Heavy quarkonia spectra and their decays in a relativistic quark model

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Abstract: Using the phenomenological relativistic harmonic model (RHM) for quarks, we have obtained the masses of S wave charmonium and bottomonium states. The full Hamiltonian used in the investigation has Lorentz scalar plus vector confinement potential, along with the confined one gluon exchange potential (COGEP). A good agreement with the experimental masses for the ground and the radially excited states is obtained both for the triplet and singlet S wave mesons. The decay properties of the ground state charmonium and bottomonium are investigated.

Key words: relativistic harmonic model, confined one gluon exchange potential, S-wave spectrum, decay rates

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1 Introduction

Heavy quark spectroscopy has undergone a great resurgence in recent years. This is mostly due to experiments at CLEO, DELPHI, BELLE, BABAR, etc., which have been continuously providing more accurate and new information about hadrons from light to heavy quark flavor sectors [1, 2].

Since the exact form of confinement from QCD is not known, one has to go for phenomenological models. The phenemenological models are either non-relativistic quark models (NRQM) [3–7] or the relativistic quark models [8]. The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term. Using the quark model the hadron spectra have been predicted successfully. The prediction of mass spectrum in accordance with experimental results doesn't guarantee the validity of a model for describing hadronic interactions. This is because different potentials have been proposed which reproduce the same spectra. Therefore using the model, one must be able to calculate other observables like radiative decay widths, leptonic decay widths, two-photon decay widths, etc. Leptonic decay widths are a test of the compactness of the quarkonium system and provide important information complementary to level spacings [9]. The decay of a heavy quark-antiquark pair into final states involving leptons, photons and light quarks can provide useful information on the strong coupling constant (α_s) [10–12]. Heavy quarkonium decays also provide a deeper insight into the exact nature of interquark forces and decay mechanisms.

Since relativistic corrections are of importance [13–16], in this work, we have used the relativistic harmonic model (RHM) [17] along with the confined one gluon exchange potential (COGEP) [18] to investigate the S wave charmonium and bottomonium spectrum. The essential new ingredient in our investigation of the mesonic states is to take into account the confinement of gluons in addition to the confinement of quarks. In this work, for the confinement of quarks we are making use of the RHM which has been successful in explaining the properties of light hadrons [8]. For the confinement of gluons we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM [19, 20]. The confined gluon propagators (CGP) are derived in CCM. Using CGP the COGEP was obtained [18]. The Hamiltonian used in the investigation has a Lorentz scalar plus a vector harmonic

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oscillator potential, in additon to the COGEP. In our earlier work, it was shown that the terms in the COGEP arising out of the confinement of gluons give the required intermediate range attraction in the nucleon-nucleon interaction and also give a good account of n-p and p-p differential cross sections [20, 21]. In this work, the total mass of the meson is obtained by calculating the energy eigenvalues of the Hamiltonian in the harmonic oscillator basis spanned over a space extending up to the radial quantum number $n_{\max}=5$. The parameters and the radial wave function employed for the prediction of the spectra are being used for the prediction of the decay properties of the ground state mesons.

In Sec. 2, we review the RHM and CCM models and give a brief description of the COGEP. The COGEP is obtained using CGP. Also the parameters used in our model are dicussed. In Sec. 3 a brief description of the various decay properties of mesons is given. The results and discussions of the calculations are presented in Sec. 4. Conclusions are given in Sec. 5.

2 Relativistic harmonic model and COGEP

In RHM [8, 17], quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic oscillator potential,

$$V_{\mathrm{c}}=\frac{1}{2}(1+\gamma_0)A^2r^2+M, \tag{1}$$

where γ_0 is the Dirac matrix:

$$\gamma_0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{2}$$

M is the quark mass and A^2 the confinement strength. They have a different value for each quark flavor. In the case of pure vector potential one would have the problem of the Klein paradox. The important feature of the scalar+vector potential is that it provides a consistent picture for both mesons and baryons [18]. On the other hand, the pure vector potential would produce only quark-antiquark bound states, whereas the scalar potential provides an attractive force for both the quark-antiquark and the quark-quark states. Thus, for the confinement of quarks, a scalar+vector potential is the more appropriate choice. The RHM with scalar and vector combination as given in Eq. (1) has been immensely successful in the prediction of hadronic properties like the light meson spectrum, prediction of N-N scattering phase shifts and differential cross sections [8, 20, 21]. Additionally, the Lorentz scalar+vector potential does not have the problem of the Klein paradox [22].

