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Low-energy $D^{*+}\bar{D}_1^0$ scattering and the nature of resonance-like structure $Z^+(4430)$ *

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Abstract Low-energy scattering of D^{*+} and $\bar{D}_1^0$ meson is studied using quenched lattice QCD with improved lattice actions on anisotropic lattices. The threshold scattering parameters, namely the scattering length a_0 and the effective range r_0, for the s-wave scattering in $J^P=0^-$ channel are extracted: $a_0=2.52(47)$ fm and $r_0=0.7(1)$ fm. It is argued that, albeit the interaction between the two charmed mesons being attractive, it is unlikely that they can form a shallow bound state in this channel. Our calculation provides some useful information on the nature of the newly discovered resonance-like structure $Z^+(4430)$ by the Belle Collaboration.

Key words D^*-$\bar{D}_1$ scattering, resonance-like structure $Z^+(4430)$, lattice QCD

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1 Introduction

Recently, a charged resonance-like structure $Z^+(4430)$ has been observed at Belle in the $\pi\psi'$ invariant mass spectrum of $B\to K\pi^+\psi'$ decays [1]. This has triggered many theoretical investigations on the nature of this structure [2–10], one possible interpretation is a shallow bound state formed by the D^* and $\bar{D}_1$ mesons [3, 4]. To further investigate this possibility, the interaction between a D^* and a $\bar{D}_1$ meson near the threshold becomes crucial.

In this paper, we briefly report our results on the scattering threshold parameters, i.e. scattering length a_0 and effective range r_0, of $D^*-\bar{D}_1$ system using quenched lattice QCD within the so-called Lüscher's formalism [11]. Details of the calculation can be found in Ref. [12]. Within this approach, it is also feasible to investigate the possible bound state of the two mesons [11, 14]. We have used improved gauge and fermion lattice actions on anisotropic lattices. The usage of anisotropic lattices with asymmetric volumes has enhanced our resolution in energy and the momentum. The computation is carried out in all possible angular momentum channels, although only the $J^P=0^-$ channel yields definite results. We find that, in this particular channel, the interaction between a D^* and a $\bar{D}_1$ meson is attractive in nature. The scattering length after continuum and chiral extrapolation is $a_0=2.52(47)$ fm while the effective range is $r_0=0.7(1)$ fm. Our simulation results suggest that the two-particle system resembles more like an ordinary scattering state rather than a bound state.

2 Theoretical framework, hadron operators and correlation functions

Consider two interacting particles with mass m_1

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and m_2 enclosed in a cubic box of size L, with periodic boundary conditions. The spatial momentum $\boldsymbol{k}$ is quantized according to:

$$\boldsymbol{k}=\left(\frac{2\pi}{L}\right)\boldsymbol{n}$$

with $\boldsymbol{n}$ being a three-dimensional integer. By taking the center-of-mass frame of the system, the two particles then have three-momentum $\boldsymbol{k}$ and $-\boldsymbol{k}$. The exact energy of the two-particle system in this finite volume is denoted as: $E_{1\cdot2}(\boldsymbol{k})$. We now define a variable $\bar{\boldsymbol{k}}^2$ via:

$$E_{1\cdot2}(\boldsymbol{k})=\sqrt{m_1^2+\bar{\boldsymbol{k}}^2}+\sqrt{m_2^2+\bar{\boldsymbol{k}}^2}\,. \tag{1}$$

It is also convenient to further define a variable q^2 as: $q^2=\bar{\boldsymbol{k}}^2L^2/(2\pi)^2$ which differs from $\boldsymbol{n}^2$ due to interaction. Lüscher's formula then says [11]:

$$\tan\delta(q)=\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1;q^2)}\,, \tag{2}$$

where $\mathcal{Z}_{00}(1;q^2)$ is the zeta-function. Therefore, by extracting the exact two-particle energy $E_{1\cdot2}(\boldsymbol{k})$ from numerical simulations, one could infer the elastic scattering phase shift $\delta(q)$.

In the case of attractive interaction, one obtains the phase $\delta(q)$ by analytic continuation: $\tan\sigma(q)=-\mathrm{i}\tan\delta(q)$ [11, 14]. In this case we have: $\tan\sigma(q)=\pi^{3/2}(-\mathrm{i}q)/\mathcal{Z}_{00}(1;q^2)$. The condition for a bound state at that particular energy requires: $\tan\sigma(q)=-1$ in the infinite volume and continuum limit.

In order to access more low-momentum modes, it is advantageous to use asymmetric volumes in the study of hadron scattering [15–17]. If the rectangular box is of size $L\times(\eta_2L)\times(\eta_3L)$, Eq. (2) is modified to:

$$\tan\delta(q)=\frac{\pi^{3/2}q\eta_2\eta_3}{\mathcal{Z}_{00}(1;q^2;\eta_2,\eta_3)}\,, \tag{3}$$

where $\mathcal{Z}_{00}(1,q^2;\eta_2,\eta_3)$ is the modified zeta-function defined in Refs. [15, 16].

