Uncertainty principle in larmor clock*

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Abstract: It is well known that the spin operators of a quantum particle must obey uncertainty relations. We use the uncertainty principle to study the Larmor clock. To avoid breaking the uncertainty principle, Larmor time can be defined as the ratio of the phase difference between a spin-up particle and a spin-down particle to the corresponding Larmor frequency. The connection between the dwell time and the Larmor time has also been confirmed. Moreover, the results show that the behavior of the Larmor time depends on the height and width of the barrier.

Key words: tunneling time, Larmor clock, Larmor time, uncertainty principle

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1 Introduction

The time that a quantum particle takes to tunnel through a barrier is still a controversial topic. After MacColl [1] pointed out this interesting problem, lots of approaches have been developed to solve the problem. Many definitions have been given, for example, the phase time [2], the dwell time [3], and the Larmor time [4]. However, some of them in the literature [2–18] cannot agree. Moreover, some experiments [19–22] have been done, but not all of the results are consistent.

One of the methods that can be used to measure tunneling time is the Larmor clock [5, 6]. Consider a particle that is spin-$\frac{1}{2}$ polarized in the x direction tunneling through a square barrier which is in a weak uniform magnetic field pointing in the z direction. Due to the existence of the magnetic field in the barrier region, the spin of the particle starts to precess when the particle enters the barrier and stops precessing when the particle leaves the barrier. In this way, we can get some information on the so-called Larmor time [4–6] by dividing the precess angles with the precess frequency ω_{L}. Because the main effect of the magnetic field is to align the particle spin with the field, Büttiker [4] modified the previous idea given by Baz' [5] and introduced a traversal time, i.e., the Büttiker-Landauer time [16]. Subsequently, some questions have been put forward in the literature [15, 16]. Whether the Larmor clock could determine the tunneling time is still in doubt.

Although the uncertainty principle has been referred to in the literature [10–12], it has not been used to study the spin expectation values of the transmitted particle and the reflected particle in the Larmor clock. In this paper we will use the uncertainty principle to reconsider the Larmor clock. It is found that the uncertainty principle is violated when we make weak magnetic field approximations carelessly. To avoid this problem, we may only assert that the phase difference between the spin-up particle and the spin-down particle is connected by time.

This article is organized as follows. In Sec. 2, we briefly review the modified Larmor clock theory [4]. In Sec. 3, the results derived by Büttiker [4] are investigated in terms of the uncertainty principle, and some discussions are made. Finally, a summary is given in Sec. 4.

2 Theory basis

Let us consider the following one-dimensional

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scattering problem. A spin-$\frac{1}{2}$ particle polarized in the x direction penetrates a square barrier where a weak uniform magnetic field exists. The height of the barrier is V_0 and the width of the barrier is a. The incident direction of the particle is along the y direction and the magnetic field points at the z direction. The energy, momentum and mass of the particle are E, p and m, respectively. The Hamiltonian of the problem is

$$H=\begin{cases}\dfrac{p^2}{2m}+V_0-\dfrac{\hbar\omega_L\sigma_z}{2}, & |y|<\dfrac{a}{2},\\[2ex] \dfrac{p^2}{2m}, & |y|>\dfrac{a}{2},\end{cases} \tag{1}$$

where $\omega_{\mathrm{L}}=\dfrac{g_{\mathrm{s}}\mu_{\mathrm{B}}B}{\hbar}$ is the Larmor frequency, μ_B is the Bohr magneton, $g_{\mathrm{s}}=2$ is the gyromagnetic ratio, B is the magnetic strength, and $\hbar$ is the Planck constant. If we choose σ_z representation, then the Hamiltonian will be diagonal, so we can solve the scattering problem respectively for both spin-up and spin-down cases. We use the subscripts $\uparrow$ and $\downarrow$ to represent the spin-up and spin-down particles. Then the wave function of the left barrier is given by

$$\psi_{\mathrm{left}}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\mathrm{e}^{\mathrm{i}ky}+\frac{1}{\sqrt{2}}\begin{bmatrix}A_\uparrow\\A_\downarrow\end{bmatrix}\mathrm{e}^{-\mathrm{i}ky},\ y<-\frac{a}{2}, \tag{2}$$

