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An estimate of two-photon exchange effect on deuteron electromagnetic form factors*

DONG Yu-Bing(董宇兵)[1,2;1)]

[1] Institute of High Energy Physics, CAS, Beijing 100049, China
[2] Theoretical Physics Center for Science Facilities (TPCSF), CAS, China

Abstract The effect of the two-photon exchange on the deuteron electromagnetic form factors is estimated based on an effective Lagrangian approach. A numerical estimate calculation of the effect is discussed. In particular, the effect on the polarization observables is analyzed.

Key words keyword, eD elastic scattering, two-photon-exchange, deuteron electromagnetic form factors

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1 Introduction

The electromagnetic (EM) form factors of the proton and deuteron are usually extracted from the measurements of the differential cross sections of ep and eD elastic scatterings and from the Rosenbluth separation method [1], which is based on one-photon-exchange (OPE) approximation. For a long time, the extracted Q^2-dependences of the nucleon EM form factors are believed to be a simple dipole form. For the proton electric and magnetic form factors, $G^{\rm p}_{\rm E,M}$, one conventionally assumes

$$G^{\rm p}_{\rm E}(Q^2)=G^{\rm p}_{\rm M}(Q^2)/\mu_{\rm p}\simeq 1/(1+Q^2({\rm GeV}^2)/0.71)^2, \quad (1)$$

where $\mu_{\rm p}$ is the proton magneton. Recently, the new experiments of the polarized ep elastic scattering were precisely carried out at Jefferson Laboratory [2]. These polarization transfer scattering experiments show that the ratio $R^{\rm p}=\mu_{\rm p}G^{\rm p}_{\rm E}(Q^2)/G^{\rm p}_{\rm M}(Q^2)$ behaves like $R^{\rm p}(Q^2)\sim 1-0.158Q^2$. It means that $R^{\rm p}$ is no longer a simple constant as implied by Eq. (1). It monotonously decreases with the increase of Q^2.

One way to resolve this discrepancy in hadronic level is to take the effect of the two-photon-exchange (TPE) into account [3–8]. Usually, it is believed that TPE is strongly suppressed by EM coupling constant $\alpha_{\rm EM}$ ($\sim 1/137$). However, it was argued [8] that due to a very steep decreasing of the nucleon EM form factors, the TPE process, where the Q^2 is equally shared by the two exchanging photons, may be compatible with the OPE one. Some calculations of the TPE corrections to the ep elastic scattering have been done recently [3–7, 9]. The effect on the EM form factors of the nucleon in the time-like region was estimated in Refs. [10, 11]. According to those analyses in the literature, it is known that the TPE corrections not only modify the conventional nucleon electric and magnetic form factors, but also provide a new one.

The TPE corrections to the deuteron (spin 1 particle) EM form factors and to $e^++e^-\rightarrow D+\bar{D}$ process have been also discussed in Refs. [12–14] qualitatively. In analogy to the TPE effect on the proton EM form factors, TPE not only modifies the conventional three EM form factors of the deuteron, but also provides three new form factors with new structures. The general discussion of the structures of the three new form factors can be seen [12, 13]. In this work, we'll show the calculations [15] of the TPE effect based on an effective Lagrangian approach [16, 17]. This paper is organized as follows. In section 2 the above mentioned two-photon-exchange effect in the eD elastic scattering is briefly discussed. Numerical results and conclusions are given in section 3.

2 Two-photon-exchange in the eD elastic scattering

According to the OPE approximation, the elec-

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1) E-mail: dongyb@ihep.ac.cn

tromagnetic form factors of the deuteron are defined by the matrix element of the electromagnetic current $J_\mu(x)$

$$
\begin{aligned}
&< p'_{\rm D},\ \lambda' \,|\, J_\mu(0) \,|\, p_{\rm D},\ \lambda >= \\
&-e_{\rm D}\Bigg\{\Bigg[G_1(Q^2)\xi'^*(\lambda')\cdot\xi(\lambda) - \\
&G_3(Q^2)\frac{(\xi'^*(\lambda')\cdot q)(\xi(\lambda)\cdot q)}{2M_{\rm D}^2}\Bigg]\cdot P_\mu + \\
&G_2(Q^2)\Big[\xi_\mu(\lambda)(\xi'^*(\lambda')\cdot q) - \xi'^*_\mu(\lambda')(\xi(\lambda)\cdot q)\Big]\Bigg\}, \quad (2)
\end{aligned}
$$