In the RHM, the confined single quark wavefunction Ψ is given by:

$$\Psi=N\begin{pmatrix}\phi\\ \dfrac{\sigma.\boldsymbol{P}}{E+M}\quad\phi\end{pmatrix}. \tag{3}$$

The normalization constant is given by

$$N=\sqrt{\frac{2(E+M)}{3E+M}}. \tag{4}$$

Here E is the eigenvalue of the single particle Dirac equation with the interaction potential given by (1). We perform a similarity transformation to eliminate the lower component of Ψ such that

$$U\Psi=\phi, \tag{5}$$

where U is given by

$$\frac{1}{N\left[1+\dfrac{\boldsymbol{P}^2}{(E+M)^2}\right]}\begin{pmatrix}1 & \dfrac{\sigma.\boldsymbol{P}}{E+M}\\ -\dfrac{\sigma.\boldsymbol{P}}{E+M} & 1\end{pmatrix}. \tag{6}$$

Here U is the momentum and state (E) dependent transformation operator. With this transformation, the upper component ϕ satisfies the equation,

$$\left[\frac{\boldsymbol{P}^2}{E+M}+A^2r^2\right]\phi=(E-M)\phi, \tag{7}$$

which is like the three-dimensional harmonic oscillator equation with an energy-dependent parameter Ω_n^2:

$$\Omega_n^2=A(E_n+M)^{\frac{1}{2}}. \tag{8}$$

The eigenvalue of (7) is given by,

$$E_n^2=M^2+(2n+1)\Omega_n^2. \tag{9}$$

The total energy due to confinement is obtained by adding the individual contributions of the quarks. The spurious centre of mass (CM) is corrected [23] by using intrinsic operators for the $\Sigma_i r_i^2$ and $\Sigma_i\nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero point contribution from the E^2 expression.

The quark-antiquark interaction potential is given by the COGEP. The COGEP is obtained from the scattering amplitude [20],

$$M_{ij}=\frac{g_{\mathrm{s}}^2}{4\pi}\bar{\psi}_i\gamma^{\mu}\frac{\lambda_i^a}{2}\psi_i D_{\mu\nu}^{\mathrm{ab}}(q)\bar{\psi}_j\gamma^{\nu}\frac{\lambda_j^{\mathrm{b}}}{2}\psi_j, \tag{10}$$

where $\bar{\psi}=\psi^{\dagger}\gamma_0$, $\psi_{i/j}$ are the wave functions of the quarks in the RHM, $D_{\mu\nu}^{\mathrm{ab}}=\partial_{\mathrm{ab}}D_{\mu\nu}$ are the CCM gluon propagators [19, 24] in the momentum representation, $g_s^2/4\pi(=\alpha_s)$ is the quark-gluon coupling constant and λ_i is the color $SU(3)_c$ generator of the i^{th} quark.

In CCM [19, 24], the coupled non-linear terms in the equation of motion of a gluon are simulated by a self-induced color current $j_\mu=\theta_\mu^\nu A_\nu(=m^2A_\mu)$ or equivalently an effective mass term for all the gluons with $m^2=c^4r^2-2c^2\delta_{\mu 0}$. The equations of motion $\Box A_\mu+m^2A_\mu=0$ are easily solved using harmonic oscillator modes in the gauge $\partial^\mu A_\mu=0$. The consistency of $\partial^\mu A_\mu=0$ and $\partial^\mu j_\mu=\partial^\mu(m^2A_\mu)=0$ imposes a secondary gauge condition: $\nabla\cdot\boldsymbol{A}+c^2\boldsymbol{r}\cdot\boldsymbol{A}=\boldsymbol{a}\cdot\boldsymbol{A}$ termed "oscillator gauge", where a is the usual harmonic oscillator annihilation operator. The propagators are then obtained very simply using the properties of harmonic oscillator wave functions as follows:

$$D_1(r,r',E=0)\equiv\left\langle r\left|\frac{c}{2\boldsymbol{a}\cdot\boldsymbol{a}^\dagger+3}\right|r'\right\rangle$$

