

# Holographic models of the scalar sector of QCD<sup>\*</sup>

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**Abstract** We investigate the AdS/QCD duality for the two-point correlation functions of the lowest dimension scalar meson and scalar glueball operators, in the case of the Soft Wall holographic model of QCD. Masses and decay constants as well as gluon condensates are compared to their QCD estimates. In particular, the role of the boundary conditions for the bulk-to-boundary propagators is emphasized.

**Key words** AdS/QCD correspondence, holographic models of QCD, hadron spectroscopy

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## 1 Introduction

G. 't Hooft showed that the gauge theories in the large- $N_c$  limit, where the planar diagrams dominate, can be described according to a formalism peculiar to string theories. In 1998, J.M. Maldacena conjectured precisely such a relation [1]: the AdS/CFT correspondence which postulates a duality between the 't Hooft limit of a strongly-coupled 4d superconformal Yang-Mills theory and the supergravity limit of a weakly-coupled superstring theory defined in a 5d anti-de Sitter space-time. Strictly speaking, this correspondence cannot be applied to a confining gauge theory such as QCD since the latter is neither supersymmetric nor conformal. Nevertheless, within the so-called AdS/QCD approach, one seeks to identify the dual gravity theory able to reproduce the main features of QCD. Among the various holographic models, the Soft Wall scenario with a background dilaton field succeeds in reproducing the linear Regge behavior of the meson trajectories [2]. Here we study the scalar sector of QCD, trying to identify which properties can be properly described in this holographic model.

## 2 Light scalar mesons in the Soft Wall model of QCD

The 5d holographic space-time is described by the following conformally flat line element ( $M, N =$

$0, \dots, 4$  and  $\mu, \nu = 0, \dots, 3$  with  $x^4 \equiv z$ ):

$$ds^2 = g_{MN}(z)dx^M dx^N = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \quad (1)$$

where  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$  is the flat metric tensor of the 4d Minkowski boundary space,  $A(z) = -\ln\left(\frac{z}{R}\right)$  is the AdS<sub>5</sub> warp factor and  $R$  is the AdS radius. In the Soft Wall scenario, the 5<sup>th</sup> holographic coordinate  $z$  runs from zero to infinity while a background dilaton field  $\Phi(z) = c^2 z^2$  is introduced, the form of which being chosen in order to recover linear Regge trajectories for the light vector mesons [2].  $c = m_\rho/2 \simeq 385$  MeV is the dimensionful dilaton parameter setting the scale of QCD quantities.

The 5d effective action able to reproduce the chiral dynamics and the scalar meson sector of QCD reads [2, 3]:

$$S_{5d} = -\frac{1}{k} \int d^5x \sqrt{-g} e^{-\Phi(z)} \text{Tr} \left\{ |DX|^2 + m_5^2 |X|^2 + \frac{1}{4g_5^2} (G_L^2 + G_R^2) \right\} \quad (2)$$

with  $g = \det(g_{MN})$  the determinant of the metric tensor. The overall parameter  $k$  has the dimension of a length while  $g_5^2$  is dimensionless. The relevant QCD operators are the  $SU(2)_L \times SU(2)_R$  left- and right-handed currents

$$j_{\mu L, R}^a(x) = \bar{q}_{L, R}(x) \gamma_\mu \frac{\sigma^a}{2} q_{L, R}(x),$$

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dual to the gauge bulk fields  $A_{M,L,R}^a(x,z)$ , the quark operator  $\bar{q}_R^i(x)q_L^j(x)$  and the scalar meson operator  $\mathcal{O}_S^A(x) = \bar{q}(x)T^A q(x)$  ( $a = 1, 2, 3$  and  $i, j = 1, 2$  while  $A = 0, \dots, 8$  with  $T^0 = 1/\sqrt{6}$ ). Their associated bulk fields, respectively  $v(z)$  (responsible for the breaking of the chiral symmetry),  $\pi^a(x, z)$  the pseudoscalar chiral field and  $S^A(x, z)$  can be gathered into the form:

$$X^{ij}(x, z) = \left( \frac{v(z)}{2} + S^A(x, z)T^A \right)^{ik} \left( e^{2i\pi^a(x, z)} \right)^{kj},$$

which is tachyonic according to the AdS/CFT duality relation:  $m_S^2 R^2 = (\Delta - p)(\Delta + p - 4) = -3$  for  $\Delta = 3$  and  $p = 0$ .