where

$$k=\sqrt{\frac{2mE}{\hbar^2}}. \tag{3}$$

In the barrier, the wave function is written as

$$\psi_{\mathrm{in}}=\frac{1}{\sqrt{2}}\begin{bmatrix}B_\uparrow\mathrm{e}^{\kappa_\uparrow y}\\B_\downarrow\mathrm{e}^{\kappa_\downarrow y}\end{bmatrix}+\frac{1}{\sqrt{2}}\begin{bmatrix}C_\uparrow\mathrm{e}^{-\kappa_\uparrow y}\\C_\downarrow\mathrm{e}^{-\kappa_\downarrow y}\end{bmatrix},\ |y|<\frac{a}{2}, \tag{4}$$

where

$$\begin{aligned}\kappa_{\uparrow\downarrow}&=\sqrt{k_0^2-k^2\mp\frac{m\omega_{\mathrm{L}}}{\hbar}},\\ k_0&=\sqrt{\frac{2mV_0}{\hbar^2}}.\end{aligned} \tag{5}$$

And the transmitted wave function is as follows

$$\psi_{\mathrm{right}}=\frac{1}{\sqrt{2}}\begin{bmatrix}D_\uparrow\\D_\downarrow\end{bmatrix}\mathrm{e}^{\mathrm{i}ky},\ y>\frac{a}{2}. \tag{6}$$

The coefficients $A_{\uparrow\downarrow}$, $B_{\uparrow\downarrow}$, $C_{\uparrow\downarrow}$, $D_{\uparrow\downarrow}$ are given by

$$\begin{aligned}A_{\uparrow\downarrow}&=R_{\uparrow\downarrow}^{\frac{1}{2}}\mathrm{e}^{\frac{-\mathrm{i}\pi}{2}}\mathrm{e}^{\mathrm{i}\phi_{\uparrow\downarrow}}\mathrm{e}^{-\mathrm{i}ka},\\ B_{\uparrow\downarrow}&=\frac{\kappa_{\uparrow\downarrow}+\mathrm{i}k}{2\kappa_{\uparrow\downarrow}}\mathrm{e}^{\frac{\mathrm{i}ka}{2}}\mathrm{e}^{\frac{-\kappa_{\uparrow\downarrow}a}{2}}D_{\uparrow\downarrow},\\ C_{\uparrow\downarrow}&=\frac{\kappa_{\uparrow\downarrow}-\mathrm{i}k}{2\kappa_{\uparrow\downarrow}}\mathrm{e}^{\frac{\mathrm{i}ka}{2}}\mathrm{e}^{\frac{\kappa_{\uparrow\downarrow}a}{2}}D_{\uparrow\downarrow},\\ D_{\uparrow\downarrow}&=T_{\uparrow\downarrow}^{\frac{1}{2}}\mathrm{e}^{\mathrm{i}\phi_{\uparrow\downarrow}}\mathrm{e}^{-\mathrm{i}ka},\end{aligned} \tag{7}$$

where

$$\begin{aligned}T_{\uparrow\downarrow}+R_{\uparrow\downarrow}&=1,\\ T_{\uparrow\downarrow}&=\frac{4k^2\kappa_{\uparrow\downarrow}^2}{4k^2\kappa_{\uparrow\downarrow}^2+(k^2+\kappa_{\uparrow\downarrow}^2)^2\sinh^2(\kappa_{\uparrow\downarrow}a)},\\ \tan(\phi_{\uparrow\downarrow})&=\frac{k^2-\kappa_{\uparrow\downarrow}^2}{2k\kappa_{\uparrow\downarrow}}\tanh(\kappa_{\uparrow\downarrow}a).\end{aligned} \tag{8}$$

The spin expectation values of the transmitted particle are given by

$$\begin{aligned}\langle S_z\rangle_{\mathrm{T}}&=\frac{\hbar}{2}\frac{T_\uparrow-T_\downarrow}{T_\uparrow+T_\downarrow},\\ \langle S_y\rangle_{\mathrm{T}}&=-\hbar\sin(\phi_\uparrow-\phi_\downarrow)\frac{\sqrt{T_\uparrow T_\downarrow}}{T_\uparrow+T_\downarrow},\\ \langle S_x\rangle_{\mathrm{T}}&=\hbar\cos(\phi_\uparrow-\phi_\downarrow)\frac{\sqrt{T_\uparrow T_\downarrow}}{T_\uparrow+T_\downarrow}.\end{aligned} \tag{9}$$