For the D^* and $\bar{D}_1$ mesons, we use the local operators $Q_i=[\bar{d}\gamma^ic](x)$ and $P_i=[\bar{c}\gamma^i\gamma^5u](x)$ respectively. The Fourier transform of these operators yields the single meson operators with definite three-momentum: $Q_i(t,\boldsymbol{k})$ and $P_i(t,\boldsymbol{k})$. The two-particle operator in the A_1 channel is given by:

$$\begin{aligned}O^{(A_1)(1)}(t)\;=\;&\sum_{R\in G}[Q_1(t+1,-R\circ\boldsymbol{k})P_1(t,R\circ\boldsymbol{k})\\&+Q_2(t+1,-R\circ\boldsymbol{k})P_2(t,R\circ\boldsymbol{k})\\&+Q_3(t+1,-R\circ\boldsymbol{k})P_3(t,R\circ\boldsymbol{k})]\,,\end{aligned} \tag{4}$$

where $R\in G$ is an operation of the symmetry group G. Single meson (with and without three-momentum) correlation functions can be constructed as usual using the operators $Q_i(t,\boldsymbol{k})$ and $P_i(t,\boldsymbol{k})$. For the two-meson operator, the following correlation matrix is constructed:

$$C_{mn}^{(A_1)(1)}(t)=\langle O_m^{(A_1)(1)\dagger}(t)O_n^{(A_1)(1)}(0)\rangle, \tag{5}$$

where n and m designates different momentum modes. Applying Wick's theorem, the single- and double-meson correlation functions defined above are transformed into quark propagators which are then measured using the gauge field configurations generated in our simulation.

3 Simulation results and discussions

The gauge action use in this study is the tadpole improved gauge action on anisotropic lattices [18–20]: while the fermion action used is the tadpole improved clover Wilson action on anisotropic lattice [21, 22]. Lattices with three different lattice spacings are used in the calculation. The lattice spacing a_s in physical units is determined from a Wilson loop calculation [23]. Relevant parameters for these lattices can be found in Table 1. For each set of lattice, the calculation is done for several light quark and heavy quark hopping parameters: $(\kappa^{\mathrm{ud}},\kappa^{\mathrm{c}})$.

The mass of D^*, $\bar{D}_1$ and the dispersion relations for them are checked carefully to make sure that single charmed meson states are properly realized in the finite box [12].

The two-meson correlation matrix (5) is first diagonalized using Lüscher-Wolff method [13] with the eigenvalue $\lambda_i(t,t_0)$ obtained. To enhance the signal to noise ration, the following ratio was formed:

$$\mathcal{R}(t,t_0)=\frac{\lambda_i(t,t_0)}{C_{D^*}(t)C_{D_1}(t)}\propto \mathrm{e}^{-\delta E_i\cdot t} \tag{6}$$

where $C_{D^*}(t)$ and $C_{D_1}(t)$ are the one-particle correlation functions with zero momentum for the respective mesons. Then, $\delta E_i=E_i-m_{D^*}-m_{D_1}$ is extracted from the following effective mass:

$$M_{\mathrm{eff}}(t)=\ln\left(\frac{\mathcal{R}(t)}{\mathcal{R}(t+1)}\right)\,. \tag{7}$$

With the energy difference δE_i extracted from the simulation data, one utilizes the modified Lüscher's formula (3) to obtain the quantity $k/\tan\delta(k)$ which has the following known expansion near threshold:

$$\frac{k}{\tan\delta(k)}=\frac{1}{a_0}+\frac{1}{2}r_0k^2+\cdots\,, \tag{8}$$

with a_0 and r_0 being the scattering length and the effective range. Fitting the data for $k/\tan\delta(k)$ obtained from the simulation according to the above formula yields the desired parameters a_0 and r_0.

After getting the values of a_0 and r_0 for each pair of quark mass parameter $(\kappa^{\mathrm{ud}}, \kappa^{\mathrm{c}})$, the results are interpolated versus κ^{c} to the physical charm quark mass which is determined by the experimental value of

$$\frac{1}{4}m_{\eta_c} + \frac{3}{4}m_{\mathrm{J}/\psi}.$$

The interpolated data are then taken for the chiral extrapolation. In this step, the results for a_0 and r_0 are extrapolated versus m_π^2 towards the chiral limit using a quadratic function in m_π^2. Finally, the continuum limit is taken by a linear extrapolation in a_s^2 (since we are using an improved lattice action) for the results of a_0 and r_0 obtained after chiral extrapolation. After these extrapolations, we obtain the scattering length a_0 and the effective range r_0 in the A_1 channel:

$$a_0 = 2.53 \pm 0.47 \text{ fm},\ r_0 = 0.70 \pm 0.10 \text{ fm}. \qquad (9)$$

To check the possibility of a bound state, the values of $\cot\sigma(q)$ corresponding to the lowest (negative) q^2 are obtained as listed in Table 1. It is seen that our results for $\cot\sigma(q)$ are all positive, far from the bound state value (-1). Thus, this result is more consistent with a scattering state than a bound state.

