Effects of inhomogeneous magnetic fields and different Dzyaloshinskii-Moriya interaction on entanglement and teleportation in a two-qubit Heisenberg XYZ chain*

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Abstract: In this paper we calculate the thermal entanglement and teleportation in a two-qubit Heisenberg XYZ chain with the different Dzyaloshinskii-Moriya interaction and inhomogeneous magnetic fields. The analytical expressions of the concurrence and the average fidelity are obtained for this model. We have shown that the quantum phase transition occurs in the system and the quantum phase transition point depends on the inhomogeneity of magnetic fields. We compare the x-component Dzyaloshinskii-Moriya interaction with the z-component Dzyaloshinskii-Moriya interaction on the effects of quantum teleportation. It is found that we can take Dzyaloshinskii-Moriya interaction as one of the effective control parameters for the teleportation manipulation.

Key words: entanglement, teleportation, different Dzyaloshinskii-Moriya interaction

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1 Introduction

Entanglement [1] as a fundamental quantum-mechanical feature makes possible tasks in quantum information which are impossible without it [2]. Now entanglement is widely studied for different model systems, such as strongly correlated fermionic models, harmonic lattice, and linear cluster states [3]. The quantum entanglement in solid state systems, such as spin chain, is an important emerging field [4–6]. The Heisenberg spin chain is an effective model to describe some realistically physical systems. In semiconductor quantum dots [7] and single molecular magnets [8], the magnetic properties of these systems are usually studied by the Heisenberg spin chains with the asymmetric anisotropic couplings such as Dzyaloshinskii-Moriya (DM) interaction [9–13]. Quantum teleportation, which uses prior shared entanglement between the sender and the remote receiver as a resource, is the most fascinating feature and potential application of quantum mechanics in quantum information processing [4, 14, 15]. The quantum teleportation can also be realized in the thermal equilibrium state generated by Heisenberg interactions. In Heisenberg spin systems, an unknown state, which is placed on one site, can be transmitted to a distant site with some fidelity by using the dynamics of the system [16, 17]. Entanglement teleportation via thermal entangled states of a two-qubit Heisenberg chain has been reported [18–20]. Recently, Zhang [21] has investigated the thermal entanglement of a two-qubit spin chain with DM interaction and entanglement teleportation via the model. The results disclose that a minimal entanglement of the thermal state in the model is needed to realize the entanglement teleportation. Fardin et.al have investigated the influences of DM interaction on the entanglement teleportation of a two-qubit XYZ system at thermal equilibrium [22]. They show that, by introducing DM interaction, the entanglement of the replica state and fidelity of teleportation can increase for the case of $J_z<0$. These studies reveal several interesting aspects of quantum entanglement not reflected by the concurrence, negativity or other entanglement measures. But only the influences of the z-component DM interaction on the entanglement teleportation have been discussed, and the DM interaction along the x-axis has rarely been taken into account. Specifically, the influences of the x-component DM interaction on the output entanglement and teleportation for XYZ system have never been reported. This interaction has many important consequences and may cause a number of unconventional phenomena. For example, the x-component parameter of the DM interaction has a more remarkable influence on the entanglement and the critical temperature than the z-component parameter of the DM interaction. Thus, by changing the

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direction of DM interaction, people can get a more efficient control parameter to enhance the entanglement and the critical temperature [23]. These are the motivations of this brief report. In this paper, we investigate the thermal output entanglement and the average fidelity in a two-qubit Heisenberg XYZ chain in the presence of the different DM interaction by teleporting two qubits in an arbitrary pure state.

2 The entanglement teleportation and the fidelity of entanglement teleportation with the DM interaction parameter D_x

The Hamiltonian of a two-qubit anisotropic Heisenberg XYZ chain with the x-component DM interaction D_x and inhomogeneous magnetic fields is[23]

$$H = J_x\sigma_1^x\sigma_2^x+J_y\sigma_1^y\sigma_2^y+J_z\sigma_1^z\sigma_2^z+D_x(\sigma_1^y\sigma_2^z-\sigma_1^z\sigma_2^y) +(B+b)\sigma_1^x+(B-b)\sigma_2^x, \qquad (1)$$

where J_x, J_y, and J_z are the real coupling coefficients, and σ^i ($i=x,\ y,\ z$) are Pauli matrices. Here, all the parameters are dimensionless.