And the spin expectation values of the reflected particle are given by

$$\begin{aligned}\langle S_z\rangle_{\mathrm{R}}&=\frac{\hbar}{2}\frac{R_\uparrow-R_\downarrow}{R_\uparrow+R_\downarrow},\\ \langle S_y\rangle_{\mathrm{R}}&=-\hbar\sin(\phi_\uparrow-\phi_\downarrow)\frac{\sqrt{R_\uparrow R_\downarrow}}{R_\uparrow+R_\downarrow},\\ \langle S_x\rangle_{\mathrm{R}}&=\hbar\cos(\phi_\uparrow-\phi_\downarrow)\frac{\sqrt{R_\uparrow R_\downarrow}}{R_\uparrow+R_\downarrow}.\end{aligned} \tag{10}$$

Up to now, the weak magnetic field condition has not been taken into account. When the infinitesimal magnetic field condition is considered, Büttiker [4] found that the spin expectation values of both the transmitted particle and the reflected particle can be written as

$$\begin{aligned}\langle S_z\rangle_{\mathrm{T,R}}&=\frac{\hbar}{2}\omega_L\tau_{z\mathrm{T,R}},\\ \langle S_y\rangle_{\mathrm{T,R}}&=-\frac{\hbar}{2}\omega_L\tau_{y\mathrm{T,R}},\\ \langle S_x\rangle_{\mathrm{T,R}}&=\frac{\hbar}{2}\left(1-\frac{\omega_L^2\tau_{x\mathrm{T,R}}^2}{2}\right)^2,\end{aligned} \tag{11}$$

where $\tau_{z\mathrm{T,R}}$, $\tau_{y\mathrm{T,R}}$ and $\tau_{x\mathrm{T,R}}$ are the characteristic times defined by Büttiker [4]

$$\begin{aligned}
\tau_{z\mathrm{T}} &= -\frac{m}{\hbar\kappa}\frac{\partial \ln T^{\frac{1}{2}}}{\partial\kappa} \\
&= \frac{mk_0^2}{\hbar\kappa^2}\frac{(\kappa^2-k^2)\sinh^2(\kappa a)+\left(\frac{\kappa a k_0^2}{2}\right)\sinh(2\kappa a)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa a)}, \\
\tau_{y\mathrm{T}} &= -\frac{m}{\hbar\kappa}\frac{\partial\phi}{\partial\kappa}=\frac{mk}{\hbar\kappa}\frac{2\kappa a(\kappa^2-k^2)+k_0^2\sinh(2\kappa a)}{4k^2\kappa^2+k_0^4\sinh^2(\kappa a)}, \\
\tau_{x\mathrm{T}}^2 &= \tau_{y\mathrm{T}}^2+\tau_{z\mathrm{T}}^2, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\tau_{z\mathrm{R}} &= -\frac{T}{R}\tau_{z\mathrm{T}}, \\
\tau_{y\mathrm{R}} &= \tau_{y\mathrm{T}}, \qquad (13) \\
\tau_{x\mathrm{R}}^2 &= \tau_{y\mathrm{R}}^2+\tau_{z\mathrm{R}}^2,
\end{aligned}$$

where T is the transmission probability of the particle without the magnetic field involved, R is the reflection probability, and ϕ is the phase increase.

3 Results and discussions

Now we will use the uncertainty principle to study these characteristic times. Based on general quantum theory, we know that

$$\begin{aligned}
\Delta S_x\Delta S_y &\geqslant \frac{1}{2}|\overline{[S_x,S_y]}|, \\
\Delta S_y\Delta S_z &\geqslant \frac{1}{2}|\overline{[S_y,S_z]}|, \qquad (14) \\
\Delta S_z\Delta S_x &\geqslant \frac{1}{2}|\overline{[S_z,S_x]}|.
\end{aligned}$$

