

# RF phase jitter in a klystron amplifier<sup>\*</sup>

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**Abstract:** RF phase jitter is a very important parameter for a relativistic klystron amplifier. This parameter is closely linked with the physics processes in the klystron. RF phase jitter is theoretically studied together with Particle in Cell (PIC) simulations in the paper. The main factor is deduced and verified in the PIC simulation. RF phase jitter is significantly affected by the fluctuation of the beam voltage. The relation between the phase jitter and the voltage fluctuation is linear in certain ranges.

**Key words:** phase jitter, microwave device, klystron

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## 1 Introduction

There are several kinds of microwave devices for high power microwave (HPM) generators, such as relativistic klystron amplifier (RKA) [1, 2], magnetron, backward wave oscillator [3], traveling wave tube [4], gyrotron, vircator [5] and transit tube oscillator [6]. Some of them are oscillators and others are amplifiers. Amplifiers become increasingly important in HPM development because the power limitation becomes obvious for a single device in recent years and the power adder with amplifiers is a better way to exceed this limitation. RKA is a typical device among HPM amplifiers [1, 7]. A very low phase jitter is required in the power adder, so it is an important parameter for the RKA devices. The paper is organized as follows. Section 2 briefly introduces how the RF field is excited in a cavity. In Section 3, the RF phase jitter is theoretically studied and the RF phase jitter is significantly affected by the fluctuation of the beam voltage during the steady state. The relation between the phase jitter and the voltage fluctuation is linear in certain ranges. In Section 4, the RF phase jitter is studied with Particle in Cell (PIC) simulations, and the theoretical results are verified. The conclusion is given in Section 5.

## 2 RF field excited in a cavity

The excited RF field,  $E_z(r, z, t) = A(t)E(z)\sin\omega t$

(here the field amplitude  $A(t)$  is a slowly variable function of time), can be described by the equation that follows Maxwell equations [8–11],

$$\frac{d^2 A(t)}{dt^2} + \omega_0^2 A = -\frac{\omega_0}{Q_0} \frac{dA(t)}{dt} + C_0 V_0 \sin\omega t - \frac{C_0^2}{\varepsilon_0 Z_g} \frac{dA(t)}{dt} + \frac{1}{\varepsilon_0} \int_V \vec{J} \cdot \vec{E}_n dv, \quad (1)$$

here,  $C_0$  denotes the couple between the cavity and the input waveguide,  $V_0 = \sqrt{2\omega P/\varepsilon_0 Z_g}$  depends on the RF power  $P$  and its frequency  $\omega$  and the RF energy stored in the cavity is  $W = \frac{1}{2}\varepsilon_0 A^2$  and  $Q_0$  is the cavity's intrinsic quality factor,  $Z_g$  the input impedance,  $\varepsilon_0$  the vacuum dielectricity,  $\omega_0$  the resonant frequency of the cavity and  $\vec{J}$  the current density in the cavity.

From Eq. (1), we have Eq. (2) as follows.

$$\frac{d^2 A(t)}{dt^2} + \omega_0^2 A(t) + \frac{\omega_0}{Q} \frac{dA(t)}{dt} = C_0 V_0 \sin\omega t + \frac{1}{\varepsilon_0} \frac{d}{dt} \int_V \vec{J} \cdot \vec{E}_n dv, \quad (2)$$

here,  $Q$  is the cavity's quality factor and can be expressed by  $\frac{1}{Q} = \frac{1}{Q_0} + \frac{1}{Q_e}$  and  $\frac{1}{Q_e} = \frac{C_0^2}{\omega_0 \varepsilon Z_g}$ . The RF field in the cavity  $A(t)$  can be expressed by  $A(t) = A\sin(\omega t + \beta)$  when it is in the stationary state, then

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we have Eq. (3) from Eq. (2).

$$\begin{aligned} & \left[ (\omega_0^2 - \omega^2) \sin(\omega t + \beta) + \frac{\omega_0}{Q} \cos(\omega t + \beta) \right] A \\ & = JJ + C_0 V_0 \sin(\omega t), \end{aligned} \quad (3)$$

here,  $JJ = \frac{1}{\varepsilon} \frac{d}{dt} \int_V J^\omega \cdot E_n^\omega dr^\omega$ , is often called the source term or the gain function and depends on the electron movement. It is clearly shown from Eq. (3) that the phase  $\beta$  of the RF field in the cavity depends on  $JJ$  and is affected by the electron's movement. Because the beam thickness is very small when compared with the tube radius,  $JJ$  can be rewritten as follows.

$$JJ = \frac{1}{\varepsilon_0} \frac{d}{dt} \int_0^L I(z, t) E(z) dz. \quad (4)$$

### 3 Device's structure and its physics processes

The device's structure is shown in Fig. 1. The electron beam is modulated when it passes the modulation cavity as shown in Fig. 1. And the RF field excited by the modulated beam in the middle cavity will further modulate the electron beam in Fig. 1. The beam is intensely bunched when it enters the output cavity and the strong RF field will be excited to convert the beam energy into the RF field. Then the high RF power will be radiated out from the output hole as shown in Fig. 1 and the phase of the output RF depends on the RF field in the output cavity.