At thermal equilibrium, the density matrix of this two-qubit spin chain system has the form as follows:

$$\rho(T)=\frac{1}{2Z}\begin{bmatrix} U_1 & Q_1^* & Q_2^* & U_2 \\ Q_1 & V_1 & V_2 & Q_2 \\ Q_2 & V_2 & V_1 & Q_1 \\ U_2 & Q_2^* & Q_1^* & U_1 \end{bmatrix}, \qquad (2)$$

where

$$Z=2\left[\mathrm{e}^{\frac{-J_x}{T}}\cosh\left(\frac{\omega_1}{T}\right)+\mathrm{e}^{\frac{J_x}{T}}\cosh\left(\frac{\omega_2}{T}\right)\right],$$

$$\omega_1=\sqrt{(J_y-J_z)^2+4B^2}$$

$$\omega_2=\sqrt{(J_y+J_z)^2+4D_x^2+4b^2},$$

$$U_{1,2} = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\sin^2\varphi_1+\mathrm{e}^{-\frac{J_x-\omega_1}{T}}\sin^2\varphi_2\pm\mathrm{e}^{\frac{J_x-\omega_2}{T}}\sin^2\varphi_3 \pm\mathrm{e}^{\frac{J_x+\omega_2}{T}}\sin^2\varphi_4,$$

$$V_{1,2} = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\cos^2\varphi_1+\mathrm{e}^{-\frac{J_x-\omega_1}{T}}\cos^2\varphi_2\pm\mathrm{e}^{\frac{J_x-\omega_2}{T}}\cos^2\varphi_3 \pm\mathrm{e}^{\frac{J_x+\omega_2}{T}}\cos^2\varphi_4,$$

$$\varphi_{1,2} = \arctan\left(\frac{2B}{J_y-J_z\pm\omega_1}\right),$$

$$\varphi_{3,4} = \arctan\left(\frac{2\sqrt{b^2+D_x^2}}{-J_y-J_z\pm\omega_2}\right),$$

$$Q_{1,2} = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\sin\varphi_1\cos\varphi_1+\mathrm{e}^{-\frac{J_x-\omega_1}{T}}\sin\varphi_2\cos\varphi_2 \pm\mathrm{e}^{\frac{J_x-\omega_2}{T}}\chi\sin\varphi_3\cos\varphi_3\pm\mathrm{e}^{\frac{J_x+\omega_2}{T}}\chi\sin\varphi_4\cos\varphi_4,$$

$$\chi = \frac{-\mathrm{i}D_x-b}{\sqrt{b^2+D_x^2}}.$$

Now we take Lee and Kim's [18] two-qubit teleportation protocol using two copies of the above state $\rho(T)\otimes\rho'(T)$ as our resource. We consider inputting a two-qubit state $|\psi\rangle_{\text{in}}=\cos(\theta/2)|10\rangle+\mathrm{e}^{\mathrm{i}\varphi}\sin(\theta/2)|01\rangle(0\leqslant\theta\leqslant\pi,0\leqslant\varphi\leqslant2\pi)$.

The output replica state can be obtained by applying a joint measurement and local unitary transformation to the input state [24]

$$\rho_{\text{out}}=\sum_{i,j}p_{ij}(\sigma_i\otimes\sigma_j)\rho_{\text{in}}(\sigma_i\otimes\sigma_j), \qquad (3)$$

where σ_i (i=0, x, y, z) signify the unit matrix I and three components of the Pauli matrix respectively, $p_{ij}=\text{tr}[E^i\rho(T)]\text{tr}[E^j\rho(T)]$, $\sum p_{ij}=1$ and $\rho_{\text{in}}=|\psi\rangle_{\text{in}}\langle\psi|$. Here $E^0=|\psi^-\rangle\langle\psi^-|$, $E^1=|\Phi^-\rangle\langle\Phi^-|$, $E^2=|\Phi^+\rangle\langle\Phi^+|$, $E^3=|\psi^+\rangle\langle\psi^+|$ and $|\psi^\pm\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle)$, $|\Phi^\pm\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle)$.

