Uncertainty study of $D_S^-(D^-)\to\gamma l\bar{\nu}$ $(l=e,\mu)$ decays determined by wave function*

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Abstract: Wave function is important for determining decay constants $f_{D_S^-}$ and f_{D^-}. Using the 5 types of D meson wave functions in the heavy quark limit, we studied the uncertainties of radiative pure-leptonic decays of $D_S^-(D^-)$ mesons. The branching ratios are (1.025390–1.706812) $\times10^{-5}$ and (0.953498–1.576725) $\times10^{-6}$ for D_S^- and D^- decays, respectively, which are sensitive to the type of wave function.

Key words: D decay, wave function, branching ratio

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1 Introduction

The pure-leptonic decays of heavy meson are useful to determine the meson decay constants, and they are also sensitive to new physics beyond the Standard Model (SM) [1–3]. Many decays of B meson have been researched not only in theory but also by experiment, such as $B^0(B_S)\to\gamma\nu\bar{\nu}$, their branching ratios are about $0.7\times10^{-9}(2.4\times10^{-8})$; $B^0(B_S)\to\gamma\mu^+\mu^-$ are about $0.65\times10^{-10}(1.7\times10^{-9})$; $B^0(B_S)\to\gamma e^+e^-$ are about $0.83\times10^{-10}(1.9\times10^{-9})$ [4, 5]. But there is less research on the pure-leptonic decays of $D_S^-(D^-)$. As the heavy meson, D meson also plays an important role in determining the meson decay constants and other parameters. The pure-leptonic decays of $D_S^-(D^-)$ may be sensitive to establishing new physics beyond the SM. But due to the small mass of leptons, they are helically suppressed by $m_l^2/m_{D_S}^2$. Fortunately, similar to the pure-leptonic decays of B mesons, it will be overcome by a photon radiated from the charged particles at the cost of the electromagnetic suppression with coupling constant α [6–8].

A pilot study has been carried out in Refs. [7, 9–11]. However, either they calculate only one dominant diagram, or their results are not consistent with each other. In Ref. [12], we find that different diagrams in B^- decay are $\Gamma_a{:}\Gamma_b{:}\Gamma_c{:}\Gamma_{a+b+c}=1.40{:}0.0005{:}0.04{:}1$, obviously Γ_a is the dominant diagram compared with the other ones. So, it is enough to calculate only one dominant diagram and to neglect the others in the case of B^- decay. But in the D_S decay, the ratio is $\Gamma_a{:}\Gamma_b{:}\Gamma_c{:}\Gamma_{a+b+c}=14.72{:}3.47{:}17.32{:}1$ and the ratio of the D^- decay is $\Gamma_a{:}\Gamma_b{:}\Gamma_c{:}\Gamma_{a+b+c}=7.30{:}0.94{:}6.03{:}1$. So, the contribution of all three diagrams is large and cannot be neglected. Then, unlike the pure leptonic decays, which only depend on the meson decays constant, the radiative leptonic decays of $D_S^-(D^-)$ depend on the structure of the meson wave function heavily [4]. This makes the theoretical prediction on this type of decay more difficult with hadronic uncertainty. However, it also provides useful information about meson wave functions, which are essential for non-leptonic D decays.

In 2003, LÜ Cai-Dian et al. calculated all diagrams that are about the decays of $D_S^-(D^-)\to\gamma l\bar{\nu}$ $(l=e,\mu)$ at the tree level, using the non-relativistic constituent quark model [12]. However, they did not research the uncertainties in these decays due to the parameters and wave functions. In this paper, we will study the uncertainties in radiative decays $D_S^-(D^-)\to\gamma l\bar{\nu}$, which are due to the D meson wave functions ϕ_D^i (i=GEN, GN, KKQT, KLS, Huang) and the parameters in them. In the next section, we analyze the decays of $D_S^-(D^-)\to\gamma l\bar{\nu}$ $(l=e,\mu)$ and use

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the wave functions to calculate their decay width. In Section 3, we give the numerical results and a brief conclusion. Finally, we provide a summary.

