

Status of the lattice and τ decay determinations of α_s *

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Abstract The two highest precision determinations of $\alpha_s(M_Z^2)$, that based on the analysis of short-distance-sensitive lattice observables, and that based on an analysis of hadronic τ decay data, have, until very recently, given results which are not in good agreement. I review new versions of these analyses which bring the two determinations into excellent agreement, and discuss prospects for additional future improvements.

Key words $\alpha_s(M_Z^2)$, τ decays, lattice QCD

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1 Introduction

Until recently, the determination of $\alpha_s(M_Z^2)$ from perturbative analyses of short-distance-sensitive lattice observables (yielding 0.1170(12) [1]), and that from finite energy sum rule (FESR) analyses of hadronic τ decay data (yielding 0.1212(11) [2]), produced central values which, though nominally precise, differed from one another by $\sim 3\sigma$. In the past year, this discrepancy has been removed by new versions of both analyses. In what follows, I outline the important features of these updates which are responsible for this change.

2 The lattice determination

The original lattice determination [1] involved the perturbative analysis of various lattice observables, O_k , computed using the MILC $n_f = 2 + 1$ configurations. The $D = 0$ expansions, computed to 3-loops for the MILC action by Mason et al. [1, 3], have the form

$$O_k = \sum_{N=1} \bar{c}_N^{(k)} \alpha_V^N(Q_k) \equiv D_k \alpha_V(Q_k) \sum_{M=0} c_M^{(k)} \alpha_V^M(Q_k), \quad (1)$$

where α_V is defined in the recent update [3] (HPQCD08), and $Q_k = d_k/a$ is the relevant BLM scale. The $\bar{c}_{1,2,3}^{(k)}$ (equivalently, $D_k, c_1^{(k)}, c_2^{(k)}$), and d_k are tabulated in HPQCD08. Regarding possible higher D contributions, (i) m_q -dependent contribu-

tions were removed by extrapolation, using data, and (ii) non-perturbative (NP) m_q -independent higher D contributions were initially assumed to be dominated by $D = 4$ gluon condensate terms, which were then fitted and removed independently for each O_k . For the lattice spacings, $a \sim 0.18, 0.12$, and 0.09 fm, of the original analysis, the observed scale-dependence of the O_k could be reproduced only by also fitting at least one additional coefficient in Eq. (1) [1].

The updated HPQCD08 [3] and CSSM [4] analyses incorporate data from new MILC ensembles with $a \sim 0.15$ and 0.06 fm, with one very new $a \sim 0.045$ fm ensemble also employed in HPQCD08. Useful cross-checks are also provided by differences in (i) the choice of coupling employed and (ii) the treatment of m_q -independent NP contribution in the two re-analyses. The choice of coupling in HPQCD08 leaves residual perturbative uncertainties in the conversion from the V to \overline{MS} scheme, that in CSSM in the effects of the truncated β function, which can be suppressed by focussing on finer lattices [4]. HPQCD08 performs an improved treatment of m_q -independent NP contributions, fitting a range of $D \geq 4$ forms to data, while CSSM restricts its attention to observables where the corresponding $D = 4$ contributions, estimated using charmonium sum-rule input for $\langle \alpha_s G^2 \rangle$ [5], are found to be small. Even with the finer lattice scales of the new analyses, it turns out that at least one additional coefficient in Eq. (1) must be fit. The α_s which results provides an excellent representation of the scale dependence of all the O_k employed. The results of

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the two re-analyses are in good agreement, and differ by only $\sim 1\sigma$ from the results of the earlier lattice analysis. The equivalent $n_f = 5$ results, at the scale $\mu^2 = M_Z^2$, are shown in Table 1 for the three most- and four least-perturbative of the O_k studied in HPQCD08. W_{kl} is the $k \times l$ Wilson loop and $u_0 = W_{11}^{1/4}$. Also shown is the corresponding quantity $\delta_{D=4}$, equal to the percent shift in the scale dependence between $a \sim 0.12$ and $a \sim 0.06$ fm resulting from first computing O_k using raw simulation values and then recomputing it after subtracting the known leading order m_q -independent $D = 4$ contributions, estimated using charmonium sum rule input for $\langle \alpha_s G^2 \rangle$ [5]. $\delta_{D=4}$ provides a measure of the expected importance of the m_q -independent NP subtractions relative to the $D = 0$ contribution from which α_s is to be determined. We see that residual NP effects in $O_k = \log(W_{11})$ are expected to be tiny; the resulting subtraction in fact produces a shift of only 0.0001 in $\alpha_s(M_Z^2)$ [4]. The fact that the results for α_s obtained after fitting and subtracting what are expected to be rather sizeable NP contributions to the most non-perturbative of the O_k agree so well with those obtained from analyses of the data for those O_k where these corrections are expected to be very small gives increased confidence in the treatment of such NP contributions and, even more important, enhanced confidence in the reliability of the results obtained from the most UV-sensitive of the O_k considered.

