Constraints of unparticle physics parameters from K^0-$\bar{K}^0$ mixing*

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Abstract: The neutral kaon meson mixing plays an important role in the test of the Standard Model (SM) and new physics beyond it. Scale invariant unparticle physics induces a flavor changing neutral current (FCNC) transition of K^0-$\bar{K}^0$ oscillation at the tree level. In this study, we investigate the scale invariant unparticle physics effects on the K^0-$\bar{K}^0$ mixing. Based on the current experimental data, we give constraints of K^0-$\bar{K}^0$ mixing on the unparticle parameters.

Key words: neutral kaon mixing, unparticle

PACS: 12.90.+b, 14.40.Df **DOI:** 10.1088/1674-1137/37/12/123102

1 Introduction

The neutral kaon meson system has played an important role in history. The mixing-induced and indirect CP violation were first discovered in this system [1]. In the SM, the neutral kaon mixing occurs through a flavor-changing neutral current transition depicted by a box diagram at the loop level. Thus it provides an important place to test the SM and/or explore new physics beyond it. The interest in research of new physics in the neutral kaon system is extensive. Even recently, there are many new physics studies, for example, in Refs. [2–7]. At present, the experimental data on the neutral kaon mixing is very precise, and the theoretical studies, in particular the lattice calculations on the non-perturbative quantities have also improved a lot. The research on the neutral kaon system are going into a deeper level.

The purpose of the study is to explore the neutral kaon mixing within a new physics scenario called unparticle. This is an idea proposed by Georgi in Refs. [8, 9]. Scale invariance is the guiding principle in this scenario. A scale invariant matter, named unparticle, possesses some properties which are different from those of the ordinary particles. The dimension of the unparticle is in general fractional rather than an integral number. Another aspect is that the real unparticle has no definite mass. The interactions between the unparticle and the SM particles are described in the framework of low energy effective theory and lead to various interesting phenomena. The unparticle physics received intensive interest after the idea was proposed. Although this topic is currently not hot, it is still necessary to study its effects in different physical processes, because we don't know what is the right direction of new physics and which new physics model favors the real world.

Within the unparticle scenario, the neutral meson mixing, such as D^0-$\bar{D}^0$, B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ have been explored [10–13]. Only the neutral kaon mixing is not studied. Because the energy scale related to the kaon system is lower than other heavy mesons, the SM contribution should be more dominant and the new physics will play a less important role. However, the more precise data on the kaon system can provide a more stringent constraint on the new physics parameters. This is the reason why the kaon system is still an active research area. One difficult problem related to the kaon system is the long distance contribution to the mixing parameter which is not easy to evaluate. This problem becomes more serious in the kaon system than in that of the heavy meson. The aim of this study is to constrain the unparticle parameters from the neutral kaon mixing. Because there has been no similar study before, it is necessary to investigate whether the unparticle scenario can be applicable to the kaon system.

2 K^0-$\bar{K}^0$ mixing in the SM

At first, we give notations for the neutral kaon system. In the K^0-$\bar{K}^0$ system, the oscillation between the two neutral kaon mesons is described by the equation

$$\mathrm{i}\frac{\mathrm{d}\psi(t)}{\mathrm{d}t}=\hat{H}\psi(t), \qquad \psi(t)=\begin{pmatrix}|K^0(t)\rangle\\ |\bar{K}^0(t)\rangle\end{pmatrix}, \tag{1}$$

Received 25 February 2013, Revised 5 June 2013

* Supported by NNSFC (11175091)

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where

$$\hat{H}=\hat{M}-\mathrm{i}\frac{\hat{\Gamma}}{2}=\begin{pmatrix} M-\mathrm{i}\frac{\Gamma}{2} & M_{12}-\mathrm{i}\frac{\Gamma_{12}}{2} \\ M_{12}^{*}-\mathrm{i}\frac{\Gamma_{12}^{*}}{2} & M-\mathrm{i}\frac{\Gamma}{2} \end{pmatrix}, \tag{2}$$

with M_{ij} and Γ_{ij} being the transition matrix elements. We have assumed the CPT conservation and hermiticity for the matrices $\hat{M}$ and $\hat{\Gamma}$.

