Bound Dirac states for pseudoscalar Cornell potential: 3+1 dimensions

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Abstract: The Cornell potential consists of Coulomb and linear potentials, i.e.$-a/r+br$, that it has received a great deal of attention in particle physics. In this paper, we present exact solutions of the Dirac equation with the pseudoscalar Cornell potential under spin and pseudospin symmetry limits in 3+1 dimensions. The energy eigenvalues and corresponding eigenfunctions are given in explicit forms.

Key words: Dirac equation, pseudoscalar Cornell potential, 3+1 dimensions, spin and pseudospin symmetries

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1 Introduction

Relativistic symmetries of the Dirac Hamiltonian were discovered many years ago, but only recently these symmetries have been recognized empirically in nuclear and hadronic spectroscopy [1]. The cases in which the mean field is composed of a vector (V_{V}) and a scalar (V_{S}) potential are usually pointed out as a necessary condition for occurrence of spin and pseudospin symmetries in nuclei. Within the framework of Dirac equation, pseudospin symmetry used to feature deformed nuclei, superdeformation, to establish an effective shell-model [2–4] and spin symmetry is relevant for mesons [5]. Spin symmetry occurs when the scalar potential S is nearly equal to the vector potential V_{V} or equivalently $V_{\mathrm{S}}\approx V_{\mathrm{V}}$ and pseudospin symmetry occurs when $V_{\mathrm{S}}\approx -V_{\mathrm{V}}$ [6–9].

The radial Dirac equations containing central potentials with only a pseudo-scalar Lorentz coupling has received a great deal of attention in theoretical physics [10–14]. McKeon and Van Leeuwen considered solutions to the Dirac equation in the presence of an external pseudoscalar Coulomb potential and they found that no normalizable bound state solutions exist [15]. de Castro solved the Dirac equation for a pseudoscalar Coulomb potential in a two-dimensional world [16] and also studied a relativistic extension of a quark confinement potential model of the Schrödinger theory [5] in 1+1 Lorentz dimensions [17]. Yao et al. obtained the quantum states of a trapped Dirac particle in the presence of a pseudoscalar potential [18]. Haouat and Chetouani examined the supersymmetric path integrals in solving the problem of a relativistic spinning particle interacting with pseudoscalar potentials [19] and solved the 3+1 dimensional Dirac equation in the presence of the radial pseudoscalar Hulthén potential by using the usual approximation of the centrifugal potential [20]. They also derived a quasi-classical quantization rule for the problem of a Dirac particle interacting with a pseudoscalar power potential in (1+1) dimension using the stationary phase approximation [21]. Very recently, Thylwe discussed relativistic bound states for a linear radial pseudo-scalar potential model using accurate amplitude-phase computations and a novel semiclassical (phase-integral) approach [22].

The Cornell potential, which consists of Coulomb plus linear potentials, i.e. $V(r)=-\dfrac{a}{r}+br$, has received a great deal of attention in particle physics. The Cornell potential was used with considerable success in models describing systems of bound heavy quarks [23–26]. The potential includes the short distance Coulombic interaction of quarks, known from perturbative quantum chromodynamics (QCD), and the large distance quark confinement, known from lattice QCD, via the linear term [27,28]. It should be stressed that a specific situation can emerge when the parameter b is simultaneously small, and leads to a particular atomic description of a perturbed Coulomb problem [29]. Ghalenovi et al. studied the strange, charmed and beauty baryons masses in the Cornell potential by using the variational approach [30].

The purpose of this work is to solve the Dirac equation for the pseudoscalar Cornell potential in 3+1 dimensions. To this end, we shall first briefly introduce the Dirac equation with pseudoscalar potential in view of spin and pseudospin symmetries in Section 2. In Section 3, we solve the Dirac equation with pseudoscalar

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Cornell potential under spin and pseudospin symmetries. Finally, Section 4 contains a summary and concluding remarks.

2 Radial Dirac equations

The time-independent Dirac equation with a scalar potential $V_{\rm S}$, a time-component of a vector potential $V_{\rm V}$, and a pseudo-scalar potential $V_{\rm PS}$ reads [22, 31]

$$\frac{{\rm d}F}{{\rm d}r}=-\left(\frac{\kappa}{r}+V_{\rm PS}\right)F+((m+V_{\rm S})+E-V_{\rm V})G, \qquad (1)$$

$$\frac{{\rm d}G}{{\rm d}r}=((m+V_{\rm S})-E+V_{\rm V})F+\left(\frac{\kappa}{r}+V_{\rm PS}\right)G, \qquad (2)$$

where $\kappa=-(l+1)\leqslant-1$ for $l=0, 1, 2,\cdots$ and $\kappa=l\geqslant 1$, $l=1, 2, 3,\cdots$ where l is the orbital angular momentum quantum number. In this article we assume $V_{\rm S}=V_{\rm V}=0$. By following the procedure introduced by Thylwe in Ref. [22], Eqs. (1) and (2) can be written in matrix form with separated diagonal and off-diagonal coefficient functions, i.e.

