Nucleon/nuclei polarized structure function, using Jacobi polynomials expansion

A. Mirjalili[1,3;1)] M. M. Yazdanpanah[2,3] F. Taghavi-Shahri[3] S. Atashbar Tehrani[3]

[1] Physics Department, Yazd University, 89195-741, Yazd, Iran
[2] Physics Department, Kerman Shahid Bahonar University, Kerman, Iran
[3] School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM) P.O. Box 19395-5531, Tehran, Iran

Abstract We use the constituent quark model to extract polarized parton distributions and finally polarized nucleon structure function. Due to limited experimental data which do not cover whole (x,Q^2) plane and to increase the reliability of the fitting, we employ the Jacobi orthogonal polynomials expansion. It will be possible to extract the polarized structure functions for Helium, using the convolution of the nucleon polarized structure functions with the light cone moment distribution. The results are in good agreement with available experimental data and some theoretical models.

Key words perturbative QCD, nucleon and nuclei structure function, jacobi polynomials

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1 Introduction

The nature of the short-distance structure of polarized nucleons is one of the central questions of present day hadron physics. For more than sixteen years, polarized inclusive deep inelastic scattering has been the main source of information on how the individual partons in the nucleon are polarized at very short distances. Following that in recent years unpolarized and polarized nuclear structure functions have been discussed from theoretical and experimental viewpoints [1, 2]. The important issue in this subject is the different behavior of parton densities in free nucleons and bound nucleons, i.e. nuclei. Here the nuclear effects play an essential rule in calculating parton distribution functions in the nuclei [3]. To obtain the unpolarized and polarized nuclear structure functions from nucleon ones, we need to perform an integral in which the light cone momentum distribution of the nucleon in the nucleus is convoluted with nucleon structure functions (in both unpolarized and polarized cases) [4]. The main requirement for these calculations is well-behaved nucleon parton distributions. Consequently we are able to calculate the polarized structure function for the Helium.

2 EMC effect

Several experimental data have revealed considerable differences between the free nucleon parton distribution functions and the bound nucleon parton distribution, which was discovered several years ago by European Muon Collaboration (EMC) in 1982 [3, 5].

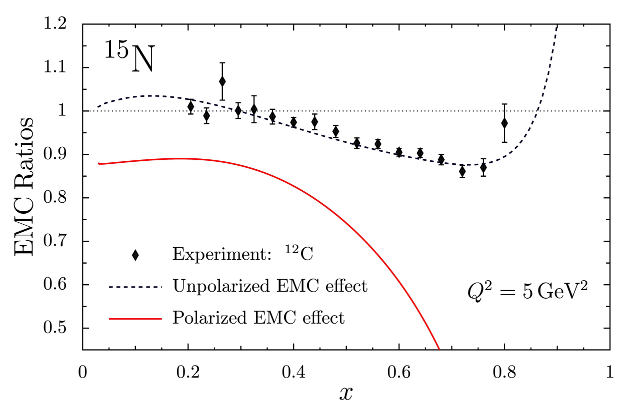

Fig. 1. The EMC effect in ^{15}N which has been adopted from [6] and references therein.

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1) E-mail: Mirjalili@mail.ipm.ir

This effect is representing in Fig. 1 for the ratio of nucleon parton distribution in ^{15}N to ^{12}C. The nuclear effects which play a role in polarized and unpolarized DIS on nuclei can be divided into coherent and incoherent contributions. Incoherent nuclear effects result from the scattering of the incoming lepton on each individual nucleon, nucleon resonance. They are present at all Bjorken x. Coherent nuclear effects arise from the interaction of the incoming lepton with two or more nucleons in the target. They are typically concentrated at low values of Bjorken x.

3 Polarized constituent quark model

Here we use the valon model as our constituent quark model [7]. Polarized valon distributions were calculated by using an improved valon model in the next-to-leading order approximation [8]. According to the improved valon model, the polarized parton distribution is related to the polarized valon distribution. On the other hand, the polarized parton distribution of a proton is obtained by convolution of two distributions: the polarized valon distributions in the proton and the polarized parton distributions in the valon.

$$\delta q_{\mathrm{i/p}}(x,Q^2)=\sum_{\mathrm{j}}\int_x^1 \delta q_{\mathrm{i/j}}\left(\frac{x}{y},Q^2\right)\delta G_{\mathrm{j/p}}(y)\frac{\mathrm{d}y}{y}\,, \quad (1)$$

where the summation is over the three valons. Here $\delta G_{\mathrm{j/p}}(y)$ indicates the probability for the polarized j-valon to have momentum fraction y in the proton. $\delta q_{\mathrm{i/p}}(x,Q^2)$ and $\delta q_{\mathrm{i/j}}\left(\frac{x}{y},Q^2\right)$ are, respectively, the polarized i-parton distribution in the proton and j-valon.