where $p'_{\rm D},\xi',\lambda'$ (or $p_{\rm D},\ \xi,\ \lambda$) denote the momentum, helicity, and polarization vector of the final (or initial) deuteron, respectively. In Eq. (2) $q=p'_{\rm D}-p_{\rm D}$ is the photon momentum, $P=p_{\rm D}+p'_{\rm D}$, $Q^2=-q^2$ is the four-momentum transfer squared, $M_{\rm D}$ is the deuteron mass, and $e_{\rm D}$ is the charge of the deuteron. In the one-photon exchange approximation or Born approximation, the unpolarized differential cross section of the eD elastic scattering, $e(k_1,s_1)+D(p_{\rm D},\xi)\to e(k'_1,s_3)+D(p'_{\rm D},\xi')$, in the laboratory frame is

$$
\frac{{\rm d}\sigma}{{\rm d}\Omega} = \frac{{\rm d}\sigma}{{\rm d}\Omega}\bigg|_{\rm Mott} I_0({\rm OPE}),
$$

$$
I_0({\rm OPE}) = A(Q^2)+B(Q^2)\tan^2\frac{\theta}{2}, \quad (3)
$$

where θ is the scattering angle of the electron, $({\rm d}\sigma/{\rm d}\Omega)_{\rm Mott}$ is the Mott cross section for a structureless particle with recoil effect, and the two structure functions are

$$
\begin{aligned}
A(Q^2) &= G_{\rm C}^2(Q^2)+\frac{2}{3}\tau_{\rm D}G_{\rm M}^2(Q^2)+\frac{8}{9}\tau_{\rm D}^2G_{\rm Q}^2(Q^2), \\
B(Q^2) &= \frac{4}{3}\tau_{\rm D}(1+\tau_{\rm D})G_{\rm M}^2(Q^2). \quad (4)
\end{aligned}
$$

In Eq. (4) $\tau_{\rm D}=Q^2/4M_{\rm D}^2$, and $G_{\rm M}$, $G_{\rm C}$ and $G_{\rm Q}$ are the deuteron magnetic, charge and quadrupole form factors, respectively. They can be expressed, in terms of G_1, G_2 and G_3, as

$$
\begin{aligned}
G_{\rm M} &= G_2,\ G_{\rm Q}=G_1-G_2+(1+\tau_{\rm D})G_3, \\
G_{\rm C} &= G_1+\frac{2}{3}\tau_{\rm D}G_{\rm Q}. \quad (5)
\end{aligned}
$$

The normalizations of the three form factors are $G_{\rm C}(0)=1$, $G_{\rm M}(0)=1.714$, and $G_{\rm Q}(0)=M_{\rm D}^2Q_{\rm D}=25.83$. Note that in Eqs. (3) and (4), there are two unpolarized structure functions A and B, and three independent form factors $G_{\rm C}$, $G_{\rm Q}$ and $G_{\rm M}$ for the deuteron. To determine the three form factors completely, one needs, at least, one polarization observable. The optimal choice is the polarization T_{20} (or P_{zz}). In fact, there are many different approaches to discuss the form factors of the deuteron in the literature. Most of them can reasonably explain the data for the electric, magnetic and quadrupole form factors and they are based on the one photon exchange approximations.

Considering both OPE ($C=-1$) and TPE ($C=+1$), and taking Lorentz, party, and charge-conjugation invariance into account, one obtains the most general form of the eD elastic scattering [12],

$$
\mathcal{M}^{\rm eD} = \frac{e^2}{Q^2}\bar{u}(k'_1,s_3)\gamma_\mu u(k_1,s_1)\sum_{i=1}^{6}G'_iM_i^\mu, \quad (6)
$$