$$=c\sum_{\{N\}}\frac{\psi_N^*(r)\psi_N(r')}{2N+3}. \qquad (11)$$

Transferring the source point r to the origin we obtain $(r-r'\to r)$

$$D_1(r,0,E=0)\equiv D_1=c\sum_{\{N\}}\frac{\psi_N^*(r)\psi_N(0)}{2N+3}$$

$$=\frac{\Gamma(3/4)c}{(4\pi cr)^{3/2}}W_{0;-1/4}(c^2r^2). \qquad (12)$$

Silmilarly,

$$D_0(r,0,E=0)\equiv D_0=c\sum_{\{N\}}\frac{\psi_N^*(r)\psi_N(0)}{2N+1}$$

$$=\frac{\Gamma(1/4)c}{(4\pi cr)^{3/2}}W_{1/2;-1/4}(c^2r^2), \qquad (13)$$

where the W's are Whittaker functions ($\sim\exp[-(rc)^2/2]/r$). The complete propagators are given by

$$D_{00}(r)=4\pi D_0(r), \qquad (14)$$

where $D_0(r)$ is given by Eq.(13). The $D_{ik}(r)$ is given by

$$D_{ik}(r)=4\pi\left(\delta_{ik}-\frac{a_i^\dagger a_k}{\boldsymbol{a}\cdot\boldsymbol{a}^\dagger}\right)D_1(r), \qquad (15)$$

where D_1 is given by Eq. (12). It should be noted that these propagators are similar to those given by Feynman et al. [25] apart from the time coordinate which is suppressed here. The closed analytical expressions for $D_0(r)$ and $D_1(r)$ were obtained in a translationally invariant ansatz [19].

We perform a similarity transformation on ψ and $\psi^\dagger$ and express ψ and $\psi^\dagger$ in terms of ϕ and $\phi^\dagger$. The details can be found in Refs. [18–20]. For example,

$$\psi_i^\dagger\psi_i=\psi_i^\dagger U^\dagger(U^\dagger)^{-1}U^{-1}U\psi_i$$

$$=\phi_i^\dagger(U^\dagger)^{-1}U^{-1}\phi_i. \qquad (16)$$

Similarly,

$$\psi_i^\dagger\alpha_i\psi_i=\phi_i^\dagger(U^\dagger)^{-1}\alpha_iU^{-1}\phi_i, \qquad (17)$$

where U is given by Eq. (6) and

$$\alpha_i=\begin{bmatrix}0&\sigma_i\\\sigma_i&0\end{bmatrix},$$

where σ_i's are the usual Pauli matrices. With this transformation the two components of ψ are eliminated without any approximation. The scattering amplitude is now expressed in terms of the two-component spinor ϕ and the momentum dependent operator U. After substituting ϕ and U in the expression for the scattering amplitude (Eq. (10)), it essentially corresponds to the Born amplitude in the momentum representation:

$$M_{ij}=\frac{1}{4}\alpha_sN^4\phi_i^\dagger\phi_j^\dagger\tilde{U}\phi_i\phi_j\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j. \qquad (18)$$

By taking the Fourier transform of each term in the Born amplitude, we obtain the COGEP. The central part of the COGEP is [18]:

$$V_{\text{COGEP}}^{\text{cent}}=\frac{\alpha_s}{4}N^4\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left[D_0(r)+\frac{1}{(E+M)^2}[4\pi\delta^3(r)\right.$$

$$\left.-c^4r^2D_1(r)]\left[1-\frac{2}{3}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\right], \qquad (19)$$

where the first term is the residual Coulomb energy and the second and the third terms are the chromomagnetic interaction leading to the hyperfine splittings.

The q$\bar{\text{q}}$ wave function for each meson state is expressed in terms of the oscillator wave functions corresponding to the center of mass (CM) and relative coordinates. The harmonic oscillator wavefunction is,

$$\psi_{nlm}(r,\theta,\phi)$$

$$=N\left(\frac{r}{b}\right)^lL_n^{l+\frac{1}{2}}\left(\frac{r^2}{b^2}\right)\exp\left[-\frac{r^2}{2b^2}\right]Y_{lm}(\theta,\phi), \qquad (20)$$

where N is the normalization constant given by,

$$|N|^2=\frac{2\ n!}{b^3\pi^{\frac{1}{2}}}\frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)!, \qquad (21)$$

and $L_n^{l+\frac{1}{2}}(x)$ are the associated Laguerre polynomials. The oscillator quantum number for the CM wavefunctions is restricted to $N_{\text{CM}}=0$. The Hilbert space of relative wavefunctions is truncated at radial quantum number $n=5$. The Hamiltonian matrix is constructed in the basis states of $|N_{\rm CM}=0,\ L_{\rm CM}=0;\ n^{2S+1}L_J\rangle$ and diagonalized. The diagonal values give the masses of the ground and radially excited states.