Table 1. $\alpha_s(M_Z^2)$ and $\delta_{D=4}$ for the 3 least- and 4 most-non-perturbative of the O_k .

O_k	$\alpha_s(M_Z^2)$	$\alpha_s(M_Z^2)$	$\delta_{D=4}$
	HPQCD08	CSSM	
$\log(W_{11})$	0.1185(8)	0.1190(11)	0.7%
$\log(W_{12})$	0.1185(8)	0.1191(11)	2.0%
$\log\left(\frac{W_{12}}{u_0^6}\right)$	0.1183(7)	0.1191(11)	5.2%
$\log\left(\frac{W_{11}W_{22}}{W_{12}^2}\right)$	0.1185(9)	N/A	32%
$\log\left(\frac{W_{23}}{u_0^{10}}\right)$	0.1176(9)	N/A	53%
$\log\left(\frac{W_{14}}{W_{23}}\right)$	0.1171(11)	N/A	79%
$\log\left(\frac{W_{11}W_{23}}{W_{12}W_{13}}\right)$	0.1174(9)	N/A	92%

3 The hadronic τ decay determination

In the SM, with $\Gamma_{V/A;ud}^{\text{had}}$ the $I = 1$ V or A current-induced width for τ to hadrons, Γ_e the τ electronic

width, $y_\tau = s/m_\tau^2$, S_{EW} a known short-distance EW correction, and $R_{V/A;ud} = \Gamma_{V/A;ud}^{\text{had}}/\Gamma_e$, one has [8]

$$dR_{V/A;ud}/dy_\tau = 12\pi^2 S_{\text{EW}} |V_{ud}|^2 \left[w_{00}(y_\tau) \rho_{V/A}^{(0+1)}(s) - w_L(y_\tau) \rho_{V/A}^{(0)}(s) \right], \quad (2)$$

where $\rho_{V/A}^{(J)}(s)$ are the spectral functions of the spin J scalar correlators, $\Pi_{V/A}^{(J)}(s)$, of the $I = 1, V/A$ current-current 2-point functions, $w_{00}(y) = (1-y)^2(1+2y)$, $w_L(y) = 2y(1-y)^2$ and, up to $O[(m_d \pm m_u)^2]$ corrections, $\rho_V^{(0)}(s) = 0$ and $\rho_A^{(0)}(s) = 2f_\pi^2 \delta(s - m_\pi^2)$, making $\rho_{V/A}^{(0+1)}(s)$ accessible from the experimental distributions $dR_{V/A;ud}/dy_\tau$ [6, 7]. For any s_0 and any analytic $w(s)$, the related finite energy sum rule (FESR)

$$\int_0^{s_0} w(s) \rho_{V/A}^{(0+1)}(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Pi_{V/A}^{(0+1)}(s) ds, \quad (3)$$

is satisfied. For large enough s_0 , the OPE can be employed on the RHS. For typical $w(s)$, and s_0 above $\sim 2 \text{ GeV}^2$, the OPE is strongly $D = 0$ dominated and hence largely determined by α_s . The 5-loop version of the $D = 0$ OPE series [9] is employed in all recent τ -based α_s analyses [2, 9–14]. Use of polynomial weights, $w(y)$, with $y = s/s_0$, helps to quantify higher D contributions, most of which must be fit to data, since (i) up to corrections of $O(\alpha_s^2)$, the integrated OPE series terminates at $D = 2N + 2$ (with N the degree of $w(y)$), and (ii) integrated OPE contributions of different D scale differently with s_0 ($D = 2k + 2$ terms scaling as $1/s_0^k$).