$$\bar{\psi}=\begin{pmatrix} F \\ G \end{pmatrix}, \qquad (3)$$

and

$$\frac{{\rm d}\bar{\psi}(r)}{{\rm d}r}=\begin{pmatrix} -\left(\frac{\kappa}{r}+V_{\rm PS}\right) & m+E \\ m-E & \frac{\kappa}{r}+V_{\rm PS} \end{pmatrix}\bar{\psi}(r). \qquad (4)$$

By differentiating Eq. (4) and using the same equation to eliminate the first derivative of $\bar{\psi}$, one obtains

$$\frac{{\rm d}^2\bar{\psi}(r)}{{\rm d}r^2}+\begin{pmatrix} E^2-m^2-\left(\frac{\kappa}{r}+V_{\rm PS}\right)^2+V'_{\rm PS}-\frac{\kappa}{r^2} & 0 \\ 0 & E^2-m^2-\left(\frac{\kappa}{r}+V_{\rm PS}\right)^2-V'_{\rm PS}-\frac{\kappa}{r^2} \end{pmatrix}\bar{\psi}(r)=0. \qquad (5)$$

In this way, one obtains two separated component equations of the Schrödinger type as follows

$$\psi''^{(\pm)}(r)+\left(\varepsilon^2-v_\kappa(r)-v_\kappa^{(\pm)}(r)\right)\psi^{(\pm)}(r)=0, \qquad (6)$$

where

$$\begin{aligned} \varepsilon^2 &= E^2-m^2, \\ v_\kappa &= \left(\frac{\kappa}{r}+V_{\rm PS}\right)^2, \\ v_\kappa^{(\pm)} &= \pm\left(V'_{\rm PS}-\frac{\kappa}{r^2}\right), \end{aligned} \qquad (7)$$

and also the $\psi^{(+)}$ ($\psi^{(-)}$) refers to the $F(G)$ component.

3 Cornell pseudoscalar potential

Let the Cornell potential be defined as

$$V_{\rm PS}=\eta_{\rm l}r-\frac{\eta_{\rm c}}{r}, \qquad (8)$$

where $\eta_{\rm l}$, $\eta_{\rm c}>0$ are the coupling constants. Then from Eq. (7), we have

$$v_\kappa=\left(\frac{\kappa}{r}+\eta_{\rm l}r-\frac{\eta_{\rm c}}{r}\right)^2=\frac{\kappa^2}{r^2}+\eta_{\rm l}^2r^2+\frac{\eta_{\rm c}^2}{r^2}+2\kappa\eta_{\rm l}-\frac{2\kappa\eta_{\rm c}}{r^2}-2\eta_{\rm l}\eta_{\rm c}, \qquad (9)$$

$$v_\kappa^{(\pm)}=\pm\left(V'_{\rm PS}-\eta_{\rm l}-\frac{\eta_{\rm c}}{r^2}\right), \qquad (10)$$

and

$$\psi''^{(\pm)}_\kappa(r)+\left(-\eta_{\rm l}^2r^2-\frac{\kappa(\kappa\pm1)+\eta_{\rm c}(\eta_{\rm c}\mp1)-2\kappa\eta_{\rm c}}{r^2}-\varepsilon^2-\eta_{\rm l}(2\kappa\mp1)+2\eta_{\rm l}\eta_{\rm c}\right)\psi_\kappa^{(\pm)}(r)=0. \qquad (11)$$

3.1 Spin symmetry limit

The exact spin symmetry occurs in the Dirac equation when

$$\frac{{\rm d}\varDelta}{{\rm d}r}=\frac{{\rm d}(V_{\rm V}-V_{\rm S})}{{\rm d}r}=0$$

or $\varDelta=C_{\rm s}$ =constant [32–35]. Here, we are taking $\varSigma=V_{\rm V}+V_{\rm S}=0$, $C_{\rm s}=0$ and pseudoscalar potential as Cornell potential in Eq. (11). Using dimensionless transformation as $x=\sqrt{\eta_{\rm l}}r$, Eq. (11) for the upper component becomes

$$\frac{{\rm d}^2\psi^{(+)}}{{\rm d}x^2}-\left[x^2+\frac{\kappa(\kappa+1)+\eta_{\rm c}(\eta_{\rm c}-1)-2\kappa\eta_{\rm c}}{x^2}+\xi^2\right]\psi^{(+)}=0, \qquad (12)$$

where $\xi^2=(2\kappa-1)-\dfrac{\varepsilon^2}{\eta_l}-2\eta_l\eta_c$. To obtain the solution of Eq. (12), by using transformation $s=x^2$, we rewrite it as follows

$$\frac{d^2\psi^{(+)}}{ds^2}+\frac{1}{2s}\frac{d\psi^{(+)}}{ds}-\frac{1}{4s^2}[s^2+\xi^2 s+a(a+1)]\psi^{(+)}=0, \quad (13)$$

where $a(a+1)=\kappa(\kappa+1)+\eta_c(\eta_c-1)-2\kappa\eta_c$. This second differential equation can be solved by the Nikiforov-Uvarov method [36]. Following Eqs. (28–35) in Ref. [37], one can obtain the energy equation as

$$(2n+1+\sqrt{4a(a+1)+1})+\xi^2=0, \quad (14)$$

or equivalently

$$E=\pm[m^2+\eta_l(2a-1)+2\eta_l(2n+a+3/2)+2\eta_l^2\eta_c]^{\frac{1}{2}}. \quad (15)$$