Let us define the Mellin transforms for any function $f(x,Q^2)$ as follows:

$$\mathcal{M}\{f(x,Q^2)\}=M(n,Q^2)=\int_0^1 x^{n-1}f(x,Q^2)\mathrm{d}x\,, \quad (2)$$

here n is the order of moments. The spin structure function has then a probabilistic interpretation, which for the proton reads

$$\mathcal{M}\{g_1^{\mathrm{N}}(x,Q^2)\}\equiv\delta M^{\mathrm{N}}(n,Q^2)\,, \quad (3)$$

where the δ symbol denotes the moments in the polarized case and N, a nucleon, refers to the proton and neutron separately. According to the valon model, the moments of polarized parton distribution functions in the nucleon which are denoted respectively by $\delta M_{\mathrm{u_v}}(n,Q^2)$, $\delta M_{\mathrm{d_v}}(n,Q^2)$, $\delta M^{\mathrm{S}}(n,Q^2)$ and $\delta M_{\mathrm{g}}(n,Q^2)$ can be obtained as follows:

$$\delta M_{\mathrm{u_v}}(n,Q^2)=2\delta M^{\mathrm{NS}}(n,Q^2)\times\delta M'_{\mathrm{U/p}}(n)\,, \quad (4)$$

$$\delta M_{\mathrm{d_v}}(n,Q^2)=\delta M^{\mathrm{NS}}(n,Q^2)\times\delta M'_{\mathrm{D/p}}(n)\,. \quad (5)$$

$$\delta M^{\mathrm{S}}(n,Q^2)=\delta M^{\mathrm{S}}_{(\mathrm{U,D})}(n,Q^2)(2\delta M''_{\mathrm{U/p}}(n)+\delta M''_{\mathrm{D/p}}(n))\,. \quad (6)$$

$$\delta M_{\mathrm{g}}(n,Q^2)=\delta M^{\mathrm{gq}}(n,Q^2)(2\delta M'_{\mathrm{U/p}}(n)+\delta M'_{\mathrm{D/p}}(n))\,, \quad (7)$$

In the above equation $M'_{\mathrm{j/p}}(N)$ is the moment of the $\delta G_{\mathrm{j/p}}(y)$ distribution, i.e. $\delta M'_{\mathrm{j/p}}(n)=\mathcal{M}\{\delta G_{\mathrm{j/p}}(y)\}$. The moment of the Non-singlet and singlet sectors for U and D valons are equal and we denoted them respectively by $\delta M^{\mathrm{NS}}(n,Q^2)$ and $\delta M^{S}_{(\mathrm{U,D})}(n,Q^2)$. The reason to use new notation for the moments of $\delta G_{\mathrm{j/p}}(y)$ distribution in Eq. (6), can be found in Ref. [8].

The moments in Eqs. (4)–(7) including $\delta M^{\mathrm{NS}}(n,Q^2)$, $\delta M^{\mathrm{S}}_{(\mathrm{U,D})}(n,Q^2)$ and $\Delta M^{\mathrm{gq}}(n,Q^2)$ (quark-to-gluon evolution function), are defined in Ref. [8]. Using the moments of parton distribution for the nucleon, the twist–2 contributions to the structure function $\delta M^{\mathrm{N}}(n,Q^2)$ can be obtained in terms of the polarized parton densities and the Wilson coefficient functions $\delta C_{\mathrm{i}}(n)$ in the moment space by [9]

$$\delta M^{\mathrm{N}}(n,Q^2)=\frac{1}{2}\sum_{\mathrm{q}}e_{\mathrm{q}}^2\Bigg\{\left(1+\frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi}\delta C_{\mathrm{q}}(n)\right)\times [\delta M_{\mathrm{q}}(n,Q^2)+\delta M_{\bar{\mathrm{q}}}(n,Q^2)]+\frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi}2\delta C_{\mathrm{g}}(n)\delta M_{\mathrm{g}}(n,Q^2)\Bigg\}. \quad (8)$$

4 Mellin integral transform of Jaccobi orthogonal polynomials

Jaccobi orthogonal polynomials is defined by [10]

$$P_{\mathrm{n}}^{(\alpha,\beta)}(x)=\frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n,n+\alpha+\beta+1,\alpha+1;\frac{1-x}{2}\right)=\frac{(\alpha+1)_n}{n!}\sum_{k=0}^{n}\frac{(-n)_k(n+\alpha+\beta+1)}{2^k k!(\alpha+1)_k}(1-x)^k\,. \quad (9)$$

where $(m)_n$ is representing the Pochhammer symbol. This polynomials allow one to factor out an essential part of the x-dependence of the structure function into the weight function. Thus, given the moments $\delta M(n,Q^2)$, a structure function $\delta f(x,Q^2)$ may be reconstructed in a form of a series [11]

$$x\delta f(x,Q^2)=x^{\beta}(1-x)^{\alpha}\sum_{n=0}^{\mu}\delta M(n,Q^2)\mathcal{P}_n^{(\alpha,\beta)}(x), \quad (10)$$

where μ is the number of polynomials needed for convergence of the summation. The relation between $\mathcal{P}_n^{(\alpha,\beta)}(x)$ and $P_n^{(\alpha,\beta)}(x)$ polynomials has been revealed in [11].