2 Theory and calculation

The calculation of D meson decay requires the hadronization of c̄s into a D meson, which is the D meson light cone wave function. So, light-cone wave function of the meson is needed in the calculation. The D_S^- meson and D^- meson have a similar structure of wave function, except for different values of parameters characterizing a small $SU(3)$ breaking effect. In general, the two-particle light-cone distribution amplitudes of D meson, up to twist-3 accuracy, are defined by [13]

$$\int \frac{d^4\omega}{(2\pi)^4} e^{ik\omega} \langle 0|\bar{c}(0)s(\omega)|D_S^-\rangle$$

$$= -\frac{i}{\sqrt{2N_C}} [\not{P} + M_D]\gamma_5 \phi_D(x,b), \tag{1}$$

$$\int \frac{d^4\omega}{(2\pi)^4} e^{ik\omega} \langle 0|\bar{c}(0)s(\omega)|D^-\rangle$$

$$= -\frac{i}{\sqrt{2N_C}} \left[(\not{P} + M_D)\not{\epsilon}_L \phi_D^L(x,b) + (\not{P} + M_D)\not{\epsilon}_T \phi_D^T(x,b)\right]. \tag{2}$$

As for the $D_S^-(D^-)$ meson, we assume that $\phi_D^L = \phi_D^T = \phi_D$ in the heavy quark limit. Then the D meson wave function is decomposed in terms of spin structure as [14]

$$\Phi_D(x,b) = -\frac{i}{\sqrt{2N_C}} [\not{P} + M_D]\gamma_5 \phi_D(x,b), \tag{3}$$

where $\not{P}$ is the D meson's momentum, M_D is the mass of D and ϕ_D is the Lorentz scalar distribution amplitude. As for the momentum distribution amplitude $\phi_D(x,b)$, we collect it as below [13, 15, 16],

$$\phi_D^{GEN}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D 6x(1-x)\left[1 + C_D(1-2x)\right], \tag{4}$$

$$\phi_D^{MGEN}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D 6x(1-x)\left[1 + C_D(1-2x)\right] \exp\left(-\frac{\omega^2 b^2}{2}\right), \tag{5}$$

$$\phi_D^{GN}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D N_D x \exp\left(-\frac{xM_D}{\omega}\right) \frac{1}{1+b^2\omega^2}, \tag{6}$$

$$\phi_D^{KKQT}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D N_D (1-x)\theta(1-x)\theta\left(\frac{2\Lambda_D}{M_D} + x - 1\right) J_0\left[b\sqrt{(1-x)\frac{2\Lambda_D}{M_D} + x - 1}\right], \tag{7}$$

$$\phi_D^{KLS}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D N_D \sqrt{x(x-1)} \exp\left[-\frac{1}{2}\left(\frac{xM_D}{\omega}\right)^2 - \frac{\omega^2 b^2}{2}\right], \tag{8}$$

$$\phi_D^{Huang}(x,b) = \frac{1}{2\sqrt{2N_C}} f_D N_D x(1-x) \exp\left(-\Lambda_D \frac{(1-x)m_d^2 + m_c^2}{x(1-x)}\right), \tag{9}$$

where J_0 is the Bessel function, f_D is the D meson decay constant and variable x is the mentum fraction of the light quark in the D meson. The first model ϕ_D^{GEN} is the Gegenbauer polynomial-like form, and $C_D = 0.7$. That the second candidate model ϕ_D^{MGEN} has an additional exponential term is for keeping the $k_\perp$ dependent. The third one ϕ_D^{GN} is an exponential model, and the fourth model ϕ_D^{KKQT} is obtained by solving the equations of motion without three-parton contributions, which were first proposed for the B meson, with $\Lambda_D = 0.75$. Here we use the heavy quark symmetry and modify the parameters to make them D meson DAs. The fifth model ϕ_D^{KLS} is a Gaussian type model. The sixth DA (we find it in the paper written by Huang, so we call it ϕ_D^{Huang}) is derived from the BHL prescription, with $m_d = 0.37$ GeV, $m_c = 1.5$ GeV. In the above candidate DAs, only the second model ϕ_D^{MGEN} has two parameters, while ϕ_D^{GEN} and ϕ_D^{Huang} are just b independent. But we think the transverse momentum dependent part of the wave function is irrelevant here, so we set $b=0$ in these D meson distribution amplitudes [4]. Then ϕ_D^{MGEN} is the same as ϕ_D^{GEN}. And we get five wave functions, which are all b independent in Fig. 1.