After diagonalizing the system and using the convention $CP|K^0\rangle=-|\bar{K}^0\rangle$, $CP|\bar{K}^0\rangle=-|K^0\rangle$, we obtain the eigenstates:

$$|K_{\mathrm{L,S}}\rangle=\frac{(1+\bar{\varepsilon})|K^0\rangle\pm(1-\bar{\varepsilon})|\bar{K}^0\rangle}{\sqrt{2(1+|\bar{\varepsilon}|^2)}}, \tag{3}$$

where the mixing parameter $\bar{\varepsilon}$ is a small complex quantity given by

$$\frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}}=\sqrt{\frac{M_{12}^{*}-\mathrm{i}\frac{1}{2}\Gamma_{12}^{*}}{M_{12}-\mathrm{i}\frac{1}{2}\Gamma_{12}}}. \tag{4}$$

In the SM, the oscillation between the flavor states K^0 and $\bar{\mathrm{K}}^0$ are caused by weak interactions, thus the above eigenstates are called the weak eigenstates. In new physics which is beyond the SM, the eigenstates represent generalized states including both the SM and new physics effects.

The eigenvalues associated with the eigenstates are

$$M_{\mathrm{L,S}}=M\pm\mathrm{Re}Q,\quad \Gamma_{\mathrm{L,S}}=\Gamma\mp2\mathrm{Im}Q, \tag{5}$$

where

$$Q=\sqrt{\left(M_{12}-\mathrm{i}\frac{1}{2}\Gamma_{12}\right)\left(M_{12}^{*}-\mathrm{i}\frac{1}{2}\Gamma_{12}^{*}\right)}. \tag{6}$$

We thus get

$$\begin{aligned}\Delta M&=M(K_{\mathrm{L}})-M(K_{\mathrm{S}})=2\mathrm{Re}Q,\\ \Delta\Gamma&=\Gamma(K_{\mathrm{L}})-\Gamma(K_{\mathrm{S}})=-4\mathrm{Im}Q.\end{aligned} \tag{7}$$

Since $\bar{\varepsilon}$ is small, at the order of $\mathcal{O}(10^{-3})$, the below relations are reasonable,

$$\mathrm{Im}M_{12}\ll\mathrm{Re}M_{12},\qquad \mathrm{Im}\Gamma_{12}\ll\mathrm{Re}\Gamma_{12}. \tag{8}$$

It should be noted that the above relations are still applicable in the unparticle physics. Thus, ignoring the small imaginary part of M_{12} and Γ_{12}, we can get a good approximation:

$$\Delta M_{\mathrm{K}}\cong2\mathrm{Re}M_{12},\quad \Delta\Gamma_{\mathrm{K}}\cong2\mathrm{Re}\Gamma_{12}. \tag{9}$$

In the SM, the matrix elements M_{12},Γ_{12} of the K^0-$\bar{\mathrm{K}}^0$ mixing contains both the short distance (SD) and long distance (LD) contributions:

$$M_{12}^{\mathrm{SM}}=M_{12}^{\mathrm{SD}}+M_{12}^{\mathrm{LD}},\quad \Gamma_{12}^{\mathrm{SM}}=\Gamma_{12}^{\mathrm{SD}}+\Gamma_{12}^{\mathrm{LD}}. \tag{10}$$

An important formula for $\bar{\varepsilon}$ including both the SD and LD contributions of the SM is [14]

$$\bar{\varepsilon}=\kappa_{\varepsilon}\frac{\mathrm{e}^{\mathrm{i}\phi_{\varepsilon}}}{\sqrt{2}}\frac{\mathrm{Im}M_{12}}{\Delta M_{\mathrm{K}}}, \tag{11}$$

where the phase $\phi_{\varepsilon}=(43.51\pm0.05)^{\circ}$ and factor $\kappa_{\varepsilon}=0.94\pm0.02$. Thus, the mixing parameter $\bar{\varepsilon}$ is approximated by

$$\bar{\varepsilon}\approx\mathrm{e}^{\mathrm{i}\phi_{\varepsilon}}\sin\phi_{\varepsilon}\frac{\mathrm{Im}M_{12}}{\Delta M_{\mathrm{K}}}. \tag{12}$$

At the quark level, the flavor changing neutral current transitions between K^0 and $\bar{\mathrm{K}}^0$ are induced through a box diagram with exchange of the intermediate up type quarks. This is the short distance contribution to M_{12}. From [15], the formula is given as

$$\begin{aligned}M_{12}^{\mathrm{SD}}&=\frac{G_{\mathrm{F}}^2}{12\pi^2}f_{\mathrm{K}}^2\hat{B}_{\mathrm{K}}m_{\mathrm{K}}m_{\mathrm{W}}^2\big[\lambda_{\mathrm{c}}^{*2}\eta_1S_0(x_{\mathrm{c}})+\lambda_{\mathrm{t}}^{*2}\eta_2S_0(x_{\mathrm{t}})\\&\quad+2\lambda_{\mathrm{c}}^{*}\lambda_{\mathrm{t}}^{*}\eta_3S_0(x_{\mathrm{c}},x_{\mathrm{t}})\big],\end{aligned} \tag{13}$$