Let us consider the quadratic part of the action (2) involving the scalar bulk fields  $S^A(x, z)$ . It is then straightforward to derive its equation of motion which is also the equation for the scalar meson bulk-to-boundary propagator defined, in the 4d Fourier space, as  $\tilde{S}^A(q^2, z) = S(q^2/c^2, c^2 z^2)\tilde{S}_0^A(q^2)$ :

$$\partial_z \left( \frac{R^3}{z^3} e^{-\Phi(z)} \partial_z S \right) + 3 \frac{R^3}{z^3} e^{-\Phi(z)} S - q^2 \frac{R^3}{z^3} e^{-\Phi(z)} S = 0. \quad (3)$$

The general solution involves the Tricomi and the Kummer confluent hypergeometric functions,  $U$  and

${}_1F_1$  respectively:

$$S \left( \frac{q^2}{c^2}, c^2 z^2 \right) = \frac{1}{Rc} \Gamma \left( \frac{q^2}{4c^2} + \frac{3}{2} \right) (cz) U \left( \frac{q^2}{4c^2} + \frac{1}{2}; 0; c^2 z^2 \right) + B \left( \frac{q^2}{c^2} \right) (cz)^3 {}_1F_1 \times \left( \frac{q^2}{4c^2} + \frac{3}{2}; 2; c^2 z^2 \right) \xrightarrow{z \rightarrow 0} \frac{z}{R}, \quad (4)$$

where the integration constant  $B(q^2/c^2)$  is an undetermined function of  $q^2/c^2$ . If we impose the standard boundary condition that the action is finite in the IR region ( $z \rightarrow +\infty$ ), the solution with  $B = 0$  must be chosen.

According to the AdS/CFT dictionary, any correlation function can be computed using the equivalence between the 4d generating functional of the connected correlators and the 5d partition function:

$$\left\langle e^{i \int_{\text{AdS}_5} d^4x \mathcal{O}(x) \phi_0(x)} \right\rangle_{\text{CFT}} = e^{iS_{5d}[\phi(x, z)]} \Big|_{\phi_z \rightarrow \phi_0} \quad (5)$$

for any generic bulk field  $\phi(x, z)$  dual to the operator  $\mathcal{O}(x)$ , for which  $\phi_0(x)$  is the source defined on the boundary. At the end of the day, the two-point correlation functions calculated in AdS are expressed in terms of the bulk-to-boundary propagators. For the scalar meson operator, we obtain [3]:

$$\begin{aligned} \Pi_{\text{AdS}}^{\text{AB}}(q^2) &= \delta^{\text{AB}} \frac{1}{k} S \left( \frac{q^2}{c^2}, c^2 z^2 \right) \frac{R^3}{z^3} e^{-\Phi(z)} \partial_z S \left( \frac{q^2}{c^2}, c^2 z^2 \right) \Big|_{z \rightarrow 0}, \\ &= \delta^{\text{AB}} \frac{4c^2 R}{k} \left[ \left( \frac{q^2}{4c^2} + \frac{1}{2} \right) \ln(c^2 z^2) + \left( \gamma_E - \frac{1}{2} \right) + \frac{q^2}{4c^2} \left( 2\gamma_E - \frac{1}{2} \right) + \left( \frac{q^2}{4c^2} + \frac{1}{2} \right) \psi \left( \frac{q^2}{4c^2} + \frac{3}{2} \right) \right] \Big|_{z=\epsilon}. \end{aligned} \quad (6)$$

The correlator (7) shows the presence of a discrete set of poles, corresponding to the poles of the Euler function  $\psi$ , with masses  $m_{S_n}^2 = c^2(4n+6)$  for all radial states labeled by  $n$ . The residues correspond to the scalar meson decay constants

$$F_{S_n}^2 = \frac{R}{k} 16c^4(n+1) = \frac{N_c}{\pi^2} c^4(n+1)$$

where the overall factor  $R/k$  is fixed by matching (7) in the short-distance limit  $q^2 \rightarrow +\infty$ , expanded in powers of  $1/q^2$ , with the QCD perturbative contribution [4]:

$$\frac{R}{k} = \frac{N_c}{16\pi^2}.$$

Thus, scalar mesons turn out to be heavier than vector mesons (for which  $m_{\rho_n}^2 = c^2(4n+4)$  [2]) if  $a_0(980)$  and  $f_0(980)$  are identified as the lightest scalar mesons. The agreement is also quantitative since