The parameters of the RHM are the masses of the heavy quarks ($m_{\rm c}$ and $m_{\rm b}$) and the oscillator size parameter $b_n(=1/\Omega_n)$. The mass of charm and bottom quarks are fixed to reproduce the J/ψ and $\Upsilon(1S)$ masses. The $\alpha_{\rm s}$ is fixed from $J/\psi-\eta_{\rm c}$ splitting. The parameters of the COGEP are the same as those used in [26]. The parameter b is fixed so as to reproduce the ground state masses. The parameters used in our calculations are listed in Table 1.

Table 1. Parameters used in our model.

$m_{\rm c}$	1.48 GeV
$m_{\rm b}$	4.62 GeV
$\alpha_{\rm s}$	0.2

3 Decay properties of the ground state quarkonium

The leptonic decay constants are a simple probe of the short distance structure of hadrons and hence a useful observable for testing quark dynamics [27]. The decay constants of the pseudoscalar and vector mesons are given by the Van Royen and Weisskopf formula [27, 28]:

$$f^2_{\rm P/V}=12\frac{|\psi_{\rm P/V}(0)|^2}{m_{\rm P/V}}, \tag{22}$$

where $\psi_{\rm P/V}(0)$ is the wave function of the pseudoscalar/vector meson calculated at the origin and $m_{\rm P/V}$ is the mass of the pseudoscalar/vector meson.

Leptonic decay widths are a test of the compactness of the quarkonium system, and provide important information complementary to level spacings. The leptonic decay widths of the heavy quarkonia are proportional to the squares of the wave functions at the origin. The partial width for a n^3S_1 state to decay to a lepton pair including the first order QCD radiative corrections is given by [9, 27, 28]:

$$\Gamma_{\rm l^+l^-}=16\pi\alpha^2e_{\rm q}^2\frac{|\psi(0)|^2}{m_{\rm V}^2}\left(1-\frac{16\alpha_{\rm s}}{3\pi}\right), \tag{23}$$

where $\alpha(=1/137)$ is the electromagnetic fine structure constant, $e_{\rm q}$ is the quark charge, $m_{\rm V}$ is the mass of the vector meson and $\psi(0)$ is the wavefunction of the meson calculated at the origin.

The $^1S_0$, $^3P_0$ and $^3P_2$ levels of charmonium and upsilon systems can decay into two photons. The same states can also decay into two gluons, which accounts for a substantial portion of the hadronic decays for states below the $c\bar{c}$ or the $b\bar{b}$ threshold [29]. The width for two-photon decay is [30]:

$$\Gamma_{\gamma\gamma}=\frac{12\pi e_{\rm q}^4\alpha^2}{m_{\rm q}^2}|\psi(0)|^2\left(1-\frac{3.4\alpha_{\rm s}}{\pi}\right), \tag{24}$$

where $e_{\rm q}$ is the quark charge ($e_{\rm q}=2/3$ for c and $-1/3$ for b). The term in parentheses is the first order QCD correction factor. The two-gluon decay width is given by [30]:

$$\Gamma_{\rm gg}=\frac{8\pi\alpha_{\rm s}^2}{3m_{\rm q}^2}|\psi(0)|^2\times CF, \tag{25}$$

where CF is the first order QCD correction factor. It is given by: $(1+4.8\alpha_{\rm s}/\pi)$ for $\eta_{\rm c}$ and $(1+4.4\alpha_{\rm s}/\pi)$ for $\eta_{\rm b}$ [30].