where f_{K} is the K-meson decay constant, m_{K} is the K-meson mass, and $\hat{B}_{\mathrm{K}}$ is the renormalization group invariant parameter. The parameters $\lambda_{\mathrm{i}}=V_{\mathrm{is}}^{*}V_{\mathrm{id}}$ where V_{is} and V_{id} are the CKM matrix elements. The functions S_0 are

$$\begin{aligned}S_0(x_{\mathrm{t}})&=\frac{4x_{\mathrm{t}}-11x_{\mathrm{t}}^2+x_{\mathrm{t}}^3}{4(1-x_{\mathrm{t}})^2}-\frac{3x_{\mathrm{t}}^3\ln x_{\mathrm{t}}}{2(1-x_{\mathrm{t}})^3},\\ S_0(x_{\mathrm{c}})&=x_{\mathrm{c}},\\ S_0(x_{\mathrm{c}},x_{\mathrm{t}})&=x_{\mathrm{c}}\left[\ln\frac{x_{\mathrm{t}}}{x_{\mathrm{c}}}-\frac{3x_{\mathrm{t}}}{4(1-x_{\mathrm{t}})}-\frac{3x_{\mathrm{t}}^2\ln x_{\mathrm{t}}}{4(1-x_{\mathrm{t}})^2}\right].\end{aligned} \tag{14}$$

The values of η_i are taken to be the next-to-leading-order (NLO) results given in [16–19]

$$\eta_1=1.38\pm0.20,\ \eta_2=0.57\pm0.01,\ \eta_3=0.47\pm0.04. \tag{15}$$

For the Γ_{12}, the SD contribution arises from the virtual quark interactions and is expected to be very small. The LD contribution coming from the intermediate hadron states should dominate. However, the theoretical uncertainties due to the non-perturbative dynamics are very large. Usually, it's difficult to separate the hadron uncertainties and the new physics effects. As will be shown, the unparticle parameters can be constrained from the term M_{12}. So, in the discussion later, we will not use the Γ_{12} in order to reduce the theoretical uncertainties.

In the above derivations and the forthcoming discussions in the unparticle physics, the below relations are useful

$$\langle \bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma^\mu(1-\gamma_5)d|K^0\rangle = \frac{8}{3}f_{\rm K}^2 m_{\rm K}^2\hat{B}_{\rm K},$$

$$\langle \bar{K}^0|\bar{s}(1-\gamma_5)d\bar{s}(1-\gamma_5)d|K^0\rangle = -\frac{5}{3}f_{\rm K}^2 m_{\rm K}^2\hat{B}_{\rm K} \times\left(\frac{m_{\rm K}}{m_{\rm s}+m_{\rm d}}\right)^2, \quad (16)$$

where $m_{\rm s,d}$ are the strange and down quark masses.

3 K^0-$\bar{K}^0$ mixing in unparticle physics

In this section, we turn to study the K^0-$\bar{K}^0$ mixing in unparticle physics. In the low energy effective theory, unparticle fields will emerge below an energy scale $\Lambda_{\mathcal{U}}$ [8]. The relevant low energy effective interaction for the s and d quarks is described by

$$\frac{C_{\rm S}^{\rm ds}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{s}\gamma_\mu(1-\gamma_5)d\ \partial^\mu O_{\mathcal{U}}+\frac{C_{\rm V}^{\rm ds}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\bar{s}\gamma_\mu(1-\gamma_5)d\ O_{\mathcal{U}}^\mu+\text{h.c.}, \quad (17)$$

where $O_{\mathcal{U}}$ and $O_{\mathcal{U}}^\mu$ denote operators of the scalar and vector unparticle fields respectively. The $C_{\rm S}$ and $C_{\rm V}$ are dimensionless coefficients and they depend on quark and lepton flavors in general. Since only quarks s and d are of concern in this study, we will simplify $C_{\rm S}^{\rm ds}\to C_{\rm S}$ and $C_{\rm V}^{\rm ds}\to C_{\rm V}$ in the later discussions.