τ decay determinations of α_s have conventionally been based on a combined analysis of the $s_0 = m_\tau^2$, $km = 00, 10, 11, 12, 13$, $w_{km}(y) = w_{00}(y)(1-y)^k y^m$ “spectral weight FESRs” [2, 6, 7]. This analysis relies crucially on the assumption that $D = 10, \dots, 16$ contributions, each in principle present for one or more of the w_{km} employed, can, in all cases, be safely neglected. A number of recent analyses employ either this strategy directly [2] or make use of nominal $D = 6, 8$ NP contributions extracted by doing so [9–11, 14]. This assumption is, however, potentially dangerous since (i) a $\sim 1\%$ determination of $\alpha_s(M_Z^2)$ requires control of $D > 4$ NP contributions to $\lesssim 0.5\%$ of the leading $D = 0$ term and (ii) the $km = 11, 12$ and 13 FESRs have strongly suppressed $D = 0$ OPE contributions. The validity of the assumption that all $D > 8$ contributions can be neglected was tested in MY08 [12] by (i) studying, as a function of s_0 , the match between the variously weighted OPE integrals, evaluated using fitted OPE parameters, and the corresponding experimental spectral integrals, and (ii)

using the same data and fitted OPE parameters as input to FESRs for different $w(y)$ involving the same set of OPE parameters. The quality of the match produced by the results of the optimized w_{km} analysis of the ALEPH data was found to be typically rather poor in the window $\sim 2 \text{ GeV}^2 < s_0 \leq m_\tau^2$, not just for the w_{km} employed in the analysis, but also for other degree ≤ 3 weights, whose OPE integrals should depend only on the $D=0, 4, 6, 8$ OPE parameters included in the ALEPH fit [12]. Similar discrepancies exist for the w_{km} analysis based on the OPAL data, and for the BJ08 [11] treatment of the w_{00} FESR, which employs a different set of assumed values for the $D=6, 8$ contributions [12].

In view of these problems, MY08 performed analyses based on alternate weights,

$$w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N,$$

designed to suppress $D > 4$ relative to the leading $D = 0$ contributions and hence optimize the determination of α_s ¹⁾ One finds (i) excellent consistency among the α_s values obtained using different $w_N(y)$ and/or different channels (V , A or $V+A$); (ii) (as intended) a significantly reduced impact of $D > 4$ OPE contributions; and (iii) in contrast to the results of the combined $\{w_{km}\}$ analysis, an excellent quality match between OPE and corresponding spectral integrals for other degree ≤ 3 weights (including the kinematic weight w_{00}) over the whole of the s_0 window noted above. The results of MY08 are the only ones in the literature to satisfy this set of self-consistency constraints. Since (i) the $\{w_{km}\}$ analyses, which should produce results in agreement with those of the corresponding w_N analyses when using the same data, instead produce significantly larger α_s , and (ii) (as shown in the Figures of MY08) the results of the $\{w_{km}\}$ analyses, considered at lower s_0 , produce optimized OPE integrals not in agreement within errors with the corresponding experimental spectral integrals, and, moreover, significantly inferior to the matches obtained using the $\{w_N\}$ analysis fit parameters, we take the results for α_s to be those obtained from the $\{w_N\}$ analysis of MY08. The central value, $\alpha_s(m_\tau^2) = 0.321(5)(12)$ (where the first error is experimental and the second theoretical) is based on the CIPT treatment of the $D=0$ series, which yields better consistency among the results of the different w_N FESRs than does the corresponding FOPT treatment

[15]. This corresponds to

$$\alpha_s(M_Z^2) = 0.1187(6)(15)(3), \quad (4)$$

where the errors are, respectively, experimental, theoretical, and that due to evolution. The τ result is now in excellent agreement with the lattice determination. Note that effects associated with the breakdown of the integrated OPE representation, if any, are not reflected in the errors quoted here. There are, in fact, no signs for the presence of such effects in the match between the optimized OPE and spectral integrals at present. Further discussion of this issue may be found in the next section.

4 Discussion and prospects

Given the sizeable estimated $D = 4$ NP subtractions for the most non-perturbative of the lattice observables O_k , and the necessity of fitting analogous $D > 4$ contributions to data in an approximate way, the results corresponding to those O_k expected to have very small m_q -independent NP contributions should be the most reliable sources of α_s in the lattice analysis. The difference between the result obtained from the most UV-sensitive of the observables, $O_k = \log(W_{11})$, $\alpha_s(M_Z^2) = 0.1185(8)$, and that representing the average over the results for all the O_k considered in HPQCD08, $\alpha_s(M_Z^2) = 0.1183(8)$, is however, in fact, very small on the scale of the uncertainties of the analysis. We note also that one would have to reduce the lattice spacing by roughly an order of magnitude before perturbative coefficients beyond 3-loops, which must be fitted at present, could be plausibly neglected. This is certainly not feasible. Fortunately, the fitting procedures employed, when applied to input pseudo-data generated using a known high-order input expansion, return the input α_s value with very good accuracy. As a result, it appears highly unlikely that the errors associated with the fitting of the higher order coefficients play any significant role. The uncertainty in α_s for what we would argue is the most favorable of the O_k analyses, that based on $O_k = \log(W_{11})$, as can be seen from the results quoted by HPQCD08, will thus be dominated by the uncertainties in the determination of the lattice spacings in physical units for the various ensembles, with comparable contributions coming from the uncertainties in r_1/a and r_1 itself [3]. It appears unlikely that these