Their normalization relation is

$$\int_0^1 \phi_D^i(x, b=0)dx = \frac{f_D}{2\sqrt{2N_C}}, \tag{10}$$

where $N_C = 3$ is the color degree of freedom and f_D is the D meson decay constant.

As shown in Fig. 1, the five wave functions all have a small sharp, excepting their different shape. Besides the variable x, all of the functions have only one parameter, the C_D, ω or Λ_D. In the following, we will study the uncertainties due to these wave functions and the parameters in them.

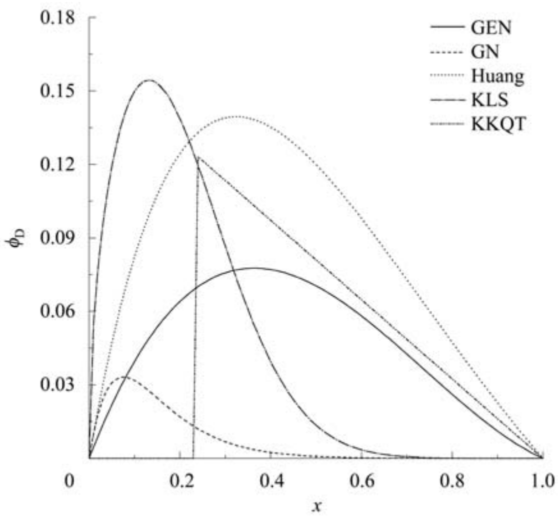

Fig. 1. D meson wave functions ϕ_D^i (i=GEN, GN, Huang, KKQT, KLS).

In the SM, the Feynman diagram for pure-leptonic decays of $D_S^-(D^-)\to l\bar{\nu}$ is in Fig. 2. However, it is helically suppressed by m_l^2/m_W^2, as we mentioned in Section 1. But the helicity suppression would be overcome if a photon were emitted from the charged particles [6]. We see that there are four charged particle lines in Fig. 2. So the radiative decays of $D_S^-(D^-)\to\gamma l\bar{\nu}$ have four diagrams, such as Fig. 3.

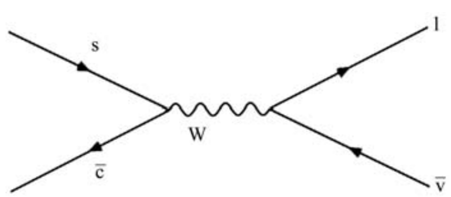

Fig. 2. Feynman diagrams in the Standard Model for $D_s^-\to l\bar{\nu}$ decay.

However, the photon line is emitted from the internal charged line of W boson in Fig. 3(d), and there is a suppression factor of m_c^2/m_W^2 at this time. Compared with the process $D_S^-(D^-)\to l\bar{\nu}$, it can be neglected [8]. So, we get three decay amplitudes $\mathcal{H}_a$, $\mathcal{H}_b$ and $\mathcal{H}_c$ for the photon radiating from the quarks, $\bar{c}$ and l, which correspond to Fig. 3(a), (b) and (c),

$$\mathcal{H}_a = -i\sqrt{2}G_F eV_{cs}\bar{c}\left[Q_c \not{\epsilon}_\gamma \frac{\not{p}_\gamma-\not{p}_c+m_c}{(p_c\cdot p_\gamma)}\gamma_\mu P_L\right] \times s(\bar{l}\gamma^\mu P_L\nu), \tag{11}$$

$$\mathcal{H}_b = -i\sqrt{2}G_F eV_{cs}\bar{c}\left[Q_s P_R\gamma_\mu \frac{\not{p}_s-\not{p}_\gamma+m_s}{(p_s\cdot p_\gamma)}\not{\epsilon}_\gamma\right] \times s(\bar{l}\gamma^\mu P_L\nu), \tag{12}$$

$$\mathcal{H}_c = -i\sqrt{2}G_F eV_{cs}(\bar{c}\gamma^\mu P_L s) \times\left[\bar{l}\not{\epsilon}_\gamma \frac{\not{p}_\gamma+\not{p}_l+m_l}{(p_l\cdot p_\gamma)}\gamma_\mu P_L\nu\right]. \tag{13}$$

We use the interpolating field techniques [17] that relate the hadronic matrix elements to the decay constants of the D_S^- mesons. The decay constant $f_{D_S^-}$ for a charged pseudoscalar meson is defined by $\langle 0|\bar{c}\gamma^\mu\gamma_5 s|D_S^-\rangle = i f_{D_S^-} p^\mu_{D_S}$ [18].