The propagators for the scalar and vector unparticle fields with the time-like momentum P are given by [9, 20]

$$\int {\rm d}^4x{\rm e}^{{\rm i}P\cdot x}\langle 0|TO_{\mathcal{U}}(x)O_{\mathcal{U}}(0)|0\rangle = {\rm i}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{1}{(P^2+{\rm i}\epsilon)^{2-d_{\mathcal{U}}}}{\rm e}^{-{\rm i}d_{\mathcal{U}}\pi},$$

$$\int {\rm d}^4x{\rm e}^{{\rm i}P\cdot x}\langle 0|TO_{\mathcal{U}}^\mu(x)O_{\mathcal{U}}^\nu(0)|0\rangle = {\rm i}\frac{A_{d_{\mathcal{U}}}{\rm e}^{-{\rm i}d_{\mathcal{U}}\pi}}{2\sin(d_{\mathcal{U}}\pi)}\frac{-g^{\mu\nu}+2(d_{\mathcal{U}}-2)P^\mu P^\nu/(d_{\mathcal{U}}-1)P^2}{(P^2+{\rm i}\epsilon)^{2-d_{\mathcal{U}}}}, \quad (18)$$

where

$$A_{d_{\mathcal{U}}}=\frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \quad (19)$$

Here we consider the vector unparticle which is not transverse: $\partial_\mu O_{\mathcal{U}}^\mu\neq 0$ unless $d_{\mathcal{U}}=0$ due to unitarity constraint [21]. Another property of $d_{\mathcal{U}}$ requested by the unitarity constraint is that $d_{\mathcal{U}}\geqslant 3$ for vector unparticles and $d_{\mathcal{U}}\geqslant 1$ for scalar unparticles [20]. The scale dimension $d_{\mathcal{U}}$ is fractional in general, and it cannot be integral (except $d_{\mathcal{U}}=1$, where the singularity is canceled by $A_{d_{\mathcal{U}}}$ in the nominator) due to the singularity of the function $\sin(d_{\mathcal{U}}\pi)$ in the denominator. The factor ${\rm e}^{-{\rm i}d_{\mathcal{U}}\pi}$ provides a CP conserving phase which produces peculiar interference effects in high energy scattering processes [9], Drell-Yan process [22] and CP violation in B decays [12].

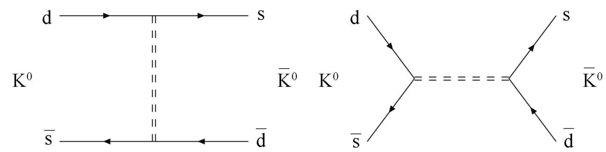

Fig. 1. The Feynman graphs of the K^0-$\bar{K}^0$ mixing in unparticle physics. The double dashed lines represent the unparticle fields.

Unlike the SM where the $\Delta S=2$ FCNC transitions occurred at loop level, the unparticle contribution can induce tree level FCNC transitions between K^0 and $\bar{K}^0$ which are depicted in Fig. 1. There are two diagrams corresponding to t- and s-channel unparticle exchanges. For the s-channel, the momentum of the unparticle $P^2=m_{\rm K}^2$. For the t-channel, the momentum of the unparticle is not fixed which introduces an uncertainty in the calculation. Because strange quark mass is much greater than that of the down quark, $m_{\rm s}\gg m_{\rm d}$, the momentum of the kaon meson is mostly carried by the strange quark. The momentum of the exchanged unparticle will be at the order of $m_{\rm K}$, and we make an approximation that $P^2\approx m_{\rm K}^2$.

Using the Feynman rules given above, we obtain the final expression for the transition matrices in the unparticle physics. For the scalar unparticle,

$$M_{12}^{\mathcal{U}} = \frac{5C_{\rm S}^2}{12}\frac{f_{\rm K}^2\hat{B}_{\rm K}A_{d_{\mathcal{U}}}}{m_{\rm K}}\left(\frac{m_{\rm K}}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}}\left(\frac{m_{\rm K}}{m_{\rm s}+m_{\rm d}}\right)^2\cot(d_{\mathcal{U}}\pi),$$

$$\Gamma_{12}^{\mathcal{U}} = \frac{5C_{\rm S}^2}{6}\frac{f_{\rm K}^2\hat{B}_{\rm K}A_{d_{\mathcal{U}}}}{m_{\rm K}}\left(\frac{m_{\rm K}}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}}\left(\frac{m_{\rm K}}{m_{\rm s}+m_{\rm d}}\right)^2, \quad (20)$$

and

$$M_{12}^{\mathcal{U}} = \frac{C_{\rm V}^2}{4}\frac{f_{\rm K}^2\hat{B}_{\rm K}A_{d_{\mathcal{U}}}}{m_{\rm K}}\left(\frac{m_{\rm K}}{\Lambda_{\rm u}}\right)^{2d_{\mathcal{U}}-2}\left[\frac{8}{3}-\frac{10(d_{\mathcal{U}}-2)}{3(d_{\mathcal{U}}-1)}\left(\frac{m_{\rm K}}{m_{\rm s}+m_{\rm d}}\right)^2\right]\cot(d_{\mathcal{U}}\pi),$$

