short distance) and not the exact form. Ultimately there is only the QCD scale parameter Λ, which can be measured in a hard large Q^2 process. But, still the connection between the confinement strength and Λ is not direct, and only rigorous lattice QCD results can establish the connection. In our work, we have fitted the bottomonium spectrum and several excited states with the harmonic oscillator wave function and the same has been employed in the calculation of $|\psi(0)|^2$.

Also, in calculating the masses and decay widths of quarkonium there are ambiguities in the value of α_s. These ambiguities are both theoretical and experimental. In theory there is uncertainty in the scale μ to be considered in computing $\alpha_s(\mu^2)$. On the other hand on the experimental side, there is uncertainty in the scale parameter Λ of the QCD. To estimate the leptonic decay widths, it is necessary to know the value of α_s unambiguously. For example, for the scale $\mu=m_\psi$ and $\Lambda=0.1$ GeV, $\alpha_s=0.2$ and for $\Lambda=0.2$ GeV, $\alpha_s=0.3$. These two values lead to the correction factor for leptonic decay widths, $(1-16\alpha_s/3\pi)\approx 0.66$ or 0.49. Thus the reduction factor is very sensitive to the value of α_s [49].

Due to the above uncertainties, we can make only a qualitative prediction of the decay rates. Also there is a reduction of the leptonic decay widths due to the QCD correction factors as expected.

5 Results and discussions

In this article, we have carried out a comprehensive study of the $b\bar{b}$ spectra within a semi-relativistic formalism. Using the relativistic Hamiltonian for the quarks, we first calculated the bottomonium spectrum.

The $b\bar{b}$ spectrum obtained from our analysis is listed in Table 2. From Table 2, we see that the predicted masses of the $b\bar{b}$ states are in good agreement with the experimental results. The important results from this present work are the prediction of masses of the recently observed spin singlet states $\eta_b(1S)$, $h_b(1P)$, and $h_b(2P)$ and the spin-weighted average of the $\chi_b(3P)$ state.

Table 2. Bottomonium mass spectrum (in GeV).

state	our	Exp. [51]	[34]	[20]
$\eta_b(1S)$	9.377	9.3909 ± 0.0028	9.389	9.414
$\eta_b(2S)$	10.008		9.987	9.999
$\eta_b(3S)$	10.366		10.330	10.345
$\eta_b(4S)$	10.634			10.623
$\Upsilon(1S)$	9.430	9.46030 ± 0.00026	9.460	9.461
$\Upsilon(2S)$	10.028	10.02326± 0.00031	10.016	10.023
$\Upsilon(3S)$	10.381	10.3552± 0.0005	10.351	10.364
$\Upsilon(4S)$	10.646	10.5794± 0.0012	10.611	10.643
$\Upsilon(5S)$	10.863	10.865± 0.008	10.831	
$\Upsilon(6S)$	11.05	11.019± 0.008	11.023	
$\Upsilon(7S)$	11.215		11.193	
$h_b(1P)$	9.918		9.903	9.900
$\chi_{b0}(1P)$	9.886	9.85944± 0.00042	9.865	9.861
$\chi_{b1}(1P)$	9.916	9.89278± 0.00026	9.897	9.891
$\chi_{b2}(1P)$	9.928	9.91221± 0.00026	9.918	9.912
$h_b(2P)$	10.303		10.256	10.262
$\chi_{b0}(2P)$	10.283	10.2325± 0.0004	10.226	10.231
$\chi_{b1}(2P)$	10.299	10.25546± 0.00022	10.251	10.255
$\chi_{b2}(2P)$	10.309	10.26865± 0.00022	10.269	10.272
$h_b(3P)$	10.582		10.529	10.544
$\chi_{b0}(3P)$	10.566		10.502	10.516
$\chi_{b1}(3P)$	10.578		10.524	10.538
$\chi_{b2}(3P)$	10.587		10.540	10.553
$1\ ^1D_2$	10.196		10.152	10.158
$1\ ^3D_1$	10.189		10.145	10.149
$1\ ^3D_2$	10.195	10161.1 ± 0.6	10.151	10.157
$1\ ^3D_3$	10.200		10.156	10.163

The $\eta_b(1S)$ was first observed in the radiative decay $\Upsilon(3S)\rightarrow\gamma\eta_b(1S)$ by the BaBar Collaboration [50]. Its PDG mass is 9.390±0.0028 GeV [51]. The mass obtained from our analysis is 9.377 GeV. The P wave singlet states $h_b(1P)$ and $h_b(2P)$ were first discovered by the Belle Collaboration in the reaction $e^+e^-\rightarrow h_b(nP)\pi^+\pi^-$ [52], with masses $M[h_b(1P)] = 9898.3\pm1.1^{+1.0}_{-1.1}$ MeV and $M[h_b(2P)] = 10259.8\pm0.6^{+1.4}_{-1.0}$ MeV. The predictions from our model are $M[h_b(1P)]=9.918$ GeV and $M[h_b(2P)]=10.303$ GeV. Our predictions for the newly observed singlet states are in fairly good agreement with the experimental results.