The partial decay rate of quarkonia to hadrons through three-gluon and two-gluon final states provides information on the strong coupling constant $\alpha_{\rm s}$. The decay widths for $J/\psi\to{\rm ggg}$, $J/\psi\to\gamma{\rm gg}$, $\Upsilon\to{\rm ggg}$ and $\Upsilon\to\gamma{\rm gg}$ are given by [30]:

$$\Gamma_{\rm ggg}(J/\psi)=\frac{40(\pi^2-9)\alpha_{\rm s}^3|\psi(0)|^2}{81m_{\rm q}^2}\left(1-\frac{3.7\alpha_{\rm s}}{\pi}\right), \tag{26}$$

$$\Gamma_{\gamma{\rm gg}}(J/\psi)=\frac{32(\pi^2-9)\alpha\alpha_{\rm s}^2e_{\rm q}^2|\psi(0)|^2}{9m_{\rm q}^2}\left(1-\frac{6.7\alpha_{\rm s}}{\pi}\right), \tag{27}$$

$$\Gamma_{\rm ggg}(\Upsilon)=\frac{40(\pi^2-9)\alpha_{\rm s}^3|\psi(0)|^2}{81m_{\rm q}^2}\left(1-\frac{4.9\alpha_{\rm s}}{\pi}\right), \tag{28}$$

$$\Gamma_{\gamma{\rm gg}}(\Upsilon)=\frac{32(\pi^2-9)\alpha\alpha_{\rm s}^2e_{\rm q}^2|\psi(0)|^2}{9m_{\rm q}^2}\left(1-\frac{7.4\alpha_{\rm s}}{\pi}\right), \tag{29}$$

where in the above equations the terms in parentheses are the first order QCD correction factor. The three photon decay width for $^3S_1\to\gamma\gamma\gamma$ is given by [30]:

$$\Gamma_{\gamma\gamma\gamma}=\frac{16(\pi^2-9)\alpha^3e_{\rm q}^6|\psi(0)|^2}{3m_{\rm q}^2}\left(1-\frac{12.6\alpha_{\rm s}}{\pi}\right). \tag{30}$$

4 Results and discussions

The masses of the singlet and the triplet S wave mesons after diagonalization in harmonic oscillator basis with $n_{\rm max}=5$ are listed in Table 2 in comparison with experiment [1]. The J/ψ was the first charmonium state discovered [31, 32]. It is the lowest $^3S_1$ $c\bar{c}$ state that can couple directly to the virtual photons produced in e^-e^+ collisions [9]. The $\psi(2S)$ resonance was discovered at SLAC in e^-e^+ collisions [33]. The most precise measurement of $\psi(1S)$ and $\psi(2S)$ mass to date comes from the KEDR collaboration, with $m(\mathrm{J}/\psi) = 3096.917 \pm 0.010 \pm 0.007$ MeV with a relative uncertainity 4×10^{-6} and $m(\psi(2S)) = 3686.093 \pm 0.034$ MeV with a relative uncertainity of 7×10^{-6} [34]. The PDG [1] values are $m(\mathrm{J}/\psi) = 3097$ MeV and $m(\psi(2S)) = 3686$ MeV. In this work, we obtain $m(\mathrm{J}/\psi) = 3097$ MeV and $m(\psi(2S)) = 3646$ MeV.

The $\eta_c(1\ ^1S_0)$ is the lightest charmonium state which was first observed in the radiative decays of the J/ψ and $\psi(2S)$ by the Mark II [35] and the Crystal Ball [36] experiments. The Belle Collaboration [37] has observed the η_c in hadronic B decays and the BES experiment [38] has studied the ground state of charmonium produced in radiative decay $\mathrm{J}/\psi \to \gamma\eta_c$.

Table 2. Mass Spectrum (in MeV).

meson	present	PDG[1]	[53]
J/ψ	3097	3096.916± 0.011	3100
$\psi(2S)$	3646	3686.09±0.04	3730
$\psi(3S)$	4102		4180
$\psi(4S)$	4687		4560
$\psi(5S)$	4892		
$\eta_c(1S)$	2980	2980.3±1.2	3000
$\eta_c(2S)$	3391	3637±4	3670
$\eta_c(3S)$	3725		4130
$\eta_c(4S)$	4169		
$\Upsilon(1S)$	9460	9460.30±0.26	9460
$\Upsilon(2S)$	9862	10023.26±0.00031	10020
$\Upsilon(3S)$	10193	10355.2±0.0005	10390
$\Upsilon(4S)$	10699	10579.4±0.0012	10680
$\Upsilon(5S)$	10978		10930
$\eta_b(1S)$	9436	9390.9±2.8	9410
$\eta_b(2S)$	9744		10000
$\eta_b(3S)$	9982		10370
$\eta_b(4S)$	10320		10660