$$\Gamma_{12}^{\mathcal{U}} = \frac{C_{\rm V}^2}{2}\frac{f_{\rm K}^2\hat{B}_{\rm K}A_{d_{\mathcal{U}}}}{m_{\rm K}}\left(\frac{m_{\rm K}}{\Lambda_{\rm u}}\right)^{2d_{\mathcal{U}}-2}\left[\frac{8}{3}-\frac{10(d_{\mathcal{U}}-2)}{3(d_{\mathcal{U}}-1)}\left(\frac{m_{\rm K}}{m_{\rm s}+m_{\rm d}}\right)^2\right], \quad (21)$$

for the vector unparticle. For both cases, we have

$$M_{12}^{\mathcal{U}}=\frac{\Gamma_{12}^{\mathcal{U}}}{2}\cot(d_{\mathcal{U}}\pi). \tag{22}$$

The above relation has been given in Ref. [13].

For the mixing parameter $\bar{\varepsilon}$, it is straightforward to obtain

$$\bar{\varepsilon}^{\mathcal{U}}=\mathrm{e}^{\mathrm{i}\phi_\varepsilon}\sin\phi_\varepsilon\frac{\mathrm{Im}M_{12}^{\mathcal{U}}}{\Delta M_{\mathrm{K}}^{\mathrm{exp}}}, \tag{23}$$

where we have used the $\Delta M_{\mathrm{K}}^{\mathrm{exp}}$ to replace the ΔM_{K} in Eq. (12). The above formula is applicable for both the scalar and vector unparticles.

4 Constraints of the unparticle parameters

In the K^0-$\bar{\mathrm{K}}^0$ system, the mass difference ΔM_{K}, width difference $\Delta\Gamma_{\mathrm{K}}$ and the CP violating parameter $\bar{\varepsilon}$ are the most important experimental parameters. We will make use of these data to constrain the phenomenological parameters of the unparticle physics. From PDG [23], the experimental results are

$$\begin{aligned}\Delta M_{\mathrm{K}}^{\mathrm{exp}}&=(3.483\pm0.006)\times10^{-15}\ \mathrm{GeV},\\ |\bar{\varepsilon}|^{\mathrm{exp}}&=(2.228\pm0.001)\times10^{-3}.\end{aligned} \tag{24}$$

The relations between the experiment and theory have been obtained as

$$\begin{aligned}\Delta M_{\mathrm{K}}^{\mathrm{exp}}&=2\mathrm{Re}\left(M_{12}^{\mathrm{SD}}+M_{12}^{\mathrm{LD}}+M_{12}^{\mathcal{U}}\right),\\ |\bar{\varepsilon}|^{\mathrm{exp}}&=\frac{|\mathrm{e}^{\mathrm{i}\phi_\varepsilon}|}{\Delta M_{\mathrm{K}}^{\mathrm{exp}}}\left[\frac{\kappa_\varepsilon}{\sqrt{2}}\mathrm{Im}M_{12}^{\mathrm{SD}}+\sin\phi_\varepsilon\mathrm{Im}M_{12}^{\mathcal{U}}\right].\end{aligned} \tag{25}$$

The SD contribution of M_{12}^{SM} is given in Eq. (13) and can be calculated reliably. The main uncertainty in the SM comes from the parameter $\hat{B}_{\mathrm{K}}$. We use the value $\hat{B}_{\mathrm{K}}=0.724\pm0.024$ given in Ref. [24] calculated from a lattice method. For the LD part $\mathrm{Re}M_{12}^{\mathrm{LD}}$, it is taken from [14]:

$$\begin{aligned}\mathrm{Re}M_{12}^{\mathrm{LD}}&=\frac{\Delta M_{\mathrm{K}}^{\mathrm{LD}}}{2},\\ \frac{\Delta M_{\mathrm{K}}^{\mathrm{LD}}}{\Delta M_{\mathrm{K}}^{\mathrm{exp}}}&\approx0.1\pm0.2.\end{aligned} \tag{26}$$

After subtracting the SM contribution, the remainder is the new physics effect. Thus, we can know the unparticle contributions $\mathrm{Re}M_{12}^{\mathcal{U}}$ and $\mathrm{Im}M_{12}^{\mathcal{U}}$. Using Eqs. (20, 21), we obtain the constraints of C_{S}, C_{V} as follows. For the coupling coefficients C_{S}^2 and C_{V}^2, they are obtained as

$$C_{\mathrm{S}}^2=\frac{12(m_{\mathrm{s}}+m_{\mathrm{d}})^2\sin(d_{\mathcal{U}}\pi)M_{12}^{\mathcal{U}}}{5A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2m_{\mathrm{K}}}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}},\quad C_{\mathrm{V}}^2=\frac{4m_{\mathrm{K}}\sin(d_{\mathcal{U}}\pi)M_{12}^{\mathcal{U}}}{A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2\left[\frac{8}{3}-\frac{10(d_{\mathcal{U}}-2)}{3(d_{\mathcal{U}}-1)}\left(\frac{m_{\mathrm{K}}}{m_{\mathrm{s}}+m_{\mathrm{d}}}\right)^2\right]}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}}. \tag{27}$$