The $\chi_b(3P)$ state was recently observed by the ATLAS detector in the proton-proton collisions at the LHC [1]. Its spin state was not resolved in this experiment and only the spin-weighted average mass of $\chi_b(3P)$ was measured, which is at 10.530±0.005(stat.)±0.009(syst.) GeV. From Table 2, we can see that the spin-weighted average mass of the $3P$ triplet states is at 10.582 GeV, in reasonably good agreement with the ATLAS measurement.

The $\Upsilon(11020)$ with $J^{PC}=1^{--}$ is assumed to be a strong candidate for the $\Upsilon(6S)$ state. Its PDG mass is 11.019±0.008 GeV. As noted in Ref. [34], the mass of $\Upsilon(6S)$ is overestimated by the quenched potential models by about 100 MeV. Our prediction for the $\Upsilon(6S)$ mass is 11.05 GeV, which is close to the PDG value. Thus the power law confining potential predicts the excited states very well.

Having obtained the mass spectra, we have used the model wave functions and model parameters to determine some of the decay properties of S and P wave bottomonia.

Table 3. Leptonic decay widths (in keV).

state	$\Gamma_{l^+l^-}$	$\bar{\Gamma}_{l^+l^-}$	Exp. [51]	[39]	[34]
$\Upsilon(1S)$	1.174	0.768	1.340± 0.018	1.587	1.60
$\Upsilon(2S)$	0.398	0.261	0.612± 0.011	0.390	0.64
$\Upsilon(3S)$	0.265	0.173	0.443± 0.008	0.211	0.44
$\Upsilon(4S)$	0.206	0.135	0.272± 0.029		0.35
$\Upsilon(5S)$	0.176	0.115	0.31± 0.07		0.29
$\Upsilon(6S)$	0.153	0.100	0.130± 0.030		0.25

The leptonic decay widths were calculated using the Van Royen-Weisskopff relation. The calculated decay widths without ($\Gamma_{l^+l^-}$) and with ($\bar{\Gamma}_{l^+l^-}$) the QCD correction factor are listed in Table 3. The two-photon and two-gluon decay widths for S and P wave states were calculated using Eqs. (4) and (5). The calculated widths without (Γ) and with ($\bar{\Gamma}$) the QCD correction factor are given in Tables 4 and 5 in comparison with the results from other theoretical models. The obtained decay widths are in reasonably good agreement with the experiment and with other theoretical models.

Table 4. Two-photon decay widths (in eV).

state	$\Gamma_{\gamma\gamma}$	$\bar{\Gamma}_{\gamma\gamma}$	[53]	[34]
$\eta_b(1S)$	396	308	300	527
$\eta_b(2S)$	133	104	140	263
$\eta_b(3S)$	88	69	100	172
$\eta_b(4S)$	69	54		105
$\chi_{b0}(1P)$	23	23	33	37
$\chi_{b2}(1P)$	6	4	7	6.6
$\chi_{b0}(2P)$	15	15	34	37
$\chi_{b2}(2P)$	4	3	8	6.7

Table 5. Two-gluon decay widths (in MeV).

state	Γ_{gg}	$\bar{\Gamma}_{gg}$	[53]	[54]
$\eta_b(1S)$	5.5	7.1	11.5	
$\eta_b(2S)$	1.9	2.4	5.2	
$\eta_b(3S)$	1.2	1.6	3.8	
$\eta_b(4S)$	1.0	1.2		
$\chi_{b0}(1P)$	0.3	0.5	0.96	0.431
$\chi_{b2}(1P)$	0.09	0.08	0.33	0.214
$\chi_{b0}(2P)$	0.2	0.3	0.99	0.122
$\chi_{b2}(2P)$	0.06	0.05	0.35	0.092

In summary, from our calculations we conclude that the semi-relativistic analysis proposed in the present model gives a reasonably good prediction for the spectra and decay rates of bottomonium in agreement with the experiment.

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