Hence, one can obtain ρ_{out} as

$$\rho_{\text{out}}(T)=\frac{1}{Z^2}\begin{bmatrix} V_1U_1 & 0 & 0 & V_2U_2C_{\text{in}} \\ 0 & V_1^2\sin^2\left(\frac{\text{arc}C_{\text{in}}}{2}\right)+U_1^2\cos^2\left(\frac{\text{arc}C_{\text{in}}}{2}\right) & \frac{\left(V_2^2+U_2^2\right)C_{\text{in}}}{2} & 0 \\ 0 & \frac{\left(V_2^2+U_2^2\right)C_{\text{in}}}{2} & U_1^2\sin^2\left(\frac{\text{arc}C_{\text{in}}}{2}\right)+V_1^2\cos^2\left(\frac{\text{arc}C_{\text{in}}}{2}\right) & 0 \\ V_2U_2C_{\text{in}} & 0 & 0 & V_1U_1 \end{bmatrix}. \qquad (4)$$

Then by the standard procedure, the corresponding concurrence [25] quantifying the output entanglement of this model is readily obtained as

$$\begin{aligned} C_{\text{out}} &= 2\max\{0,|C_{\text{in}}V_2U_2| \\ &\quad -\sqrt{\left(V_1^2\sin^2\left(\frac{\arcsin C_{\text{in}}}{2}\right)+U_1^2\cos^2\left(\frac{\arcsin C_{\text{in}}}{2}\right)\right)\left(U_1^2\sin^2\left(\frac{\arcsin C_{\text{in}}}{2}\right)+V_1^2\cos^2\left(\frac{\arcsin C_{\text{in}}}{2}\right)\right)}, \\ &\quad \left|\frac{(V_2^2+U_2^2)C_{\text{in}}}{2}\right|-V_1U_1\Big\}. \end{aligned} \tag{5}$$

The concurrence of the initial state is $C_{\text{in}} = 2|e^{i\varphi}\cos(\theta/2)\sin(\theta/2)|=\sin\theta$. When the input is a pure state, we can apply the concept of fidelity as a useful indicator of teleportation performance of a quantum channel. The fidelity between ρ_{in} and ρ_{out} characterizes the quality of the teleported state ρ_{out}. The fidelity is defined as [26]

$$F(\rho_{\text{in}},\rho_{\text{out}})=\left\{\text{tr}[\sqrt{(\rho_{\text{in}})^{1/2}\rho_{\text{out}}(\rho_{\text{in}})^{1/2}}]\right\}^2. \tag{6}$$