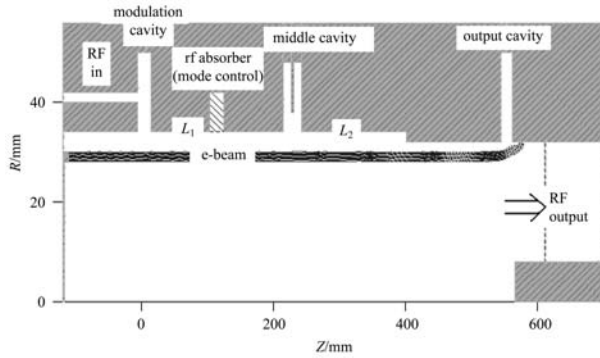


Fig. 1. Device's structure.

#### 3.1 Beam's modulation in the modulation cavity

The beam is not bunched before it enters the modulation cavity, so  $JJ \approx 0$  in Eq. (3). Then the RF field in the modulation cavity depends on the input RF power. When  $\omega = \omega_0$ , we have  $\beta \approx \pi/2$  from Eq. (3). And from Eq. (3) the stationary RF field

can be expressed by:

$$A_1 = \frac{Q}{\omega_0} C_0 V_0. \quad (5)$$

Then the electron movement in the modulation cavity can be expressed by [10]:

$$\frac{d(mv)}{dt} = e A_1 E_1 \cos(\omega t), \quad (6)$$

here,  $e$ ,  $m$ ,  $v$  is the charge, mass, velocity of electron respectively.

The gap voltage acting on electron in the modulation cavity is small when compared with the beam voltage, so the electron's velocity, when it passes the modulation cavity, can be expressed by:

$$v \approx v_0 \left[ 1 + \frac{V_1}{2V_0} M_1(\theta_1) \sin\left(\frac{\theta_1}{2} + \omega t\right) \right], \quad (7)$$

here,  $V_0 = \frac{m_0 v_0^2}{e}$  is the beam voltage,  $V_1 = A_1 E_1 d_1$  is the gap voltage acting on electron in the modulation cavity,  $\theta_1 = \omega d_1 / v_0$  is the transit angle in the modulation cavity,  $M_1(\theta_1) = \frac{\sin(\theta_1/2)}{\theta_1/2}$  is the couple between the beam and the modulation cavity.

#### 3.2 Interaction between the beam and the middle cavity

It is shown in Eq. (7) that beam is modulated in the modulation cavity. The beam is slowly bunched in the drifting tube before it enters the middle cavity as shown in Fig. 1. The electron's movement in the drifting tube can be expressed by:

$$\frac{d}{dz} \varphi = \frac{\omega}{v}, \quad (8)$$

here,  $\varphi = \omega t$ .

From Eq. (7) and (8), we have

$$\varphi \approx \varphi_0 + \frac{\omega}{v_0} z - \frac{\omega}{v_0} k_1 z \sin\left(\varphi_0 + \frac{\theta_0}{2}\right),$$

where  $k_1 \approx \frac{V_1}{2V_0} M_1(\theta_1)$  and  $\varphi_0 = \omega t_0$  ( $t_0$  denotes the initial time).

In the drifting tube, the beam current can be expressed by

$$I(z, t) = \sum_{n=-\infty}^{\infty} I_n(z) e^{jn\varphi_0},$$

so the first harmonic beam current  $I_1(z, t)$ <sup>8</sup> can be calculated and be given by:

$$I_1(z, t) = 2I_0 J_1\left(\frac{\omega}{v_0} k_1 z\right) \sin\left(\varphi_0 + \frac{\omega}{v_0} z + \frac{\theta_0}{2}\right).$$

So, the first harmonic beam current in the middle

cavity can be given by:

$$I_1(t) = 2I_0J_1\left(\frac{\omega}{v_0}k_1L_1\right)\sin\left(\varphi_0 + \frac{\omega}{v_0}L_1 + \frac{\theta_0}{2}\right), \quad (9)$$

From Eq. (4) and (9), the source term  $JJ$  can be given by:

$$JJ = \frac{\omega E_2 d_2}{\varepsilon_0} I_0 J_1\left(\frac{\omega}{v_0}k_1L_1\right) e^{j\left(\frac{\omega}{v_0}L_1 + \frac{\theta_0}{2}\right)}. \quad (10)$$

When  $\omega = \omega_0$ , we have  $\beta \approx \frac{\omega}{v_{01}}L_1 + \frac{\theta_0}{2}$  from Eqs. (3) and (10).