One needs to note that Eqs. (8, 9) are used in deriving the above equations. The vector coupling is more suppressed due to an additional factor $\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^2$. Similarly, the real and imaginary parts of the scalar coupling C_{S}^2 are given by

$$\begin{aligned}\mathrm{Re}C_{\mathrm{S}}^2&=\frac{12(m_{\mathrm{s}}+m_{\mathrm{d}})^2\tan(d_{\mathcal{U}}\pi)\mathrm{Re}M_{12}^{\mathcal{U}}}{5A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2m_{\mathrm{K}}}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}},\\ \mathrm{Im}C_{\mathrm{S}}^2&=\frac{12(m_{\mathrm{s}}+m_{\mathrm{d}})^2\tan(d_{\mathcal{U}}\pi)\mathrm{Im}M_{12}^{\mathcal{U}}}{5A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2m_{\mathrm{K}}}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}},\end{aligned} \tag{28}$$

and for the vector coupling C_{V}^2, the real and imaginary parts are given by

$$\begin{aligned}\mathrm{Re}C_{\mathrm{V}}^2&=\frac{4m_{\mathrm{K}}\tan(d_{\mathcal{U}}\pi)\mathrm{Re}M_{12}^{\mathcal{U}}}{A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2\left[\frac{8}{3}-\frac{10(d_{\mathcal{U}}-2)}{3(d_{\mathcal{U}}-1)}\left(\frac{m_{\mathrm{K}}}{m_{\mathrm{s}}+m_{\mathrm{d}}}\right)^2\right]}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}},\\ \mathrm{Im}C_{\mathrm{V}}^2&=\frac{4m_{\mathrm{K}}\tan(d_{\mathcal{U}}\pi)\mathrm{Im}M_{12}^{\mathcal{U}}}{A_{d_{\mathcal{U}}}\hat{B}_{\mathrm{K}}f_{\mathrm{K}}^2\left[\frac{8}{3}-\frac{10(d_{\mathcal{U}}-2)}{3(d_{\mathcal{U}}-1)}\left(\frac{m_{\mathrm{K}}}{m_{\mathrm{s}}+m_{\mathrm{d}}}\right)^2\right]}\left(\frac{m_{\mathrm{K}}}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}}.\end{aligned} \tag{29}$$

4.1 Input parameters

Here, we collect all the input parameters used in the numerical analysis. Most parameters are taken from PDG [23].

CKM parameters (A, λ, $\overline{\rho}$, $\overline{\eta}$):

$$A=0.804^{+0.022}_{-0.015},\quad \lambda=0.2253\pm0.0007, \qquad (30)$$

$$\overline{\rho}=0.132^{+0.022}_{-0.014},\quad \overline{\eta}=0.341\pm0.013. \qquad (31)$$

Decay constant of kaon meson:

$$f_{\rm K}=160\ {\rm MeV}. \qquad (32)$$

Quark and gauge boson masses:

$$m_{\rm d}=4.1-5.8\ {\rm MeV},\quad m_{\rm s}=101^{+29}_{-21}\ {\rm MeV},$$
$$m_{\rm c}=1.27^{+0.07}_{-0.09}\ {\rm GeV},\quad m_{\rm W}=80.384\pm0.014\ {\rm GeV}, \qquad (33)$$
$$m_{\rm t}=171.2\pm0.9\pm1.3\ {\rm GeV}.$$

4.2 Bound on the unparticle parameters

After the above preparations, we are now ready to discuss the numerical results. The phenomenological parameters of unparticle physics are: scale dimension $d_{\mathcal{U}}$, energy scale $\Lambda_{\mathcal{U}}$ and the coupling coefficients $C_{\rm S}(C_{\rm V})$. At first, we provide an estimate on the magnitude of the scalar coupling parameter $C_{\rm S}$ from the neutral kaon mixing and then compare it with the values constrained from the neutral B and D systems. As [11], we fix the energy scale and dimension by $\Lambda_{\mathcal{U}}=1$ TeV and $d_{\mathcal{U}}=3/2$. The numerical results of the absolute value of the scalar coupling parameter $|C_{\rm S}|$ are given in Table 1. We also include the upper bound of the coupling from neutral B and D systems given in Ref. [11] for comparison. Because the unitarity constraint for the vector unparticle is not considered in Ref. [11], it is impossible to compare the results for the vector coupling coeffients. Thus, only the scalar coupling parameter is given. From Table 1, the scalar coupling coefficient $C_{\rm S}$ is constrained to be $C_{\rm S}=7.1\times10^{-3}$. Under the assumption that the coupling parameter is flavor independent, the kaon mixing provides more stringent constraints on unparticle coupling than other systems.