The Fermilab E835 Collaboration has reported measurements of η_c directly formed in $\mathrm{p\bar{p}}$ annihilations [39]. BABAR [40] and CLEO [41] have studied η_c produced in two-photon fusion in e^-e^+ annihilations. The PDG value of $\eta_c(1S)$ is 2980 MeV and the mass obtained from our calculation is 2980 MeV. The first experimental claim for the η_c $(2\ ^1S_0)$ state was made by the Crystal Ball Collaboration [42]. The BELLE Colaboration observed a η_c $(2S)$ candidate in the $e^+e^- \to \mathrm{J}/\psi$+X reaction [43] which was confirmed by BABAR [40] and by CLEO [41] in $\gamma\gamma$ collisions. The PDG value of $\eta_c(2S)$ is 3637 MeV and our value is 3391 MeV.

The spin singlet states of quarkonia are of particular importance because they give a direct measurement of the hyperfine splittings between the energy levels. The hyperfine separations are directly related to the spin-spin interaction. The hyperfine mass splittings also provide a test of the Lorentz nature of the $\mathrm{Q\bar{Q}}$ confining potential. For splitting of the observed energy levels, we obtain for $\mathrm{c\bar{c}}$: $m(\mathrm{J}/\psi) - m(\eta_c(1S)) = 117$ MeV, $m(\psi(2S)) - m(\eta_c(2S)) = 255$ MeV, $m(\psi(2S)) - m(\mathrm{J}/\psi) = 549$ MeV, $m(\eta_c(2S)) - m(\eta_c(1S)) = 411$ MeV and the corresponding splittings from experiment are 117 MeV, 49 MeV, 591 MeV and 657 MeV respectively. The observed splittings are in fairly good agreement with experimental results.

Soon after the discovery of the charmonium $(\mathrm{c\bar{c}})$ states, the bottomonium $(\mathrm{b\bar{b}})$ states, $\Upsilon(1S)$, $\Upsilon(2S)$, were discovered in the proton-nucleon collisions [44, 45]. The states $\Upsilon(1S)$, $\Upsilon(2S)$ were later confirmed in e^-e^+ experiments at the DORIS storage ring [46, 47]. The narrow state $\Upsilon(3S)$ and a fourth state $\Upsilon(4S)$ were identified at CESR, Cornell [48, 49]. The obtained masses for the Υ system are $m(\Upsilon(1S)) = 9460$ MeV, $m(\Upsilon(2S)) = 9862$ MeV, $m(\Upsilon(3S)) = 10193$ MeV, $m(\Upsilon(4S)) = 10699$ MeV. The PDG values are $m(\Upsilon(1S)) = 9460$ MeV, $m(\Upsilon(2S)) = 10023$ MeV, $m(\Upsilon(3S)) = 10355$ MeV, $m(\Upsilon(4S)) = 10579$ MeV respectively. The discovery of $\eta_b(1S)$, the lowest member of the bottomonium family was reported recently by the BABAR collaboration [50]. The mass of the above state is reported to be 9391 MeV and the value obtained in our model is 9436 MeV.

We have also calculated the partial decay widths for various decays of quarkonium ground states. The results are listed in Table 3 and compared with experimental values. The leptonic decay constants for the ground state pseudoscalar and vector mesons were calculated using the Van Royen Weisskopf formula, Eq. (22). The calculated values for the charmonium states, $\eta_c(1S)$ (MeV) and J/ψ (MeV), are in good agreement with experimental results. The calculated decay constant for $\Upsilon(1S)$ (356 MeV), is lower than the experimental value (708 MeV). The leptonic decay width of J/ψ was calculated using Eq. (23). The value obtained including the QCD correction factor is 4.04 keV and the value without the correction factor is 6.11 keV. The PDG value is 5.55 keV. Hence the QCD correction factor is found to decrease the leptonic decay of J/ψ. The decay widths for the ggg, γgg and $\gamma\gamma\gamma$ decays for the $1\ ^3S_1$ states were calculated using Eqs. (26–30). The results are listed in Table 3.