$$R_{f_0(a_0)} = \frac{m_{f_0(a_0)}^2}{m_\rho^2} = \frac{3}{2},$$

to be compared to  $R_{f_0}^{\text{exp}} = 1.597 \pm 0.033$  and  $R_{a_0}^{\text{exp}} = 1.612 \pm 0.004$ . Considering the first radial excitations, the predictions

$$R'_{f_0(a_0)} = \frac{5}{4}$$

should be compared to the measurements  $R_{f_0}^{\prime \text{exp}} = 1.06 \pm 0.04$  and  $R_{a_0}^{\prime \text{exp}} = 1.01 \pm 0.04$ , having identified  $a_0(1450)$ ,  $f_0(1505)$  and  $\rho(1405)$  as radial excitations. As for the decay constants, the AdS prediction is  $F_{a_0} = \frac{\sqrt{3}}{\pi} c^2 = 0.08 \text{ GeV}^2$ , to be compared with the QCD estimates of the current-vacuum matrix elements  $F_{a_0} = \langle 0 | \mathcal{O}_S^3 | a_0(980)^0 \rangle = 0.21 \pm 0.05 \text{ GeV}^2$  and  $\langle 0 | \bar{s}s | f_0(980) \rangle = 0.18 \pm 0.015 \text{ GeV}^2$  for the  $f_0(980)$ . For the first radial excitation, we have  $F_{a_0'} = 0.12 \text{ GeV}^2$  while for large values of  $n$ , the ratio  $\frac{F_n^2}{m_n^2}$  becomes independent of the radial quantum number.

The AdS/QCD duality can also be checked for the various terms in the  $1/q^2$  power expansion, compar-

ing [3]:

$$\begin{aligned} \Pi_{\text{AdS}}^{\text{AB}}(q^2) = & \delta^{\text{AB}} \frac{R}{k} \left[ q^2 \ln \left( \frac{q^2}{\nu^2} \right) + q^2 \left( 2\gamma_{\text{E}} - \ln 4 - \frac{1}{2} \right) + \right. \\ & 2c^2 \left( \ln \left( \frac{q^2}{\nu^2} \right) - \ln 4 + 2\gamma_{\text{E}} + 1 \right) + \\ & \left. \frac{2}{3} \frac{c^4}{q^2} + \frac{4}{3} \frac{c^6}{q^4} + O(1/q^6) \right] \end{aligned} \quad (8)$$

with the QCD result (in (8), the UV cutoff  $\epsilon$  has been identified with the renormalization scale  $1/\nu$ ). For  $m_q = 0$ , the 4d gluon condensate can be computed and we find  $\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{2}{\pi^2} c^4 \simeq 0.004 \text{ GeV}^4$  which is smaller than the commonly accepted value  $\langle \frac{\alpha_s}{\pi} G^2 \rangle \simeq 0.012 \text{ GeV}^4$ , the estimated uncertainty of which being about 30%. Considering the  $O(1/q^4)$  terms in QCD, one can use the factorization approximation such that  $\langle (\bar{q}\sigma_{\mu\nu}^a q)^2 \rangle \simeq -\frac{16}{3} \langle \bar{q}q \rangle^2$  and  $\langle (\bar{q}\gamma_{\mu}^a q)^2 \rangle \simeq -\frac{16}{9} \langle \bar{q}q \rangle^2$  for the dimension six operators. Within such an approximation, the  $O(1/q^4)$  term do not match since it is positive in (8) while it is negative in QCD [4]. There is also a contribution in (8) interpreted in terms of a dimension two condensate, while an analogous term expressed as the vacuum expectation value of a local gauge invariant operator is absent in QCD. However, it is possible to cancel the dimension two condensate in AdS. If the UV subleading solution in the scalar meson bulk-to-boundary propagator (4) plays a role, its coefficient can be tuned to cancel the dimension two contribution. In such a Soft Wall scenario, in which the AdS dual theory needs to be regularized in the IR [5], the subleading solution modifies some terms in the power expansion of the two-point correlation function, leaving the perturbative term unaffected.