As a next step, we consider the general case where the scale dimension is varied. At present, the exact knowledge about the range of the scale dimension is still

Table 1. The constraints on the coupling parameter $|C_{\rm S}|$ with $d_{\mathcal{U}}=3/2$ and $\Lambda_{\mathcal{U}}=1$ TeV.

	from B-system	from D-system	from K-system
$\lvert C_{\rm S}\rvert$	3.4×10^{-2}	2.1×10^{-2}	7.1×10^{-3}

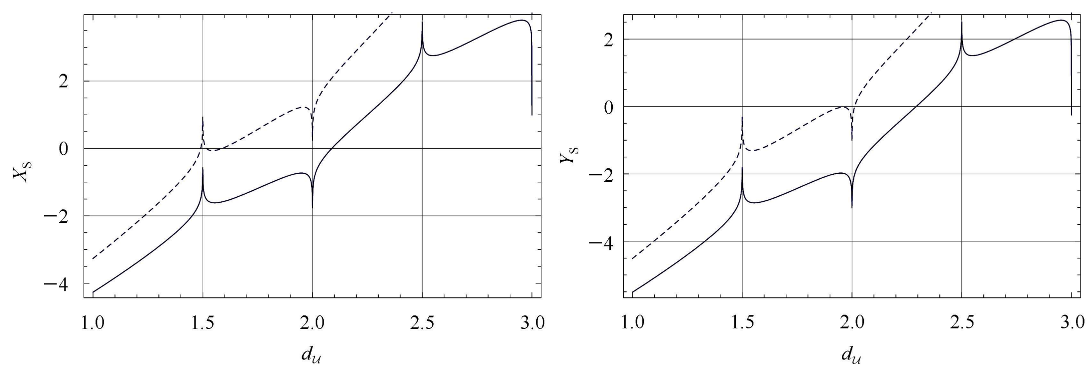

Fig. 2. The scalar unparticle coupling parameter $C_{\rm S}$ versus the scale dimension $d_{\mathcal{U}}$ where the vertical variable $X_{\rm S}=\log_{10}\sqrt{|{\rm Re}C^2_{\rm S}|}$, $Y_{\rm S}=\log_{10}\sqrt{|{\rm Im}C^2_{\rm S}|}$. The solid line is given for $\Lambda_{\mathcal{U}}=1$ TeV and the dashed for $\Lambda_{\mathcal{U}}=10$ TeV.

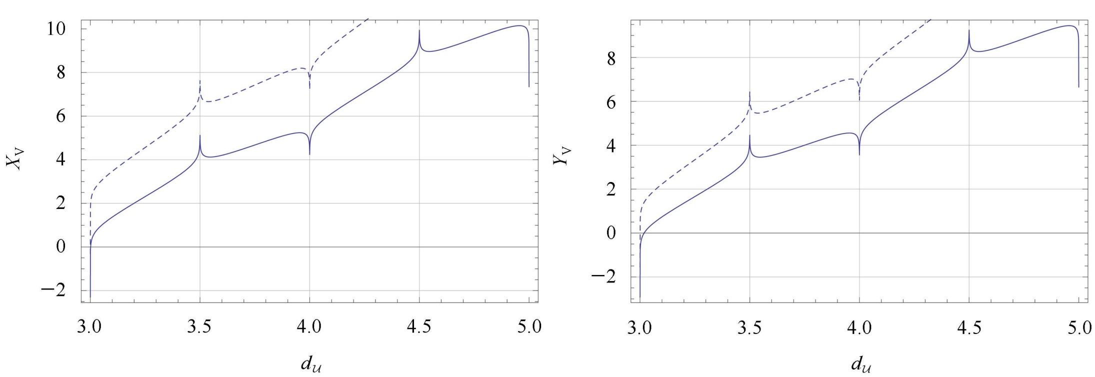

Fig. 3. The scalar unparticle coupling parameter $C_{\rm V}$ versus the scale dimension $d_{\mathcal{U}}$ where the vertical variable $X_{\rm V}=\log_{10}\sqrt{|{\rm Re}C^2_{\rm V}|}$, $Y_{\rm V}=\log_{10}\sqrt{|{\rm Im}C^2_{\rm V}|}$. The solid line is given for $\Lambda_{\mathcal{U}}=1$ TeV and the dashed for $\Lambda_{\mathcal{U}}=10$ TeV.