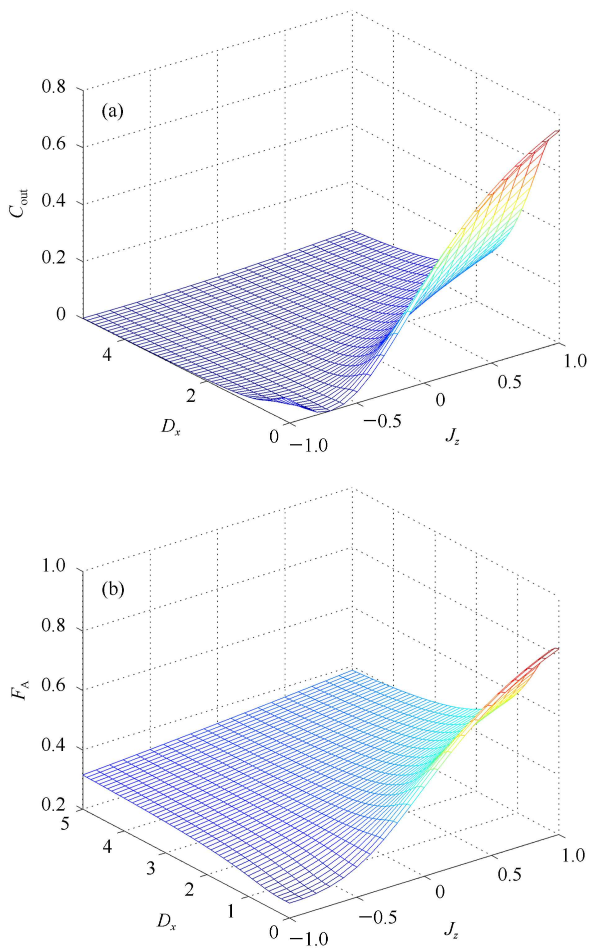

Fig. 1. (color online) The output entanglement C_{out} and the average fidelity F_A as a function of the spin coupling J_z and DM interaction D_x where $J=1$, $\gamma=0.3$, $B=1$, $b=0.5$, $T=0.1$, and $C_{\text{in}}=1$. All the parameters are dimensionless.

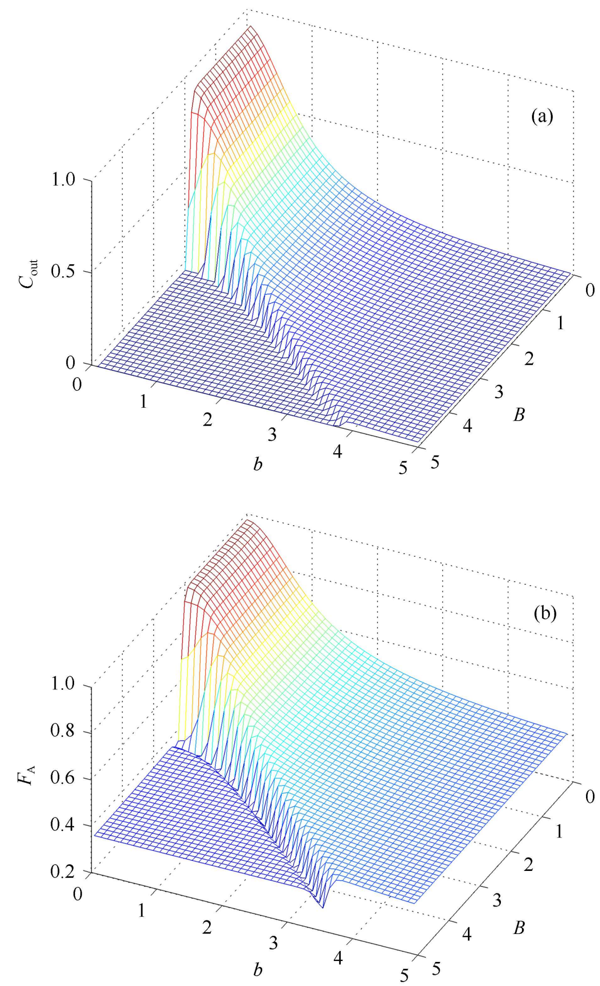

Fig. 2. (color online) The output entanglement C_{out} and the average fidelity F_A as a function of the magnetic fields B and the inhomogeneous magnetic fields b where $J=1$, $\gamma=0.3$, $D=0.2$, $T=0.1$ and $C_{\text{in}}=1$. All the parameters are dimensionless.

The average fidelity F_A is another useful concept for characterizing the quality of teleportation. F_A can be obtained by averaging F over all possible initial states

$$F_A=\frac{\int_0^{2\pi}d\varphi\int_0^{\pi}F\sin\theta d\theta}{4\pi}. \tag{7}$$

For this model $F_{\rm A}$ can be written as

$$F_{\rm A}=\frac{1}{3Z^2}(V_2^2+U_1^2+2V_1^2). \tag{8}$$

Here we are mainly concerned with the entanglement teleportation and the fidelity of entanglement teleportation caused by the D_x interaction. We introduce the mean coupling parameter J and the partial anisotropic parameter γ in the XY-plane, where $J=\dfrac{J_x+J_y}{2}$, $\gamma=\dfrac{J_x-J_y}{J_x+J_y}$.