Table 1. Lattice parameters and results for $\cot\sigma(q)$ at the lowest q^2.

β	a_s/fm	Volume	$N_{\mathrm{conf.}}$	q^2	$\cot\sigma(q^2)$
2.5	0.20	$8^2\cdot12\cdot40$	700	$-0.026(3)$	5.23(0.65)
2.8	0.14	$12^2\cdot20\cdot64$	500	$-0.064(5)$	0.16(0.18)
3.2	0.09	$16^2\cdot24\cdot80$	200	$-0.053(16)$	0.92(0.93)

One could investigate this possibility from the values of scattering length a_0. Since we are studying the scattering near the threshold, it is appropriate to study the problem using non-relativistic quantum mechanics. Within non-relativistic quantum mechanics, it is known that if the potential acquires an infinitely shallow bound state, the scattering length should approach negative infinity [14]. Our lattice results for the scattering lengths indicate that it is quite large but positive. This usually happens when the potential is on the verge of developing a shallow bound state. Note that this argument is generally valid for a wide variety of potentials. If we further approximate the potential by a square-well potential with range R and depth V_0, we find that, $R = r_0 = 0.70(10)$ fm and $V_0 = 73(21)$ MeV. These values for a square-well potential also gives no bound states. If we fix $r_0 = R = 0.7$ fm, the first bound state will occur at about $V_0 \simeq 92$ MeV.

4 Conclusions

We briefly report our quenched anisotropic lattice study for the scattering of D^* and $\bar{\mathrm{D}}_1$ mesons near the threshold in the channel $J^P = 0^-$. After the chiral and continuum extrapolations, we obtain: $a_0 = 2.53(47)$ fm and $r_0 = 0.70(10)$ fm for the scattering length and effective range, indicating that the interaction between the two mesons is attractive. Our results also suggest that it is unlikely that D^* and $\bar{\mathrm{D}}_1$ can form a genuine bound state right below the threshold. The lowest two-particle state found in the simulation is likely to be a scattering state. This result might shed some light on the nature of the recently discovered $\mathrm{Z}^+(4430)$ state by Belle. However, we should emphasize that, our lattice calculation is done in a particular channel only and it is within the quenched approximation. Obviously, to further clarify the nature of the structure $\mathrm{Z}^+(4430)$, lattice studies in other symmetry channels and preferably with dynamical fermions are much welcomed.

References

1 Choi K S et al. Phys. Rev. Lett., 2008, **100**: 142001
2 Rosner J L. Phys. Rev. D, 2007, **76**: 114002
3 LIU X et al. Phys. Rev. D, 2008, **77**: 034003
4 LIU X et al. Phys. Rev. D, 2008, **77**: 094015
5 Bugg V D, J. Phys. G, 2008, **35**: 075005
6 QIAO C F, J. Phys. G, 2008 **35**: 075008
7 LEE H S et al. Phys. Lett. B, 2008, **661**: 28–32
8 Braaten E, MENG Lu. Phys. Rev. D, 2009, **79**: 051503
9 LIU X H et al. Phys. Rev. D, 2008, **77**: 094005
10 Godfrey S, Olsen S L. Ann. Rev. Nucl. Part. Sci., 2008, **58**: 51–73
11 M LÜSCHER. Nucl. Phys. B, 1991, **354**: 531
12 MENG G Z et al. Phys. Rev. D, 2009, **80**: 034503
13 M LÜSCHER, Wolff U. Nucl. Phys. B, 1990, **339**: 222
14 Sasaki S, Yamazaki T. PoS LAT2007, 2007, 131
15 LI X, LIU C. Phys. Lett. B, 2004, **587**: 100
16 FENG X et al. Phys. Rev. D, 2004, **70**: 014505
17 LI Xin et al. JHEP, 2007, **06**: 053
18 Morningstar C, Peardon M. Phys. Rev. D, 1999, **60**: 034509
19 LIU C. Phys. Lett., 2001, **18**: 187
20 CHEN Y et al. Phys. Rev. D, 2006, **73**: 014516
21 ZHANG J H, LIU C. Mod. Phys. Lett. A, 2001, **16**: 1841
22 SU S Q et al. Int. J. Mod. Phys. A, 2006, **21**: 1015
23 LIU W et al. Mod. Phys. Lett. A, 2006, **21**: 2313