If we don't take the infinitesimal magnetic field into account, we would find

$$\begin{aligned}
&\Delta S_{x\mathrm{T}}\Delta S_{y\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{x\mathrm{T}},S_{y\mathrm{T}}]}| \\
&\rightarrow 4\sin^2[2(\phi_\uparrow-\phi_\downarrow)]\frac{(T_\uparrow T_\downarrow)^2}{(T_\uparrow+T_\downarrow)^4}\geqslant 0, \\
&\Delta S_{y\mathrm{T}}\Delta S_{z\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{y\mathrm{T}},S_{z\mathrm{T}}]}| \\
&\rightarrow 4\sin^2(\phi_\uparrow-\phi_\downarrow)\frac{T_\uparrow T_\downarrow(T_\uparrow-T_\downarrow)^2}{(T_\uparrow+T_\downarrow)^4}\geqslant 0, \\
&\Delta S_{z\mathrm{T}}\Delta S_{x\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{z\mathrm{T}},S_{x\mathrm{T}}]}| \\
&\rightarrow 4\cos^2(\phi_\uparrow-\phi_\downarrow)\frac{T_\uparrow T_\downarrow(T_\uparrow-T_\downarrow)^2}{(T_\uparrow+T_\downarrow)^4}\geqslant 0, \\
&\Delta S_{x\mathrm{R}}\Delta S_{y\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{x\mathrm{R}},S_{y\mathrm{R}}]}| \\
&\rightarrow 4\sin^2[2(\phi_\uparrow-\phi_\downarrow)]\frac{(R_\uparrow R_\downarrow)^2}{(R_\uparrow+R_\downarrow)^4}\geqslant 0, \\
&\Delta S_{y\mathrm{R}}\Delta S_{z\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{y\mathrm{R}},S_{z\mathrm{R}}]}| \\
&\rightarrow 4\sin^2(\phi_\uparrow-\phi_\downarrow)\frac{R_\uparrow R_\downarrow(R_\uparrow-R_\downarrow)^2}{(R_\uparrow+R_\downarrow)^4}\geqslant 0, \\
&\Delta S_{z\mathrm{R}}\Delta S_{x\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{z\mathrm{R}},S_{x\mathrm{R}}]}| \\
&\rightarrow 4\cos^2(\phi_\uparrow-\phi_\downarrow)\frac{R_\uparrow R_\downarrow(R_\uparrow-R_\downarrow)^2}{(R_\uparrow+R_\downarrow)^4}\geqslant 0. \qquad (15)
\end{aligned}$$

One can see that all the inequalities are satisfactory. When we take the infinitesimal magnetic field into consideration and make some approximations given by Büttiker [4], we find

$$\begin{aligned}
&\Delta S_{x\mathrm{T}}\Delta S_{y\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{x\mathrm{T}},S_{y\mathrm{T}}]}| \rightarrow \omega_{\mathrm{L}}^2\tau_{y\mathrm{T}}^2-\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{T}}^4}{4} \\
&+\frac{\omega_{\mathrm{L}}^6\tau_{x\mathrm{T}}^4\tau_{y\mathrm{T}}^2}{4}-\omega_{\mathrm{L}}^4\tau_{x\mathrm{T}}^2\tau_{y\mathrm{T}}^2\geqslant 0, \qquad (16) \\
&\Delta S_{y\mathrm{T}}\Delta S_{z\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{y\mathrm{T}},S_{z\mathrm{T}}]}| \rightarrow -\omega_{\mathrm{L}}^2\tau_{z\mathrm{T}}^2-\omega_{\mathrm{L}}^2\tau_{y\mathrm{T}}^2 \\
&+\omega_{\mathrm{L}}^4\tau_{y\mathrm{T}}^2\tau_{z\mathrm{T}}^2\geqslant -\omega_{\mathrm{L}}^2\tau_{x\mathrm{T}}^2+\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{T}}^4}{4}, \qquad (17) \\
&\Delta S_{z\mathrm{T}}\Delta S_{x\mathrm{T}} \geqslant \frac{1}{2}|\overline{[S_{z\mathrm{T}},S_{x\mathrm{T}}]}| \rightarrow \omega_{\mathrm{L}}^2\tau_{z\mathrm{T}}^2-\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{T}}^4}{4} \\
&+\frac{\omega_{\mathrm{L}}^6\tau_{x\mathrm{T}}^4\tau_{z\mathrm{T}}^2}{4}-\omega_{\mathrm{L}}^4\tau_{x\mathrm{T}}^2\tau_{z\mathrm{T}}^2\geqslant 0, \qquad (18) \\
&\Delta S_{x\mathrm{R}}\Delta S_{y\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{x\mathrm{R}},S_{y\mathrm{R}}]}| \rightarrow \omega_{\mathrm{L}}^2\tau_{y\mathrm{R}}^2-\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{R}}^4}{4} \\
&+\frac{\omega_{\mathrm{L}}^6\tau_{x\mathrm{R}}^4\tau_{y\mathrm{R}}^2}{4}-\omega_{\mathrm{L}}^4\tau_{x\mathrm{R}}^2\tau_{y\mathrm{R}}^2\geqslant 0, \qquad (19) \\
&\Delta S_{y\mathrm{R}}\Delta S_{z\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{y\mathrm{R}},S_{z\mathrm{R}}]}| \rightarrow -\omega_{\mathrm{L}}^2\tau_{z\mathrm{R}}^2-\omega_{\mathrm{L}}^2\tau_{y\mathrm{R}}^2 \\
&+\omega_{\mathrm{L}}^4\tau_{y\mathrm{R}}^2\tau_{z\mathrm{R}}^2\geqslant -\omega_{\mathrm{L}}^2\tau_{x\mathrm{R}}^2+\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{R}}^4}{4}, \qquad (20) \\
&\Delta S_{z\mathrm{R}}\Delta S_{x\mathrm{R}} \geqslant \frac{1}{2}|\overline{[S_{z\mathrm{R}},S_{x\mathrm{R}}]}| \rightarrow \omega_{\mathrm{L}}^2\tau_{z\mathrm{R}}^2-\frac{\omega_{\mathrm{L}}^4\tau_{x\mathrm{R}}^4}{4} \\
&+\frac{\omega_{\mathrm{L}}^6\tau_{x\mathrm{R}}^4\tau_{z\mathrm{R}}^2}{4}-\omega_{\mathrm{L}}^4\tau_{x\mathrm{R}}^2\tau_{z\mathrm{R}}^2\geqslant 0. \qquad (21)
\end{aligned}$$