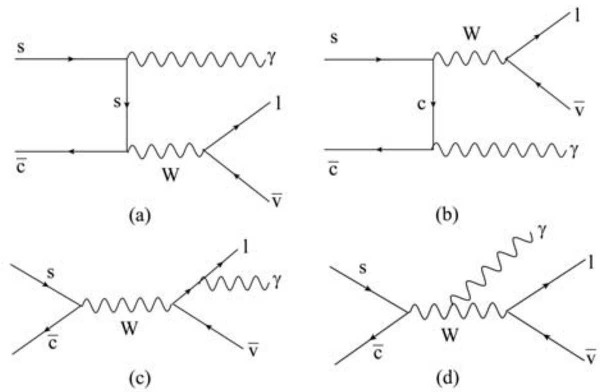

Fig. 3. Feynman diagrams in the Standard Model for $D_s^-\to\gamma\, l\bar{\nu}$ decay.

After calculation and simplification, we get the whole decay amplitude for $D_S^-(D^-)\to\gamma l\bar{\nu}$ ($l=e,\mu$),

$$\mathcal{A} = \frac{\sqrt{2}eG_F V_{cs}}{6} f_D\left[\left(\frac{1}{(p_s\cdot p_\gamma)}-2\frac{1}{(p_c\cdot p_\gamma)}\right) \times i\epsilon_{\alpha\beta\mu\nu}p_\gamma^\alpha\epsilon_\gamma^\beta p_B^\nu + \left(6-\frac{1}{(p_s\cdot p_\gamma)}-2\frac{1}{(p_c\cdot p_\gamma)}\right) \times(p_{\gamma\nu}\epsilon_{\gamma\mu}-p_{\gamma\mu}\epsilon_{\gamma\nu})p_{D_S}\right](\bar{l}\gamma^\mu P_L\nu), \tag{14}$$

where we neglect all of the terms suppressed by m_l/m_c.

Using the normalization relation of the wave function in Eq. (10), the decay amplitude is then

$$\mathcal{A} = \frac{\sqrt{2}eG_F V_{cs}}{6P_B\cdot p_\gamma}2\sqrt{2N_c}\left[\left(\int_0^1\frac{\phi(x)}{(p_s\cdot p_\gamma)}dx -2\int_0^1\frac{\phi(x)}{(p_c\cdot p_\gamma)}dx\right)i\epsilon_{\alpha\beta\mu\nu}p_\gamma^\alpha\epsilon_\gamma^\beta p_B^\nu + \left(6-\int_0^1\frac{\phi(x)}{(p_s\cdot p_\gamma)}dx-2\int_0^1\frac{\phi(x)}{(p_c\cdot p_\gamma)}dx\right) \times(p_{\gamma\nu}\epsilon_{\gamma\mu}-p_{\gamma\mu}\epsilon_{\gamma\nu})p_{D_S}\right](\bar{l}\gamma^\mu P_L\nu). \tag{15}$$

From the definition of the wave function, we have $p_s=(1-x)P_{D_S}$, $p_c=xP_{D_S}$. Then we can get the decay amplitude,

$$\mathcal{A}=\frac{\sqrt{2}eG_F V_{cs}}{6P_B\cdot p_\gamma}2\sqrt{2N_c}\left[\left(\left(\int_0^1\frac{\phi(x)}{(1-x)}dx-2\int_0^1\frac{\phi(x)}{x}dx\right)\right.\right.$$
$$\times i\epsilon_{\alpha\beta\mu\nu}p_\gamma^\alpha\epsilon_\gamma^\beta P_B^\nu+\left(6-\int_0^1\frac{\phi(x)}{(1-x)}dx\right.$$
$$\left.\left.-2\int_0^1\frac{\phi(x)}{x}dx\right)(p_{\gamma\nu}\epsilon_{\gamma\mu}-p_{\gamma\mu}\epsilon_{\gamma\nu})P_{D_S}\right](\bar{l}\gamma^\mu P_L\nu). \tag{16}$$