Let us now focus on the interaction terms involving one scalar  $S$  and two light pseudoscalar fields  $P$  (the chiral field  $\pi$  and the longitudinal component of the axial-vector bulk field  $\partial_M \phi = A_M - A_{\perp M}$ ) which only appear in (2) from the covariant derivative:

$$\begin{aligned} S_{5d}^{(\text{SPP})} = & -\frac{4}{k} \int d^5x \sqrt{-g} e^{-\Phi(z)} g^{MN} v(z) \times \\ & \text{Tr} \{ S(\partial_M \pi - \partial_M \phi)(\partial_N \pi - \partial_N \phi) \}. \end{aligned} \quad (9)$$

Then, on the basis of the AdS/CFT correspondence, the QCD three-point correlator can be obtained by functional derivation of (5) with the result [3]:

$$\Pi_{\text{AdS}\alpha\beta}^{\text{abc}}(p_1, p_2) = \frac{p_{1\alpha} p_{2\beta}}{p_1^2 p_2^2} f_{\pi}^2 d^{\text{abc}} \sum_{n=0}^{\infty} \frac{F_{S_n} g_{S_n \text{PP}}}{q^2 + m_{S_n}^2}. \quad (10)$$

The AdS expression of the coupling constant  $g_{\text{S}_0 \text{PP}}$

for the lowest radial number  $n=0$  is

$$g_{\text{S}_0 \text{PP}} = \frac{\sqrt{N_c}}{4\pi} \frac{m_{\text{S}_0}^2}{f_{\pi}^2} R c^2 \int_0^{\infty} dz e^{-\Phi(z)} v(z) \quad (11)$$

and depends linearly on the field  $v(z)$ . In the Soft Wall model considered here, the coupling turns out to be numerically small, of the order 10 MeV depending on the quark mass used as an input. On the contrary, phenomenological determinations of the SPP couplings indicate sizeable values. For example,  $g_{\text{a}_0 \eta \pi}^{\text{exp}} = 12 \pm 6 \text{ GeV}$ . The origin of the small value for the SPP couplings in the AdS/QCD Soft Wall model (2) can be traced to the expression of  $v(z)$ :

$$v(z) = \frac{m_q}{Rc} \Gamma(3/2)(cz) U(1/2; 0; c^2 z^2) \xrightarrow{z \rightarrow 0} \frac{m_q z}{R} + \frac{\Sigma z^3}{R}, \quad (12)$$

which is determined uniquely by the light quark mass. As a consequence, the chiral condensate  $\Sigma$  is proportional to  $m_q$  at odds with what happens in QCD. This shortcoming does not appear in the Hard Wall model where the coefficients of  $z$  and  $z^3$  terms of  $v(z)$  are independent [6, 7] and in an improved Soft Wall model [8].

### 3 Investigating the AdS/QCD duality through the scalar glueball correlation function

The lowest dimension QCD operator describing the scalar glueballs is  $\mathcal{O}_S = \beta(\alpha_s) G_{\mu\nu}^a G^{\mu\nu a}$  ( $a = 1, \dots, 8$  a color index) with  $\beta(\alpha_s) = \beta_1 \left( \frac{\alpha_s}{\pi} \right) + \beta_2 \left( \frac{\alpha_s}{\pi} \right)^2$  the Callan-Symanzik function ( $\beta_1 = -\frac{11}{6} N_c + \frac{1}{3} n_F$  with  $N_c$  and  $n_F$  the number of colors and of active flavors respectively. In the sequel, we use  $n_F = 0$ ). According to the AdS/CFT correspondence, the associated scalar bulk field  $Y(x, z)$  is massless and is described by the action [9]:

$$S_{5d} = -\frac{1}{2\kappa} \int d^5x \sqrt{-g} e^{-\Phi(z)} g^{MN} (\partial_M Y)(\partial_N Y). \quad (13)$$

The scalar glueball bulk-to-boundary propagator is solution of the equation of motion derived from (13) and reads  $(\tilde{Y}(q, z) = K(q^2/c^2, c^2 z^2) \tilde{Y}_0(q))$ :

$$\begin{aligned} K \left( \frac{q^2}{c^2}, c^2 z^2 \right) = & \Gamma \left( \frac{q^2}{4c^2} + 2 \right) U \left( \frac{q^2}{4c^2}; -1; c^2 z^2 \right) + \\ & B \left( \frac{q^2}{c^3} \right) L \left( -\frac{q^2}{4c^2}; -2; c^2 z^2 \right), \end{aligned} \quad (14)$$

where  $L$  is the generalized Laguerre function. Then, the AdS representation of the two-point correlation function writes [5]:

$$\Pi_{\text{AdS}}(q^2) = -\frac{1}{\kappa} K\left(\frac{q^2}{c^2}, c^2 z^2\right) \frac{R^3}{z^3} e^{-\Phi(z)} \partial_z K\left(\frac{q^2}{c^2}, c^2 z^2\right) \Big|_{z=0}^{z \rightarrow +\infty}, \quad (15)$$

$$= \frac{R^3}{8\kappa} \left\{ 2Bc^4 - q^2(q^2 + 4c^2) \left( \ln(c^2 \epsilon^2) + \psi\left(\frac{q^2}{4c^2} + 2\right) + \gamma_E - 3 \right) \right\}. \quad (16)$$