To find the eigenfunction of the spin symmetry case, we follow Eqs. (36)–(39) in Ref. [37] and we obtain the upper spinor as

$$\psi^{(+)}_{n\kappa}(x)=N e^{-\frac{1}{2}x^2}x^{a+1}L_n^{a+\frac{1}{2}}(x^2), \quad (16a)$$

or equivalently

$$\psi^{(+)}_{n\kappa}(r)=N e^{-\frac{1}{2}\eta_l r^2}(\sqrt{\eta_l}r)^{a+1}L_n^{a+\frac{1}{2}}(\eta_l r^2), \quad (16b)$$

where N is normalization constant determined as [38]

$$N=\sqrt{\frac{2n!}{\left(n+\frac{1}{2}a+\frac{3}{2}\right)!}}. \quad (17)$$

The lower spinor can be obtained from (1) as

$$\psi^{(-)}_{n\kappa}=\frac{1}{m+E}\left[\frac{d\psi^{(+)}_{n\kappa}}{dr}+\left(\frac{\kappa}{r}+\eta_l r\right)\psi^{(+)}_{n\kappa}\right]. \quad (18)$$

3.2 Pseudospin symmetry limit

Ginocchio showed that there is a connection between pseudospin symmetry and the time component of a vector potential and the scalar potential are nearly equal, i.e., $S(r)\approx -V(r)$ [1, 39, 40]. Also, Meng et al. derived that if

$$\frac{d[V(r)+S(r)]}{dr}=\frac{d\Sigma(r)}{dr}=0$$

or $\Sigma(r)=C_{ps}$=constant, pseudospin symmetry is exact in the Dirac equation [41,42]. In this subsection, we are taking $\Delta=V_V-V_S=0$, $C_{ps}=0$ and pseudoscalar potential as Cornell potential. Under the pseudospin symmetry case, solution of the Dirac equation can be obtained from the previous subsection ones by doing $\psi^{(+)}\leftrightarrow\psi^{(-)}$, $\kappa\to-\kappa$, $E\to-E$, $\Sigma\to\Delta$ and $V_p\to -V_p$. Therefore the energy equation can be obtained as

$$(2n+1+\sqrt{4\tilde{a}(\tilde{a}-1)+1})+\tilde{\xi}^2=0, \quad (19)$$

where $\tilde{\xi}^2=(2\kappa+1)-\dfrac{\varepsilon^2}{\eta_l}-2\eta_l\eta_c$ and $\tilde{a}(\tilde{a}+1)=\kappa(\kappa-1)+\eta_c(\eta_c+1)-2\kappa\eta_c$. We can also write the energy eigenvalues as

$$E=\pm[m^2+\eta_l(2\tilde{a}+1)+2b\eta_l(2n+\tilde{a}+1/2)+2\eta_l^2\eta_c]^{\frac{1}{2}}. \quad (20)$$

On the other hand, the corresponding lower spinor eigenfunctions can be obtained as

$$\psi^{(-)}_{n\kappa}(x)=\tilde{N}e^{-\frac{1}{2}x^2}x^{\tilde{a}}L_n^{\tilde{a}-\frac{1}{2}}(x^2), \quad (21a)$$

or equivalently

$$\psi^{(-)}_{n\kappa}(r)=\tilde{N}e^{-\frac{1}{2}\eta_l r^2}(\sqrt{\eta_l}r)^{\tilde{a}}L_n^{\tilde{a}-\frac{1}{2}}(\eta_l r^2), \quad (21b)$$

where $\tilde{N}$ is normalization constant determined as [38]

$$\tilde{N}=\sqrt{\frac{2n!}{\left(n+\frac{1}{2}\kappa+\frac{1}{2}\right)!}}. \quad (22)$$

The upper spinor can be obtained from (2) as

$$\psi^{(+)}_{n\kappa}(r)=\frac{1}{m-E}\left[\frac{d\psi^{(-)}_{n\kappa}(r)}{dr}-\left(\frac{\kappa}{r}+\eta_l r\right)\psi^{(-)}_{n\kappa}(r)\right]. \quad (23)$$

4 Conclusions

In this paper, we have investigated the exact energy levels and corresponding wave functions of the Dirac equation with the pseudoscalar Cornell potential under spin and pseudospin symmetry limits in 3+1 dimensions. We found that the Cornell potential with pseudoscalar potential has an exact solution however there is no exact solution when the Cornell potential is studied as scalar or vector potential [26, 30].

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