unknown. As we have pointed out before, unitarity constraints request $d_{\mathcal{U}} \geqslant 1$ for scalar and $d_{\mathcal{U}} \geqslant 3$ for vector unparticles. For the coupling parameters $C_{\rm S}(C_{\rm V})$, their real and imaginary parts will be explored separately unlike the analysis in Refs. [11, 12]. Because the coupling coefficients $C_{\rm S}(C_{\rm V})$ are very sensitive to the scale dimension $d_{\mathcal{U}}$ and change rapidly as $d_{\mathcal{U}}$ varies, we introduce logarithmic functions $X_{\rm (S,V)}=\log_{10}\sqrt{|{\rm Re}C^2_{\rm (S,V)}|}$ and $Y_{\rm (S,V)}=\log_{10}\sqrt{|{\rm Im}C^2_{\rm (S,V)}|}$ (here the subscripts "S, V" represent the scalar and vector cases). The numerical results of the functions $X_{\rm (S,V)}$ and $Y_{\rm (S,V)}$ for the scalar and vector unparticle versus the scale dimension $d_{\mathcal{U}}$ are plotted in Figs. 2 and 3. The range of the dimension is chosen to be $1<d_{\mathcal{U}}<3$ for the scalar and $3<d_{\mathcal{U}}<5$ for the vector cases.

Since $X_{\rm (S,V)}$ and $Y_{\rm (S,V)}$ represent the order of ${\rm Re}C_{\rm (S,V)}$ and ${\rm Im}C_{\rm (S,V)}$, negative values mean that the coupling coefficients are smaller than 1. If the coupling parameters are too large, one may meet the non-perturbative problem. If we require that the coupling parameters are smaller than 1, we obtain $1<d_{\mathcal{U}}<2$ for scalar unparticle. For the vector unparticle, $d_{\mathcal{U}}$ has to lie very close to 3. This unnatural thing indicates that the vector unparticle contribution is either too small or a large coupling parameter is required. From Figs. 2 and 3, it is shown that the values of X are larger than Y by about 1–2, thus the magnitude of the real part of the coupling parameters $C_{\rm (S,V)}$ is larger than their imaginary part by 1–2 orders for both the scalar and vector unparticles. The physical reason is that the real part is proportional to $\Delta M_{\rm K}^{\rm exp}$ while the imaginary part contributes to the small kaon mixing parameter $\bar{\varepsilon}$. Another property of these figures is that the parameters X and Y are increasing in nearly the whole range except at the integral and half integral points of $d_{\mathcal{U}}$. This is because the $\tan(d_{\mathcal{U}}\pi)$ function will break to $+\infty$ or $-\infty$ when the dimension is getting half integral, or become zero when the dimension is integral. In both cases there will be no constraints on $C_{\rm (S,V)}$. The dependence of $C_{\rm (S,V)}$ on the energy scale $\Lambda_{\mathcal{U}}$ is simple because $C_{\rm (S,V)}$ are proportional to $\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}$ or $\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}$. This can also be seen clearly from figures where we give two cases of $\Lambda_{\mathcal{U}}$=1 TeV and $\Lambda_{\mathcal{U}}$=10 TeV.

5 Conclusions and discussions

In this study, the new physics effects from the scale invariant unparticle sector on the K^0-$\bar{K}^0$ mixing are explored. The SM contribution, in particular the long distance part, will produce large uncertainties which are not under good control. This difficulty exists for any new physics search in the kaon system. We have considered the long distance contributions in a simple and maybe a slightly crude way. This treatment can be improved in the future if we have better knowledge of the non-perturbative hadron dynamics.

With the unparticle scenario, the flavor changing neutral current transitions of K^0-$\bar{K}^0$ mixing can occur at tree level. Thus, the neutral kaon system provides a sensible probe to unparticle physics. We observe that the kaon system gives very stringent constraint on the parameters. The coupling parameter for the scalar unparticle and quarks is obtained to be at the order of 10^{-3}. For the vector unparticle, if the unitarity condition is imposed, there is nearly no parameter space to let the coupling coefficient be smaller than 1. When the scale dimension $d_{\mathcal{U}}$ is larger than 3, the vector unparticle contribution will be small, otherwise, large coupling parameters are required which may induce a non-perturbative problem. Our numerical results show that the coupling parameters are very sensitive to the choice of the scale dimension.

The unparticle physics effects on the neutral B and D mixing had been studied a lot in literature but there had been no study on the neutral kaon system up to now. Our results show that the neutral kaon is also important for testing the unparticle scenario.

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