Finally, the amplitude can be written simply as

$$\mathcal{A}=\frac{\sqrt{6}eC}{6}[C_1 i\epsilon_{\alpha\beta\mu\nu}p_\gamma^\alpha\epsilon_\gamma^\beta p_B^\nu+C_2(p_{\gamma\nu}\epsilon_{\gamma\mu}$$
$$-p_{\gamma\mu}\epsilon_{\gamma\nu})p_{D_S}](\bar{l}\gamma^\mu P_L\nu), \tag{17}$$

and the parameters in Eq. (17) are

$$C=2\sqrt{2}G_F V_{cs}\alpha, \tag{18}$$

$$C_1=\int_0^1\frac{\phi(x)}{(1-x)}dx-2\int_0^1\frac{\phi(x)}{x}dx, \tag{19}$$

$$C_2=6-\int_0^1\frac{\phi(x)}{(1-x)}dx-2\int_0^1\frac{\phi(x)}{x}dx, \tag{20}$$

After squaring the amplitudes, and then performing the phase space integration over one of the two Dalitz variables, we get the differential decay width versus the photon energy E_γ [19],

$$\frac{d\Gamma}{dE_\gamma}=\frac{6C^2}{12\pi}(C_1^2+C_2^2)(M_{D_S}-E_\gamma)E_\gamma. \tag{21}$$

Integrating the variable E_γ, we obtain the decay width,

$$\Gamma=\frac{M_{D_S}^3C^2\alpha}{(24\pi)^2}(C_1^2+C_2^2). \tag{22}$$

Finally, we obtain the branching ratio,

$$B_r=\frac{\Gamma\cdot\tau_{D_S}}{\hbar}. \tag{23}$$

3 Numerical results

In the numerical calculations, we use the following input parameters [9, 10, 19, 20],

$$\begin{aligned}
M_{D_S^-} &= 1.97\text{ GeV},\ \ |V_{cs}|=0.974,\ \ \alpha=1/137,\\
M_{D^-} &= 1.87\text{ GeV},\ \ |V_{cd}|=0.22,\ \ \omega=0.4,\\
\hbar &= 6.582122\times10^{-25},\ \ G_F=1.66\times10^{-5}\text{ GeV}^{-2},\\
\tau_{D^-} &= 1.05\times10^{-12},\ \ f_{D_-}=0.23\text{ GeV},\ \ \Lambda_D=0.75,\\
\tau_{D_S^-} &= 0.5\times10^{-12},\ \ f_{D_S^-}=0.23\text{ GeV},\\
\pi &= 3.14159265359.
\end{aligned}$$

In Fig. 4, we show the differential decay width of $D_S^-\to\gamma l\bar{\nu}$ and $D^-\to\gamma l\bar{\nu}$ versus the photo energy E_γ, respectively. Obviously, different wave functions have the same shape. But the photo energy has a different peak value – it peaks in the range of 2.2 GeV to 2.8 GeV for $D_S^-\to\gamma l\bar{\nu}$ and 1.0 GeV to 1.4 GeV for $D^-\to\gamma l\bar{\nu}$, respectively. The uncertainties here are from the style of D meson wave functions and their parameters. We show them in Table 1, Table 2 and Table 3. Table 1 contains all of the influences from the wave function style. Table 2 shows the uncertainty due to the parameters of the wave function. By calculation, we find that the influences of the parameters in wave function are very small. And we fit the parameters for the wave functions. Table 3 shows the branching ratios of the different wave functions. From the table, we also see that different wave functions give different branching ratios. From Fig. 4, we find that the line of ϕ_D^{KKQT} is higher and the ϕ_D^{KLS} is lower.