The quantity $C_{\rm out}$ and $F_{\rm A}$ as a function of the x-component DM interaction D_x and the spin coupling J_z are plotted in Fig. 1. From Eq. (5) we know that $C_{\rm out}$ is dependent on the entanglement of the initial state and the parameters of the channel. The Fig. 1(a) shows that $C_{\rm out}$ decreases monotonically with the increasing D_x in both antiferromagnetic and ferromagnetic cases. It is particularly worth mentioning that $C_{\rm out}$ is not zero regardless of the ferromagnetic cases ($J_z=-1$), which is different from Ref. [22]. In Fig. 1(b) we find that the average fidelity $F_{\rm A}$ becomes greater than 2/3 for $J_z>0.1$, which makes the channel useful for performance in a teleportation protocol. Clearly the channel is inferior to the classical communication for teleportation when J_z is in ferromagnetic cases. The entanglement in antiferromagnetic cases is more robust than that in ferromagnetic cases. This is the reason why the system in antiferromagnetic cases can get better fidelity.

The quantity $C_{\rm out}$ and $F_{\rm A}$ as a function of the magnetic fields B and the inhomogeneous magnetic fields b are plotted in Fig. 2. We consider first the case as b=0. With increasing B, the $C_{\rm out}$ and $F_{\rm A}$ are maximal and constants, but it drops suddenly as B crosses the critical value $B_{\rm c}$. At the critical point (b=0, $B=B_{\rm c}$) the entanglement becomes a nonanalytic function of B and a quantum phase transition occurs [27, 28]. It means that there will be a quantum phase transition in the absence of the inhomogeneous field. We also see that the quantum phase transition field depends on the inhomogeneity. From Fig. 2(a) we also find that increasing inhomogeneous magnetic fields can induce the output entanglement, which means we can make entanglement birth using the inhomogeneity of the magnetic field. In