where

$$
\begin{aligned}
M_1^\mu &= (\xi'^*\cdot\xi)P^\mu, \\
M_2^\mu &= \Big[\xi^\mu(\xi'^*\cdot q)-(\xi\cdot q)\xi'^{*\mu}\Big], \\
M_3^\mu &= -\frac{1}{2M_{\rm D}^2}(\xi\cdot q)(\xi'^*\cdot q)P^\mu, \quad (7)
\end{aligned}
$$

and

$$
\begin{aligned}
M_4^\mu &= \frac{1}{2M_{\rm D}^2}(\xi\cdot K)(\xi'^*\cdot K)P^\mu, \\
M_5^\mu &= \Big[\xi^\mu(\xi'^*\cdot K)+(\xi\cdot K)\xi'^{*\mu}\Big], \\
M_6^\mu &= \frac{1}{2M_{\rm D}^2}\Big[(\xi\cdot q)(\xi'^*\cdot K)-(\xi\cdot K)(\xi'^*\cdot q)\Big]P^\mu, \quad (8)
\end{aligned}
$$

where $K=k_1+k'_1$. Generally speaking, the form factors G'_i, with $i=1,6$, are complex functions of $s=(p_{\rm D}+k_1)^2$ and $Q^2=-(k_1-k'_1)^2$. They can be expressed as

$$
G'_i(s,Q^2)=G_i(Q^2)+G_i^{(2)}(s,Q^2), \quad (9)
$$

where G_i correspond to the contributions arising from the one-photon exchange and $G_i^{(2)}$ stand for the rest which would come mostly from TPE. In the OPE approximation, $G'_4=G'_5=G'_6=0$. It is easy to see that G_i $(i=1,2,3)$ are of order of $(\alpha_{\rm EM})^0$ and $G_i^{(2)}$ $(i=1,...,6)$ are of order $\alpha_{\rm EM}$.

To consider that a deuteron is a weakly bound state of a proton and a neutron, we take the following effective interaction between the deuteron and its composites (pn) [17]

$$
\begin{aligned}
\mathcal{L}_{\rm D} = g_{\rm D}D^{\mu+}(x)\int{\rm d}y\Phi_{\rm D}(y^2)\bar{p}\left(x+\frac{1}{2}y\right)\times \\
C\gamma_\mu n\left(x-\frac{1}{2}y\right)+{\rm H.c.}, \quad (10)
\end{aligned}
$$

where C is the charge conjugate matrix, D^μ, p and n are the fields of the deuteron, proton and neutron, respectively. The correlation function $\Phi_{\rm D}$ in Eq. (10) characterizes the finite size of the deuteron as a pn

bound state and depends on the relative Jacobi coordinate y, in addition, x being the center-of-mass (CM) coordinate. The Fourier transformation of the correlation function reads

$$\Phi_{\rm D}(y^2)=\int\frac{{\rm d}^4p}{(2\pi)^4}\,e^{-{\rm i}py}\,\widetilde{\Phi}_{\rm X}(-p^2)\,. \tag{11}$$

A basic requirement for the choice of an explicit form of the correlation function is that it vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. Here, we adopt a Gaussian form $\tilde{\Phi}_{\rm D}(p_{\rm E}^2)\doteq\exp(-p_{\rm E}^2/\Lambda_{\rm D}^2)$ for the vertex function, where $p_{\rm E}$ is the Euclidean Jacobi momentum of the deuteron, and $\Lambda_{\rm D}$ is a size parameter. It characterizes the distribution of the constituents inside the deuteron. In our approach, the coupling $g_{\rm D}$ in Eq. (10) is determined by the compositeness condition [18, 19]. It implies that the renormalization constant of the deuteron wave function is set equal to zero. The mass operator of the deuteron in our approach is described by Fig. 1. If the size parameter $\Lambda_{\rm D}$ is fixed, the coupling ${\rm g_D}$ is fixed too according to the compositeness condition (see detail in Ref. [17]).

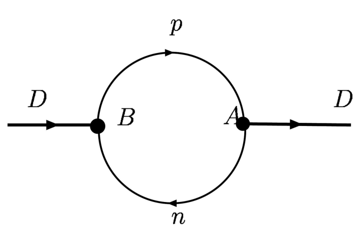

Fig. 1. Deuteron mass operator.