Using the solution (14), the expression (15) is singular both in the UV and in the IR. A regularization prescription consists then in considering a new effective action  $S_{5d}^{\text{reg.}} = S_{5d} - S_{\text{c.t.}}|_{z=\epsilon} - S_{\text{c.t.}}|_{z=\Lambda}$  where the two counterterm actions are introduced to subtract the UV ( $z = \epsilon \rightarrow 0$ ) and IR ( $z = \Lambda \rightarrow +\infty$ ) divergences. The first one is the usual term considered in the AdS/CFT procedure while the second one defines the IR Soft Wall model: it involves  $B$  in (14) and vanishes when  $B = 0$  as in the standard procedure.

If the AdS/QCD duality holds then the two-point correlation function (16) should match the QCD result. Let us consider the short-distance regime. In the limit  $q^2 \rightarrow +\infty$ , (16) can be expressed in terms of a perturbative contribution and a series of power corrections in  $1/q^2$ . Identifying the perturbative term gives  $\kappa = \frac{\pi^4}{16\alpha_s^2 \beta_1^2} R^3$ . Moreover, if  $B$  is a function having a polynomial behavior at large space-like  $q^2$ , namely  $B(q^2/c^2) \xrightarrow{q^2 \rightarrow +\infty} \eta_1 \frac{q^2}{c^2} + \eta_0$ , the parameters  $\eta_0$  and  $\eta_1$  can be fixed by matching  $\Pi_{\text{AdS}}(q^2)$  with the OPE expansion of  $\Pi_{\text{QCD}}(q^2)$ . The constant term  $\eta_0$  turns out to contribute to the 4d gluon condensate:

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = \frac{4\alpha_s}{\pi^3} \left( 2\eta_0 - \frac{5}{6} \right) c^4. \quad (17)$$

On the other hand, in the region close to  $q^2 = 0$ , one finds  $\Pi_{\text{AdS}}(0) = \frac{R^3}{\kappa} 2B(0)c^4$ . For a constant coefficient function  $B(0) = \eta_0$ , we find:

$$\Pi_{\text{AdS}}(0) = \frac{\alpha_s}{4\pi} (-\beta_1) \frac{\eta_0}{\eta_0 - \frac{5}{12}} \left( -16\beta_1 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right). \quad (18)$$

Imposing that (18) coincides with the Low Energy Theorem  $\Pi_{\text{QCD}}(0) = -16\beta_1 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$ , it is possible

to constrain the value of  $\eta_0 = \frac{5}{12} \left( \frac{1}{1 + \frac{\alpha_s}{4\pi} \beta_1} \right)$ . Using this expression in (17) together with  $\alpha_s = 1.5$ , we have  $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \simeq 0.007 \text{ GeV}^4$ . Without the contribution of  $\eta_0$ , the 4d gluon condensate would be negative which seems to indicate that the general solution (14) plays a role in order to reconstruct a bulk-to-boundary propagator able to implement the AdS/QCD duality. As for the 6d and 8d gluon condensates, we have the following AdS expressions [5]:

$$\langle g_s f_{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle = \frac{4}{3\pi^2} c^6, \quad (19)$$

$$14 \langle (f_{abc} G_{\mu\alpha}^a G_{\nu\alpha}^b)^2 \rangle - \langle (f_{abc} G_{\mu\nu}^a G_{\alpha\beta}^b)^2 \rangle = -\frac{8}{15\alpha_s \pi^3} c^8, \quad (20)$$

which are different in size (and in sign for the latter) from their commonly used values, respectively  $0.045 \text{ GeV}^6$  and  $\frac{9}{16} \left( \frac{\pi}{\alpha_s} \right)^2 \left( \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right)^2$ . However, the values of these gluon condensates are very uncertain.

Besides, in the time-like  $q^2 < 0$  region, a discrete set of poles appears according to the spectral relation [9]  $m_{G_n}^2 = c^2(4n + 8)$  with residues  $F_{G_n}^2 = \frac{R^3}{\kappa} 8(n+1)(n+2)c^6$ . Scalar glueballs are heavier than scalar mesons:  $\frac{m_G^2}{m_{f_0}^2} = \frac{4}{3}$  for the lowest-lying states while the hierarchy among the hadron species is reduced for higher radial states, which become degenerate when the quantum number  $n$  increases.

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