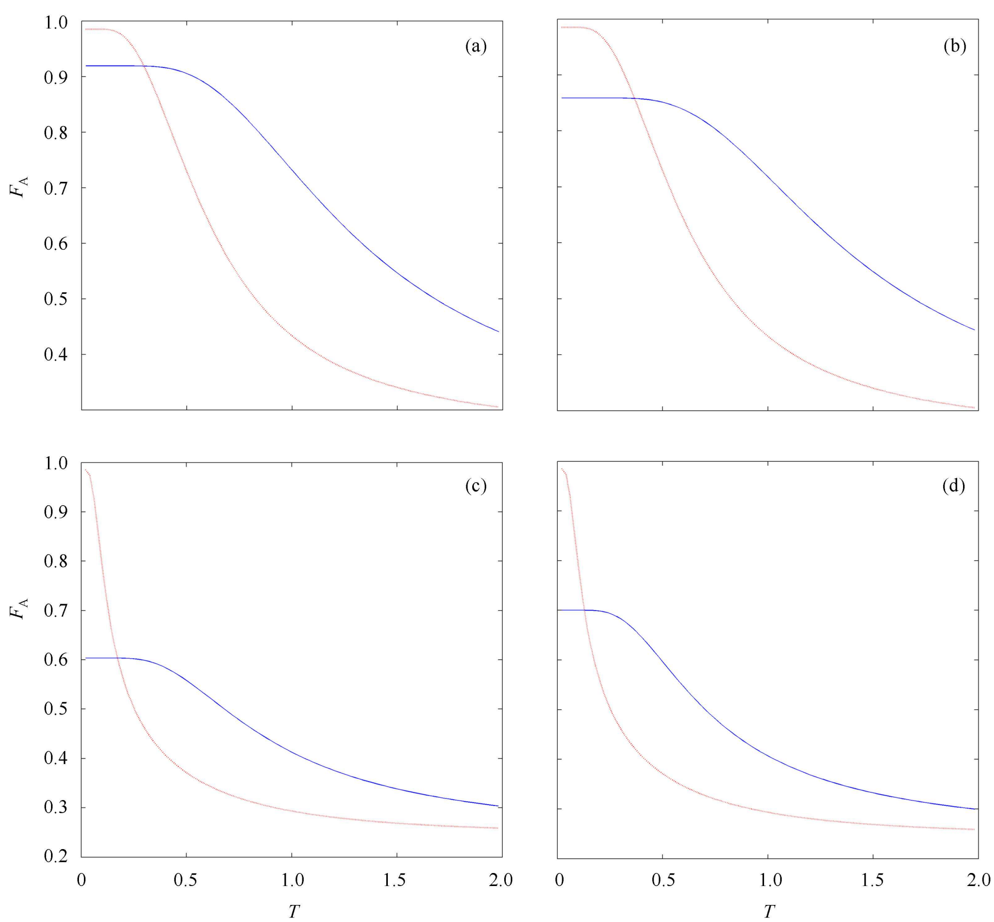

Fig. 3. (color online) The average fidelity $F_{\rm A}$ as a function of temperature T for different values of spin coupling J and anisotropy coupling J_z, (a) J=1, J_z=0.4 (b) $J=-1$, J_z=0.4 (c) $J=-1$, $J_z=-0.4$ (d) J=1, J_z=−0.4. γ=0.1, B=0.4, b=0.2. D_z=0.2 (red, dotted line), D_x=0.2 (blue, solid line). All the parameters are dimensionless.

Fig. 2(b) the trend for average fidelity accords with the $C_{\rm out}$. The model is not superior to classical communication for teleportation when $B>2.2$. These results show that the fidelity of teleportation and entanglement of the replica state are tunable by the channel parameters such as D_x, b, B [12].

3 The comparison between different DM interactions

The Hamiltonian of the two-qubit Heisenberg XYZ chain with the z-component DM interaction D_z is

$$H = J_x\sigma_1^x\sigma_2^x+J_y\sigma_1^y\sigma_2^y+J_z\sigma_1^z\sigma_2^z+D_z(\sigma_1^x\sigma_2^y-\sigma_1^y\sigma_2^x) +(B+b)\sigma_1^z+(B-b)\sigma_2^z. \quad (9)$$

Fardin et al. has investigated the influences of the z-component DM interaction on the entanglement and the entanglement teleportation of a two-qubit system at thermal equilibrium [22]. We mainly study the differences between the two different DM interactions D_x and D_z.

The effects of the different DM interactions on the average fidelity are shown in Fig. 3. We depict the average fidelity as a function of temperature T for different values of J and J_z. With the increase of temperature, the average fidelity of the overall trend is downward. But we also note that the x-component fidelity will maintain a steady value first and then decrease with the increase of temperature. It is found that the z-component DM interaction can get a higher average fidelity than the x-component DM interaction for some regions. In addition, we also see some other novel properties, such as the z-component DM interaction, can get optimal teleportation when the system is at ground state. For certain values of parameters, $F_{\rm A}$ becomes greater than 2/3, which makes the channel more useful for the performance in a teleportation protocol. Especially for the x-component, fidelity can be even greater than 2/3 for a higher temperature T (for example the case T=1 in Fig. 3(b)). These results show that we can get proper fidelity by introducing D_x or D_z interaction for different conditions.

4 Conclusion

In summary, we have investigated the effects of the different DM interaction on the entanglement and teleportation in a two-qubit Heisenberg XYZ chain under inhomogeneous magnetic fields. We analyze in detail the effects of different DM interaction and other parameters of the system on the entanglement and the average fidelity. By introducing the x-component DM interaction, the output entanglement can exist even for the ferromagnetic case. It also shows that we can get optimal entanglement teleportation ($F_{\rm A}\approx 1$) for certain values of parameters. In particular, we also find ways of how to birth entanglement when we need it. All of the above results might be helpful in the investigation of entanglement production and teleportation manipulation in the spin system.

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