3 Numerical results and conclusions

To proceed a numerical calculation, we adopt the parametrization forms of the nucleon EM form factors given by Mergell, Meissner and Drechsel [20]. Here we follow the numerical technique of Ref. [21] to simplify our loop integration. In our calculation, we have one parameter $\Lambda_{\rm D}$ in the correlation function. According to the condition that the deuteron is bound as $<|\,r^{-2}\,|>\leqslant 0.02$ GeV^2, we select a typical value for the parameter: $\Lambda_{\rm D}=0.30$ GeV which is consistent with the one used in Refs. [15, 17].

To check TPE on the charge, magnetic and quadrupole form factors in our numerical calculation, we find the TPE effect on the three form factors are small. Moreover, one can estimate the new form factors $G_{5,6}$ based on our approach. The result show a clear θ-dependence contributed by TPE. To further study the TPE effect, we find that we can clearly see the effect from the polarizations of P_{xz} and P_z. In the one photon exchange approximation we have

$$\begin{aligned} P_{xz} &= -\tau_{\rm D}\frac{K_0}{M_{\rm D}}\tan\frac{\theta}{2}G_{\rm M}G_{\rm Q}, \\ P_z &= \frac{1}{3}\frac{K_0}{M_{\rm D}}\sqrt{\tau_{\rm D}(\tau_{\rm D}+1)}\tan^2\frac{\theta}{2}G_{\rm M}^2. \end{aligned} \tag{12}$$

When the TPE effect is taken into account, they are the contributions are

$$\delta P_{xz}\sim 2\tau_{\rm D}^2\cot\frac{\theta}{2}\left[2\left(\frac{G_1}{\tau_{\rm D}+1}+G_3\right){\rm Re}(G_5')+\right.$$
$$\left.\left(G_1-4G_2+2(\tau_{\rm D}+1)G_3\right){\rm Re}(G_6')\right] \tag{13}$$

and

$$\delta P_z\sim-\frac{2\tau_{\rm D}}{3}\sqrt{\frac{\tau_{\rm D}}{\tau_{\rm D}+1}}\left[\left(3+2(\tau_{\rm D}+1)\tan^2\frac{\theta}{2}\right)\times\right.$$
$$\left.G_2{\rm Re}(G_5')+2(\tau_{\rm D}+1)G_2{\rm Re}(G_6')\right]. \tag{14}$$

Clearly, one sees that the TPE corrections to this two polarizations, δP_{xz} and δP_z, do not vanish while the one photon exchange contributions do in the limit of $\theta\to 0$. The explicit illustrations for the TPE effect on the two polarizations are shown in Figs. 2 and 3, where $R(P_{xz,z})$ stand for the ratios of the TPE contributions of P_{xz} and P_z to the OPE ones, respectively. A clear θ dependence is seen and moreover, one may conclude that the TPE corrections play an important role, and turn out to be sizeable, especially in the polarization of P_z.

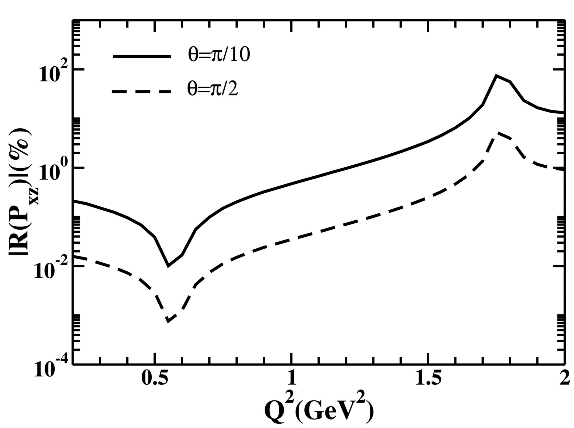

Fig. 2. Ratio for P_{xz}.