All the inequalities turn out to be satisfactory if we retain the terms of the order ω_{L}^2 in these inequalities. However, if we study the original inequalities, some surprising results will emerge from our analysis. Now we focus our attention on Eqs. (12), (13), (17), (20). Obviously, to make Eq. (17) satisfactory, $\tau_{z\mathrm{T}}^2$ must be equal to $\tau_{y\mathrm{T}}^2$ all the time. However, when we use Eq. (12) to calculate $\tau_{y\mathrm{T}}^2/\tau_{z\mathrm{T}}^2$, we find that $\tau_{y\mathrm{T}}^2/\tau_{z\mathrm{T}}^2$ is not equal to one all the time. Similar procedures also show that $\tau_{y\mathrm{R}}^2/\tau_{z\mathrm{R}}^2$ is not equal to one all the time. Fig. 1 and Fig. 2 show $\log(\tau_{y\mathrm{T}}^2/\tau_{z\mathrm{T}}^2)$ and $\log(\tau_{y\mathrm{R}}^2/\tau_{z\mathrm{R}}^2)$ in detail, respectively.

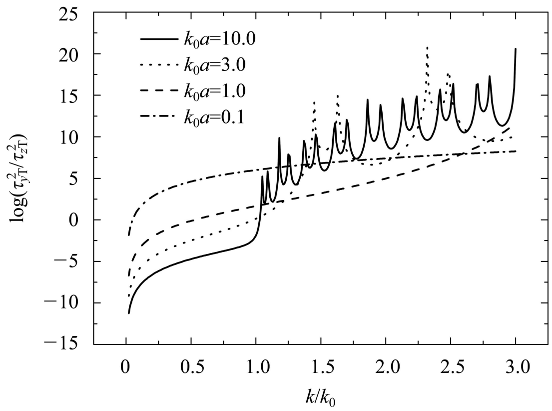

Fig. 1. $\log(\tau_{y\mathrm{T}}^2/\tau_{z\mathrm{T}}^2)$ as a function of the incident particle energy with different values of k_0a.

Since ω_{L} can be set at an arbitrarily small value, inequalities except Eqs. (17) and (20) can be verified as correct through numerical calculations. Hence, the uncertainty principle is violated under such careless approximations. Considering that the spin operators are not commutative, we cannot make approximations in a straightforward and consistent manner. Actually we must take the characteristic property of the spin operators into account when we make weak magnetic field approximations. If we only make an approximation for the phase difference instead of the transmitted probability difference and the reflected probability difference, i.e., $T_\uparrow - T_\downarrow$ and $R_\uparrow - R_\downarrow$, we can avoid breaking the uncertainty principle. Thus, the Larmor time of the transmitted particle and the reflected particle can be defined by $\tau = \tau_{\mathrm{T,R}} = (\phi_\uparrow - \phi_\downarrow)/\omega_{\mathrm{L}}$, which is identical to the definitions given by other methods in the literature [14, 18]. Apparently, $\tau = -(m/\hbar\kappa)\partial\phi/\partial\kappa = \tau_{y\mathrm{T}} = \tau_{y\mathrm{R}}$.