The RF field excited by the bunched beam will further modulate the beam as shown in Fig. 1. The electron's velocity, when it passes the middle cavity, can be expressed by:

$$v \approx v_0 + \frac{eV_2}{mv_0} M_2(\theta_2) \sin\left(\frac{\theta_2}{2} + \omega t_0 + \frac{\omega L_1}{v_0}\right), \quad (11)$$

here,  $V_2 = A_2 E_2 d_2$  is the gap voltage acting on the electron in the middle cavity.

The first harmonic beam current in the drifting tube after the middle cavity can be given by:

$$I_1(z, t) = 2I_0J_1\left(\frac{\omega}{v_0}k_2z\right)\sin\left(\varphi_0 + \frac{\omega}{v_0}z + \frac{\theta_0}{2} + \frac{\omega}{v_0}L_1\right), \quad (12)$$

here,  $k_2 \approx \frac{V_2}{2V_0} M_2(\theta_2)$ .

### 3.3 RF phase of the RF field in the output cavity

The beam is intensely bunched in the output cavity. The first harmonic beam current can be given by:

$$I_1(t) = 2I_0J_1\left(\frac{\omega}{v_0}k_2L_2\right)\sin\left(\varphi_0 + \frac{\omega}{v_0}L_2 + \frac{\omega}{v_0}L_1\right), \quad (13)$$

from Eq. (4) and (9), the source term  $JJ$  can be given by:

$$JJ \approx 2\frac{\omega I_0 E_0}{\varepsilon_0} J_1\left(\frac{\omega}{v_0}k_2L_2\right) d_3 \sin\left(\varphi_0 + \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2 + \frac{\theta_1}{2}\right), \quad (14)$$

when  $\omega = \omega_0$ , we have

$$\beta = \varphi_0 + \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2 + \frac{\theta_1}{2} + \frac{\theta_2}{2}$$

from Eq.(3) and (14). So the RF phase of the RF field in the output cavity can be given by:

$$\beta = \varphi_0 + \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2 + \frac{\theta_1}{2} + \frac{\theta_2}{2}. \quad (15)$$

The RF phase of the output RF power depends

on that in the output cavity, so it can be given by:

$$\phi = \varphi_0 + \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2 + \frac{\theta_1}{2} + \frac{\theta_2}{2}.$$

Then the phase difference between the input and output can be given by:

$$\begin{aligned} \Delta\phi &= \phi - \varphi_0 = \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2 + \frac{\theta_1}{2} + \frac{\theta_2}{2} \\ &\approx \frac{\omega}{v_0}L_1 + \frac{\omega}{v_0}L_2, \end{aligned} \quad (16)$$

and it depends on the transit angle of the drifting tube  $\left(\frac{\omega}{v_0}L_1\right)$  and  $\left(\frac{\omega}{v_0}L_2\right)$ . Ignoring the phase jitter of the input RF, the phase jitter of the output depends on the beam voltage, then we can have such a relation from Eq. (17) as follows.

$$\begin{aligned} \Delta\phi &= \frac{2\pi}{\lambda} (L_1 + L_2) \\ &\times \left[ \frac{1}{\sqrt{1 - \left[\frac{510}{510 + V_0}\right]^2}} - \frac{1}{\sqrt{1 - \left[\frac{510}{510 + V_0 + \Delta V}\right]^2}} \right]. \end{aligned} \quad (17)$$

When  $\lambda=10.5$  cm,  $V_0=600$  kV  $L_1 + L_2 = 60$  cm, then  $\Delta\phi \approx 0.52\Delta V$ , where  $\Delta V$  is the fluctuation of the beam voltage. When  $\Delta V \leq 30$  kV, the phase jitter is less than  $15.6^\circ$ .