To summarize our numerical results, we have estimated the TPE corrections to the conventional form factors of the deuteron, $G_{C,M,Q}$ and of $G_{5,6}'$. Our numerical results of the TPE contributions tell that

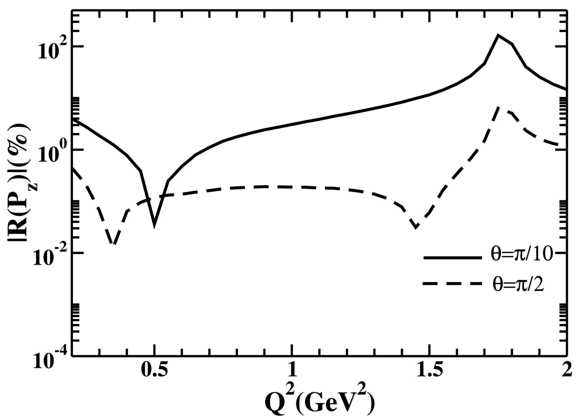

Fig. 3. Ratio for P_z.

$G^{(2)}_{C,M,Q}$ are small (less than 1%). However, $G'_{5,6}$ are clearly θ-dependent. The two additional form factors are expected to be tested in the future measurements of the double and single polarization observables of $P_{xz}(T_{21})$ and P_z (T_{10}) in the small angle limit and at some specific Q^2 values. Further work on a full calculation of the two-photon exchange effect on the deuteron system, without using the assumption of Ref. [21], is in progress.

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References

1 Rosenbluth M N. Phys. Rev., 1950, **79**: 615
2 Jones M K et al (Jefferson Lab. Hall A collaboration). Phys. Rev. Lett., 2000, **84**: 1398; Gayou O et al (Jefferson Lab. Hall A collaboration). Phys. Rev. Lett., 2002, **88**: 092301
3 Guichon P A M, Vanderhaeghen M. Phys. Rev. Lett., 2003, **91**: 142303
4 Blunden P G, Melnitchouk W, Tjon J A. Phys. Rev. Lett., 2003, **91**: 142304; Kondratyuk S, Blunden P G, Melnitchouk W, Tjon J A. Phys. Rev. Lett., 2005, **95**: 172503
5 CHEN Y C, Afanasev A, Brodsky S J, Carlson C E, Vanderhaeghen M. Phys. Rev. Lett., 2004, **93**: 122301
6 Rekalo M P, Tomasi-Gustafsson E, Dave Prout. Phys. Rev. C, 1999, **60**: 042202(R); Borisyuk D, Kobushkin A. Phys. Rev. C, 2006, **74**: 065203
7 Arrington J. Phys. Rev. C, 2003, **68**: 034325; Arrington J. Phys. Rev. C, 2004, **69**: 022201(R)
8 Gunion J, Stodolsky L. Phys. Rev. Lett., 1973, **30**: 345; Franco V. Phys. Rev. D, 1973, **8**: 826
9 Blunden P G, Melnitchouk W, Tjon J A. Phys. Rev. C, 2005, **72**: 034612
10 CHEN D Y, ZHOU H Q, Dong Y B. Phys. Rev. C, 2008, **78**: 045208
11 Gakh G I, Thomasi-Gustafsson E. Nucl. Phys. A, 2005, **761**: 120; Nucl. Phys. A, 2006, **771**: 169
12 DONG Y B, KAO C W, YANG S N, CHEN Y C. Phys. Rev. C, 2006, **74**: 064006
13 Gakh G I, Tomasi-Gustafsson E. Nucl. Phys. A, 2008, **799**: 127
14 Gakh G I, Tomasi-Gustafsson E, Adamuscin C, Dubnicka S, Dubnickova A Z. Phys. Rev. C, 2006, **74**: 025202
15 DONG Y B, CHEN D Y. Phys. Lett. B, 2009, **675**: 426
16 Ivanov A N, Troitskaya N I, Faber M, Oberhummer H. Phys. Lett. B, 1995, **361**: 74
17 DONG Y B, Faessler Amand, Gutsche Thomas, Lyubovitskij Valery E. Phys. Rev. C, 2008, **78**: 035205
18 Faessler Amand, Gutsche Th., Ivanov M A, Lyubovitskij V E, WANG P. Phys. Rev. D, 2003, **68**: 014011
19 Weinberg S. Phys. Rev., 1963, **130**: 776
20 Mergell P, Meissner U G, Dreschsel D. Nucl. Phys. A, 1996, **596**: 367
21 Bystritskiy YU M, Kuraev E A, Tomasi-Gustafsson E, Phys. Rev. C, 2007, **75**: 015207