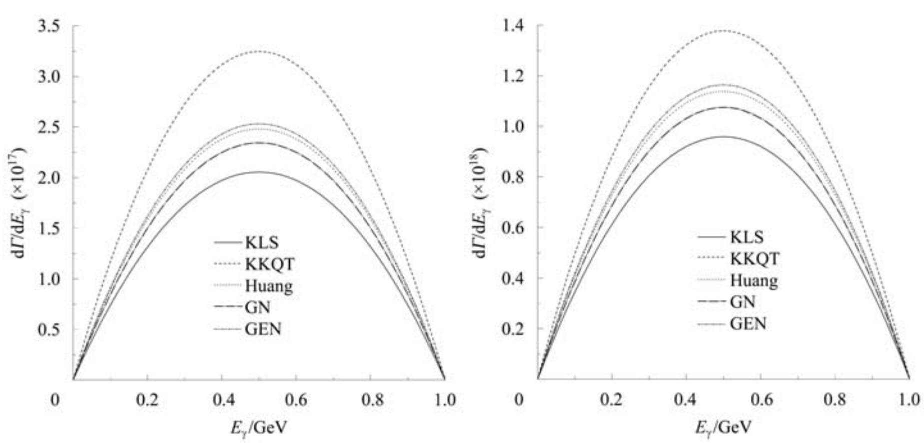

Fig. 4. Differential decay of $D_S^-\to\gamma l\bar{\nu}$ and $D^-\to\gamma l\bar{\nu}$ versus the photo energy E_γ.

Table 1. Comparison of influences from different wave functions. It can be found in Eq. (17–20) that the influences of wave functions are all in C_1 and C_2. And obviously, the value variety of the $(C_1^2+C_2^2)$ is big. It is about 24 to 42.

wave function	GEN	GN	KKQT	KLS	Huang
$(C_1^2+C_2^2)D_S^-\to\gamma l\bar{\nu}$	30.7426	28.4296	41.5979	24.0445	30.1052
$(C_1^2+C_2^2)D^-\to\gamma l\bar{\nu}$	30.7426	28.4446	41.9284	24.5063	30.0337

Table 2. Comparison of influences from different parameters of the wave functions. We find that the effects of parameters on the wave functions are small. The biggest influence is from Λ_D in ϕ^{KKQT}, and it is only of the order of 10^{-6}.

wave function	parameter	$B_r(D_S^-)\times10^{-5}$	$B_r(D^-)\times10^{-6}$
GEN	$C_D=0.7\pm0.1$	1.263488 ± 0.001868	1.157828 ± 0.001686
GN	$\omega=0.4\pm0.1$	1.169734 ± 0.08120	1.069288 ± 0.018315
KKQT	$\Lambda_D=0.75\pm0.1$	1.706812 ± 0.045922	1.576725 ± 0.068188
KLS	$\omega=0.4\pm0.1$	1.025390 ± 0.091764	0.953498 ± 0.082003
Huang	$\Lambda_D=0.75\pm0.1$	1.237763 ± 0.007077	1.132268 ± 0.006844

Table 3. Comparison of results from different wave functions. This shows that the branching ratios are sensitive to the type of wave function.

wave function	GEN	GN	KKQT	KLS	Huang	C. D. LÜ
$B_r(D_S^-\to\gamma l\bar{\nu})\times10^{-5}$	1.263488	1.169734	1.706812	1.025390	1.237763	1.8
$B_r(D^-\to\gamma l\bar{\nu})\times10^{-6}$	1.157828	1.069288	1.576725	0.953498	1.132268	4.6

The other three ϕ_D^{GEN}, ϕ_D^{Huang} and ϕ_D^{GN} are closer compared with the other wave functions. So, we conclude that these three wave functions are more fit for the pure-leptonic radiative decay of $D_S^-(D^-)\to\gamma l\bar{\nu}$. Of course, they will be measured by future experiments, such as BESIII.

4 Summary

In this paper, we find that the branching ratios are sensitive to the type of wave function. We show that the decay branching ratios in the SM for $D_S^-\to\gamma l\bar{\nu}$ ($l=e,\mu$) is of the order of 10^{-5} and for $D^-\to\gamma l\bar{\nu}$ ($l=e,\mu$) of 10^{-6}. The different type of wave functions and different input parameters affect the size of the decay branching ratios but not the shape of the photon energy spectrum. These decay channels are useful to determine the decay constants f_D and or D meson wave function. After calculation, it is found that our leading order results are the same order as other approaches, but a little smaller than the sum rule approaches [21]. Such a branching ratio for the radiative leptonic decays can be measured in future BESIII experiments.

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