1) In terms of its size relative to the leading, α_s -dependent integrated $D=0$ series, neglect of $D>8$ contributions would, in fact, be between 7 and 814 times safer for the w_N analyses than it is for the higher w_{km} FESRs of the conventional analyses [12]. The fact that only a single $D > 4$ contribution, with $D = 2N+2$, enters the w_N FESR also simplifies fitting the corresponding OPE contribution.

can be significantly reduced in the near future, so one should expect only minor improvements in the lattice determination, such as those that should result from access to a larger set of $a \sim 0.045$ fm ensembles.

The current errors on the τ determination are larger than for the lattice determination, but may be amenable to more significant near-term reduction. The major components of the quoted 0.012 uncertainty on $\alpha_s(m_\tau^2)$ are: 0.0084 from the maximum difference between CIPT and FOPT determinations over the w_2, \dots, w_6 set considered in MY08; 0.0059 from the uncertainty in the $0.009(7)$ GeV^4 charmium sum rule input employed for $\langle \alpha_s G^2 / \pi \rangle$; and 0.0056 from the assumed 100% uncertainty on the FAC estimate for the 6-loop coefficient of the $D = 0$ Adler function [9]. We discuss these contributions briefly in what follows.

BJ08 have raised an interesting question about the relative reliability of the FOPT and CIPT prescriptions for evaluating the truncated $D = 0$ series. Based on a model reflecting known general properties of the divergent $D = 0$ series, they argue that, despite better observed convergence behavior for the truncated CIPT series, the truncated FOPT series might, nonetheless, better approximate the true result. Currently, it is known that, contrary to what one would expect from this model, a combined fit to a collection of degree ≤ 3 weights using the truncated fifth order CIPT prescription yields a good combined OPE-spectral integral match, while the analogous combined fit using the truncated FOPT prescription does not [15]. The analogous calculation, however, has yet to be carried out for the fully resummed BJ08 model. This work is in progress. An interesting additional observation is that the w_2 case yields a much better agreement between the CIPT and FOPT treatments of the integrated $D = 0$ OPE series than is found for other weights, and that this agreement persists over the whole of the $2 \text{ GeV}^2 < s_0 < m_\tau^2$ window. A much better than usual agreement between the FOPT and CIPT versions of the w_2 -weighted integrated truncated $D = 0$ OPE series in the vicinity of the smallest terms is also seen in the BJ08 model, suggesting that the w_2 FESR may be especially well suited to the de-

termination of α_s . Further work, however, is needed to have any realistic hope of significantly reducing the existing FOPT vs. CIPT prescription-dependence of the results for α_s . The prescription-dependence uncertainty is, of course, intimately related to that associated with the uncertainty on the estimated 6-loop Adler function coefficient.

Reduction in the error associated with the uncertainty on the input for $\langle \alpha_s G^2 \rangle$ would require an improved determination of this condensate. Given the associated renormalon ambiguity, such a determination should be performed using the same hadronic τ decay data, in an analysis with a consistently truncated $D = 0$ series. Preliminary studies indicate that it is, in fact, likely that such an improved determination of $\langle \alpha_s G^2 \rangle$ is possible. This issue turns out to be closely linked to the question of the level of residual duality violation (integrated OPE breakdown) present in the FESRs employed in the analysis, and the modelling of duality violating contributions to the physical spectral functions.

A final potential error source for the τ decay determination is residual integrated duality violation. While (i) it is known empirically that, at the scales employed in the τ decay analyses, OPE breakdown is localized to the vicinity of the timelike point on the OPE contour [16], and (ii) the weights employed in the analysis described above all have a double zero at the timelike point, a physically well-motivated model for such duality violations [10], when fitted to the observed V and A spectral functions, allows duality-violating-induced shifts to $\alpha_s(m_\tau^2)$ in the range 0.003–0.010. An additional uncertainty at the upper end of this range would not be negligible on the scale of the other errors quoted above. Preliminary investigations, however, indicate that significant further constraints can be placed on this model and that, when they are, (i) the allowed duality-violating-induced shifts lie at the low end of the range quoted above, and (ii) an improved simultaneous determination of the gluon condensate is almost certainly possible. Further work is required before more quantitative statements on these issues can be made.

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