Table 3. Decay Properties.

observable	present	Exp.	others
$f_P(\eta_b(1S))$	356 MeV		599 MeV [27]
$f_V(\Upsilon(1S))$	356 MeV	708±8 MeV	665 MeV [27]
$\Gamma_{e^+e^-}(J/\psi)$	4.04 keV	5.55±0.14 keV	5.41 keV [54]
$\Gamma_{\gamma\gamma\gamma}(J/\psi)$	0.7 eV	seen	
$\Gamma_{ggg}(J/\psi)$	77.2 keV	seen	59.5 keV [54]
$\Gamma_{\gamma gg}(J/\psi)$	5.2 keV	seen	5.7 keV [54]
$\Gamma_{e^+e^-}(\Upsilon(1S))$	0.324 keV	1.340±0.018 keV	1.35 keV [9]
$\Gamma_{ggg}(\Upsilon(1S))$	10.2 keV	seen	
$\Gamma_{\gamma gg}(\Upsilon(1S))$	0.23 keV	seen	
$\Gamma_{\eta_c\to\gamma\gamma}$	3.61 keV	7.2±0.9 keV	3.48 keV [29]
$\Gamma_{\eta_b\to\gamma\gamma}$	44.7 eV		
$\Gamma_{\eta_c\to gg}$	5.06 MeV	26.7±3.0 MeV	10.57 MeV [29]
$\Gamma_{\eta_b\to gg}$	0.99 MeV		

The first observation of the decay $J/\psi\to 3\gamma$ was reported by the CLEO collaboration [51]. The signal corresponds to a branching fraction of $B(J/\psi\to 3\gamma = (1.2\pm0.3\pm0.2)\times10^{-5})$, in which the errors are statistical and systematic respectively. The measurements of $B_{\gamma\gamma\gamma}$, $B_{\gamma gg}$, B_{ggg}, and $B_{l^+l^-}$ relative to one another provides crucial experimental tests for QCD predictions [30]. The predicted ratios for the J/ψ state are: $\Gamma(ggg)/\Gamma(l^+l^-) = 19.19$ (11.2±0.4), $\Gamma(\gamma gg)/\Gamma(ggg) = 6.74\%$ (10%±4%), $\Gamma(\gamma\gamma\gamma)/\Gamma(ggg) = 0.9\times10^{-5}$ (1.4×10^{-5}). The experimental values [9] are given in parentheses. The ratio of the inclusive direct photon decay rate to that of the dominant three-gluon decay, $R_\gamma = B(\gamma gg)/B(ggg)$, for the upsilon system was investigated in Ref. [52]. The obtained value was $R_\gamma(1S) = (2.70\pm0.01\pm0.13\pm0.24)\%$, where the errors shown are statistical, systematic, and theoretically model-dependent respectively. From our calculation we obtain $R_\gamma = 2.25\%$.

The two-photon and the two-gluon decay widths for the pseudoscalar states $\eta_c(1S)$ and $\eta_b(1S)$ were calculated using Eqs. (24, 25). The results are listed in Table 3. For charmonium there is reasonable agreement with experimental results. The two-photon decay rates for the upsilon sysytem have not been measured.

5 Summary and conclusions

In this work we have investigated the quarkonium spectra and its decay properties. The masses of the quarkonium states were obtained in the frame work of RHM in the harmonic oscillator basis which spanned over a space extending up to the radial quantum number $n_{max} = 5$. We have also calculated the partial widths for various decays for the ground state charmonium and bottomonium. Our model has the right prediction both for the spectrum and the decay rates.