5 The statues of nuclei structure function

We are interested in the polarized deep inelastic scattering (PDIS) process $\mathrm{lT} \to \mathrm{l'X}$, where T is a polarized nucleus (nucleon), l is a polarized lepton and X is an unobserved hadronic state. The spin-dependent structure functions of nucleon are related to the target spin-dependent structure functions in Bjorken limit as:

$$g_{1,2}^{\mathrm{T}}(x,Q^2) = \sum_{\mathrm{i=p,n}} \int_{x\leqslant z} \mathrm{d}y N_{\mathrm{i}} \Delta f_{\mathrm{T}}^{i}(y) g_{1,2}^{i}\left(z=\frac{x}{y},Q^2\right) \equiv \sum_{\mathrm{i=p,n}} N_{\mathrm{i}} \Delta f_{\mathrm{T}}^{i} \otimes g_{1,2}^{i}\,. \tag{11}$$

The polarized light cone momentum distribution in nucleus $\Delta f_{\mathrm{T}}^{i}(y)$, is the spin-dependent probability to find the nucleon in the nucleus with a given fraction of the total momentum y of the nucleus on the light front and it can be related to polarized spectral functions [3, 4].

An exact investigation of the invariant spectral function and nucleon momentum distributions would require complete information about the nuclear states (wave functions), which for heavy nuclei is a complicated matter and for that reason this calculation has been limited to the light nuclei such as Helium, Tritium and nuclear matter. The three-nucleon wave function obtained using the Faddeev decomposition and Schrödinger equation is given by,

$$|\Psi\rangle = |\varphi_{\varsigma}\rangle + |\varphi_{\xi}\rangle + |\varphi_{\zeta}\rangle \equiv G_0(E)V|\Psi\rangle \equiv \sum_{i=1}^{3} G_0(E)V_i|\Psi\rangle. \tag{12}$$

In this equation $|\varphi\rangle$ refers to the Faddeev component of the wave function and the indices run from 1 to 3, where $G_0(E)$ is the three-body Green function and V_i is the two body interaction potential. Using the symmetry group properties of the three-nucleon wave function, one can derive a set of coupled integral equations for the Faddeev components,

$$|\varphi_{\varsigma}\rangle = G_0(E)T_{\varsigma}(E-\epsilon_{\varsigma})(|\varphi_{\xi}\rangle + |\varphi_{\zeta}\rangle)\,. \tag{13}$$

T_{ς} is the usual T-matrix defined by the Lippmann-schwinger equation and ϵ_{ς} is the energy of the ς particle in the three-body center of mass system. The exact solution of these coupled equations is complicated and has been dealt with using different techniques by different authors [4].

6 Conclusion

Determination of parton distributions in a nucleon in the framework of quantum chromodynamics (QCD) always involves some model-dependent procedure like AAC and BB models [12, 13]. We used the constituent quark framework (valon model) to extract polarized parton distributions and then polarized nucleon structure functions. Due to limited experimental data which do not cover the whole range of x and Q^2 values, and to increase the reliability of the fitting, we employed Jaccobi polynomials expansions.

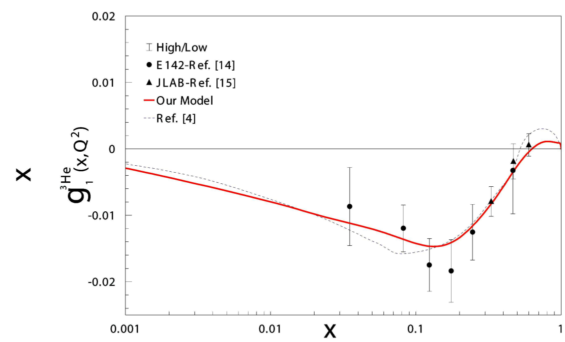

Fig. 2. Analytical result for the polarized Helium structure function in our model and [4] which has been compared with experimental data [14, 15].

Having at our disposal the analytical result for the $g_1^{\mathrm{N}}(x,Q^2)$ and convoluting it with the light cone moment distribution of nucleon in the nucleus, it is possible to extract polarized structure functions for Helium. The result has been depicted in Fig. 2 and indicates good agreement with available experimental data and the theoretical model in [4].

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