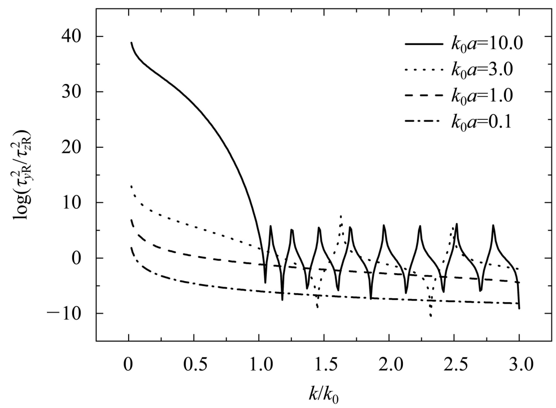

Fig. 2. $\log(\tau_{y\mathrm{R}}^2/\tau_{z\mathrm{R}}^2)$ as a function of the incident particle energy with different values of k_0a.

The dwell time [3, 4] is defined as the number of particles within the barrier region divided by the incident flux, i.e.,

$$\tau_{\mathrm{d}} = \int_{-a/2}^{a/2} |\psi_{\mathrm{in}}|^2 \mathrm{d}y/(\hbar k/m).$$

A simple calculation will show $\tau_{\mathrm{d}} = \tau_{\mathrm{T,R}}$. Since $R+T=1$, the important identity [16] $\tau_{\mathrm{d}} = R\tau_{\mathrm{R}} + T\tau_{\mathrm{T}}$

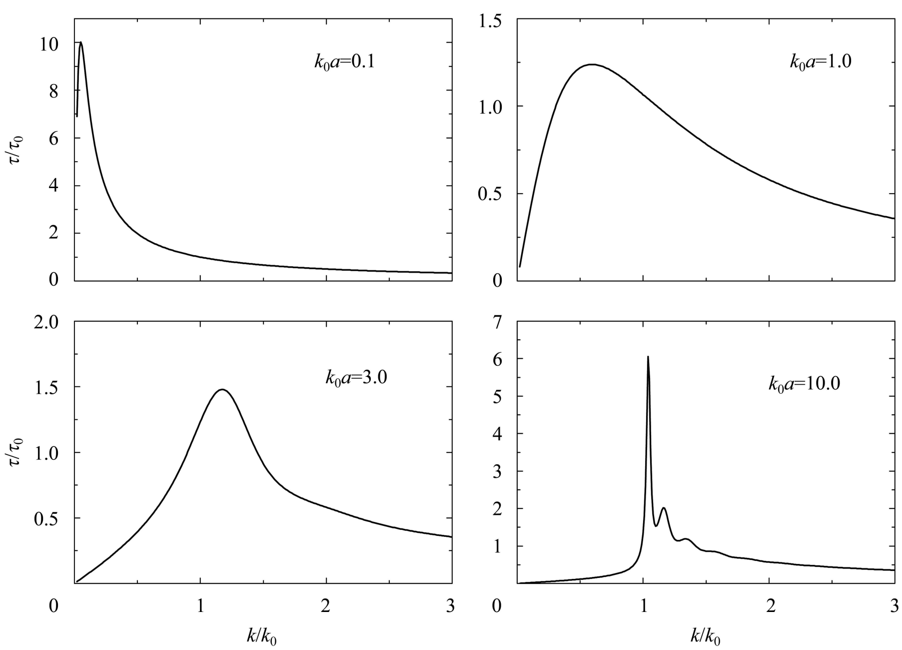

Fig. 3. τ as a function of the incident particle energy with different values of k_0a, $\tau_0 = \dfrac{ma}{\hbar k_0}$.

is obviously obeyed. The graphs of τ with different values of k_0a are shown in Fig. 3.

From Fig. 3, one can see that when the incident energy of the particle is smaller than the height of the barrier, τ doesn't always increase monotonically with k. The conclusion is that the behavior of τ depends strongly on the specific height and width of the barrier.

In conclusion, because the phase difference is included in trigonometric functions, there is still some doubt whether τ can solve the problem of measuring tunneling time accurately.

4 Summary

In summary, the Larmor time defined by Büttiker has been studied. When we make the weak magnetic field approximations, we must consider the particularity of the spin operators. The Larmor time can be defined by dividing the phase difference with the Larmor frequency. In this way, the identity $\tau_{\rm d}=R\tau_{\rm R}+T\tau_{\rm T}$ is obviously obeyed. It is also shown that the behavior of τ depends strongly on k_0a. Although τ is well defined, it still could not answer the problem of measuring tunneling time well.

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