## 4 Simulation

The PIC simulation is carried out based on the structure shown in Fig. 1 with the beam voltage 600 kV and current 5 kA. The output power is 930 MW (shown in Fig. 2) when the input rf power is 680 kW and the frequency 2.850 GHz. We can obtain the variation of the RF frequency versus time through the fast Fourier transform treatment of the output RF field shown in Fig. 3. So the curve of the RF phase versus time can be obtained by integrating the RF frequency as follows.

$$\phi(t) = \phi_0 + 2\pi \int_0^t f(t) dt, \quad (18)$$

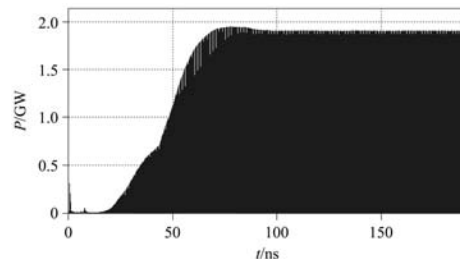


Fig. 2. Curve of the output RF power versus time.

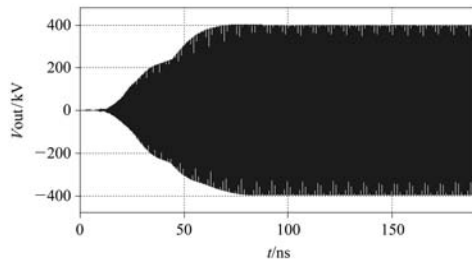


Fig. 3. Curve of the output RF field versus time.

When compared with the input RF whose frequency is  $f_0$ , the phase difference between the input and output can be expressed by:

$$\Delta\phi(t) = 2\pi \int_0^t (f(t) - f_0) dt, \quad (19)$$

the beam voltages with a step change ( $\Delta V = 30$  kV, rise time 10 ns) and a tip change ( $\Delta V = 30$  kV, duration time 20 ns as shown in Fig. 4) are used in the PIC simulations to explore how the phase jitter is affected by the beam voltage. Fig. 5 is the curve of the phase difference versus time driven by the beam voltage with a step change. It is shown in Fig. 4 that the phase difference is raised when the beam voltage rises and reaches the value calculated by Eq. (18). Fig. 6 is the curve of the phase difference versus time driven by the beam voltage with a tip. Fig. 6 shows that the phase difference increases as the beam voltage changes and decreases as the beam voltage returns to the original value and that the maximum value of the phase difference is  $11.5^\circ$ .

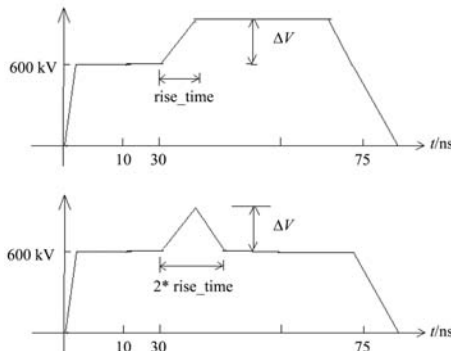


Fig. 4. Curve of the beam voltage versus time (up: with a step change, down: with a tip change).

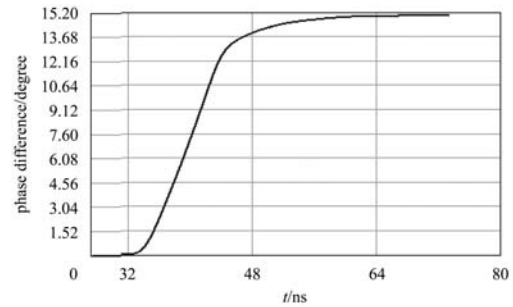


Fig. 5. Curve of the RF phase variation versus time (beam voltage with a step change).

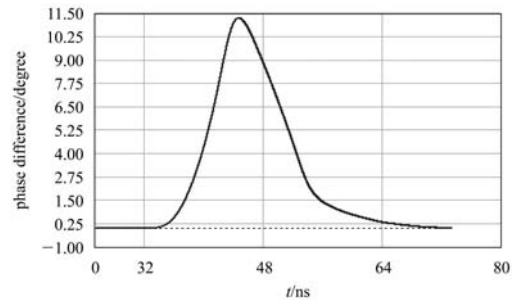


Fig. 6. Curve of the RF phase variation versus time (beam voltage with a tip change).

## 5 Conclusion

During the steady state, the interaction between the beam and the RF field in the modulation cavity depends on the input RF and determined by the phase of the input RF. The bunch center of the beam will affect the phase of the RF field in the following cavities (such as the middle cavity and the output cavity as shown in Fig. 1). The phase difference between the input and output RF depends on the transit angle of the drifting tubes. Because the frequency of the input RF and the length of the drifting tubes are fixed, the phase jitter depends on the fluctuation of the beam voltage. With the structure shown in Fig. 1 driven by the beam with a voltage 600 kV, the fluctuation of the beam voltage is required to be less than 10% to keep the phase jitter of the output RF less than  $20^\circ$ .

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