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References

1 Nakamura K et al. (Particle Data Group). J. Phys. G: Nucl. Part. Phys., 2010, **37**: 075021
2 Amsler C et al. (Particle Data Group). Phy. Lett. B, 2008, **667**: 1
3 Gromes D. Nucl. Phys. B, 1977, **130**: 18
4 Greenberg O. W. Ann. Rev. Nucl. Part. Sci., 1978 **28**: 327
5 Bhaduri R K, Cohler L E, Nogami Y. Nuovo Cim. A, 1981, **65**: 376
6 Godfrey S, Isgur N. Phys. Rev. D, 1985, **32**: 189
7 Silvestre-Brac B, Gignoux C. Phys. Rev. D, 1985, **32**: 743
8 Vijayakumar K B, Hanumaiah B, Pepin S. Eur. Phys. J. A, 2004, **19**: 247
9 Eichten E, Godfrey S, Mahlke H, Rosner J L. Rev. Mod. Phys., 2008, **80**: 1161; arXiiv: hep-ph/0701208v3
10 Appelquist T, Politzer H D. Phys. Rev. Lett., 1975, **34**: 43
11 Kwong W, Rosner J L, Quigg C. Annu. Rev. Nucl. Part. Sci., 1987, **37**: 325
12 Kwong W, Mackenzie P B. Phys. Rev. D, 1988, **37**: 3210
13 Suaya R, Townsend J. Phys. Rev. D, 1977, **16**: 3346
14 Bergström L, Snellman H, Tengstrand G. Phys. Lett. B, 1979, **82**: 419
15 LI Z P, Close F E, Barnes T. Phys. Rev. D, 1991, **43**: 2161
16 Gupta S N, Johnson J M. Phys. Rev. D, 1996, **54**: 2075; arXiv:hep-ph/9606349
17 Khadkikar S B, Gupta S K. Phys. Lett. B, 1983, **124**: 523
18 Vinodkumar P C, Vijay Kumar K B, Khadkikar S B. Pramana-J. Phys., 1992, **39**: 47
19 Khadkikar S B, Vinodkumar P C. Pramana-J. Phys., 1987, **29**: 39
20 Khadkikar S B, Vijaya Kumar K B. Phys. Lett. B, 1991, **254**: 320
21 Vijaya Kumar K B, Khadkikar S B. Nucl. Phys. A, 1993, **556**: 396
22 Fishbane P M et al. Phys. Rev. D, 1983, **27**: 2433
23 Gartenhaus G, Schwartz C. Phys. Rev., 1957, **108**: 482
24 Khadkikar S B. Pramana-J. Phys., 1985, **24**: 63
25 Feynman R P, Kislinger M, Ravndal F. Phys. Rev. D, 1976, **3**: 2706
26 Monteiro A P, Vijaya Kumar K. B. Commun. Theor. Phys., 2010, **53**: 325
27 Lakhina O, Swanson E S. Phys. Rev. D, 2006, **74**: 014012
28 Van Royen R, Weisskpof V F. Nuovo Cim. A, 1967, **50**: 617
29 Laverty J T, Radford S F, Repko W W. 2009, arXiv: hep-ph/0901.3917v3
30 Kwong W, Mackenzie P B, Rosenfeld R, Rosner J L. Phys. Rev. D, 1988, **37**: 3210
31 Aubert J J et al. Phys. Rev. Lett., 1974, **33**: 1401

32 Augustin J E et al. Phys. Rev. Lett., 1974, **33**: 1406
33 Abrams G S et al. Phys. Rev. Lett., 1974, **33**: 1453
34 Aulchenko V M et al. (KEDR collaboration). Phys. Lett. B, 2003, **573**: 63
35 Himel T et al. Phys. Rev. Lett., 1980, **45**: 1146
36 Partridge R et al. Phys. Rev. Lett., 1980, **45**: 1150
37 FANG F et al. (Belle colaboration). Phys. Rev. Lett., 2003, **90**: 071801
38 BAI J Z et al. (BES collaboration). Phys. Lett. B, 2003, **555**: 174
39 Ambrogiani M et al. (Fermilab E835 Collaboration). Phys. Lett. B, 2003, **566**: 45
40 Aubert B et al. (BABAR collaboration). Phys. Rev. Lett., 2004, **92**: 142002
41 Asner D M et al. (CLEO collaboration). Phys. Rev. Lett., 2004, **92**: 142001
42 Edwards C et al. Phys. Rev. Lett., 1982, **48**: 70
43 Abe K et al. (Belle collaboration). Phys. Rev. Lett., 2002, **89**: 142001
44 Herb S W et al. Phys. Rev. Lett., 1977, **39**: 252
45 Innes W R et al. Phys. Rev. Lett., 1977, **39**: 1240
46 Berger C et al. Phys. Lett. B, 1978, **76**: 243
47 Bienlein J K et al. Phys. Lett. B, 1978, **78**: 360
48 Andrews D et al. Phys. Rev. Lett., 1980, **44**: 1108
49 Andrews D et al. Phys. Rev. Lett., 1980, **45**: 219
50 Aubert B et al. (BABAR collaboration). 2008, arXiv: hep-ex/0807.1086
51 Adams G S et al. (CLEO collaboration). Phys. Rev. Lett., 2008, **101**: 101801
52 Besson D et al. (CLEO collaboration). 2006, arXiv: hep-ex/0512061v2
53 ZENG J, Van Orden J W, Roberts W. Phys. Rev. D, 1995, **52**: 5229; arXiv: hep-ph/9412269
54 Badalian A M, Danilkin I V. Phys. At. Nucl., 2009, **72**: 1206