Optimum condition of efficiency functions for HPGe γ-ray detectors in the 121-1408 keV energy range

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Abstract The optimum condition of three commonly used functions in the Genie 2000 γ spectra analysis software have been studied in the 121–1408 keV energy range. The three functions are applied for fitting the full-energy peak efficiency of the HPGe gamma-ray detector. A detailed procedure to obtain the optimum condition is described. The HPGe detector is calibrated at 11 cm by three radioactive sources of point form (^{152}Eu, ^{137}Cs, ^{60}Co) providing 11 energy peaks. After data processing, results shows that the three functions used in the Genie 2000 gamma spectra analysis software fit best at orders 3–5. Lastly the standard radioactive source ^{133}Ba is chosen to validate the results. Differences between the standard activity of ^{133}Ba and the result obtained from the fitting functions are below 1.5%. Therefore the optimum orders of the three functions used in the Genie 2000 γ spectra analysis software are 3–5 with the 11 energy peaks.

Key words HPGe, fit function, efficiency calibration, Genie 2000

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1 Introduction

In γ-ray spectrometry with HPGe detectors, it is a common practice to approximate the detector efficiency for full-energy peaks by an analytical function, fitted to the experimental efficiency data points, and many nuclear measurements require accurate knowledge of the shape of the efficiency curve in a particular energy range [1–3]. As the number of energy peaks obtained from standard radioactive sources are limited, functions are required to fit the detecting efficiency at a wide range of energies. Therefore the study on efficiency functions is a key point in γ-ray spectrometry. Several kinds of functions have been used to fit the efficiency of HPGe detectors in previous research [4–6]. For a wide range of energies, functions of the form $\ln\varepsilon = F\ln E$ have generally been used [7] and functions with other forms are also used to fit the efficiency. Some efficiency functions have been compared in a wide energy range [8] and least square fitting has also been studied [9]. In many cases of Prompt Gamma Activity Analysis (PGAA) the energy of gamma rays are between 121–1408 keV [10]. So, how to fit the detecting efficiency better with commonly used functions is very important to most γ-ray spectrometry analysis laboratories. In this energy range the radioactive sources ^{152}Eu, ^{137}Cs, ^{133}Ba, ^{60}Co can be applied in the calibration. In the Genie 2000 gamma spectra analysis software, three kinds of functions have been proposed to fit the peak efficiency, as shown in Table 1, where the parameter $E_{\rm m}$ in function II is the average value ($E_{\rm m} = (E_{\gamma\max} + E_{\gamma\min})/2$) of the maximum and minimum energy chosen in the calibration. The performance of the functions depends on the degree of a polynomial of order i. The order of the fit function is of great importance. The aim of

Table 1. Three kinds of functions used in the Genie 2000.

	function description	function
function Ⅰ	ln-ln scale	$\ln\varepsilon=\sum a_i(\ln E_\gamma)^i$
function Ⅱ	ln-ln scale	$\ln\varepsilon=\sum a_i(\ln\frac{E_{\rm m}}{E_\gamma})^i$
function Ⅲ	lg scale	$\lg\varepsilon=\sum a_i E_\gamma^{1-i}$

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this paper is to obtain the optimum condition for the fitting functions with the 11 energy peaks of the radioactive sources ^{152}Eu, ^{137}Cs, and ^{60}Co, and to analyze how the performance changes with the fitting orders in the energy range 121–1408 keV. All three functions are fitted with a least-square procedure. The radioactive source ^{133}Ba is chosen to validate the result.

2 Materials and methods

2.1 Measurement system

The model of the HPGe detector used in this experiment is BE3830, manufactured by the CANBERRA Company, USA. The active diameter of the detector is 69.5 mm and its active area is 3800 mm^2. The cryostat window material is A carbon composite with a thickness of 0.6 mm. 3000 V is recommended as a bias voltage for the measurement. The relative efficiency of the HPGe detector is 32% (compared with a 3" × 3" NaI crystal), and the revolution at 1332 keV (^{60}Co) is 1.754 keV. The electronic setup includes a high-voltage power supply, a preamplifier, the spectroscopy, an amplifier and a pulse generator, which are all from CANBERRA. During the experiment the radiation sources are placed 11 cm away from the detector with a standard bracket (the standard bracket is just 11 cm high).

2.2 Sources used in the calibration

A set of radioactive sources including ^{152}Eu, ^{60}Co, ^{137}Cs, are used to obtain the γ-ray response in the energy range between 121–1408 keV. ^{152}Eu is widely used in efficiency calibration because of its multi-gamma peaks and good branching ratio of each peak. Ruy M. Castro has tried to develop ^{152}Eu into a standard for the detector efficiency calibration [11]. So, in this work ^{152}Eu is used as a main isotope to calibrate the instrument within 121–1408 keV. ^{60}Co and ^{137}Cs are also used in the calibration because of their simple decay schemes for which the relationship between source activity and γ-ray emission probability can be derived without any a priori knowledge of the efficiencies. All the three radionuclides are in the form of point sources. With these radiation sources a nearly uniform distribution of the calibration peaks can be obtained in the energy range between 121–1408 keV, which will reduce the probability of the occurrence of non-physical oscillations. The applied calibration peaks have been grouped together in Table 2 for convenience. This table also contains the certified and fitted activities. The half-lives used have been taken from the 1991 IAEA recommendation [10].

Table 2. Nuclear data of the calibration sources.

sources	half-life/d	No. of peaks used	E_γ /keV	P_γ/per decay	activity /kBq
^{60}Co	1925±0.5	2	1173.24	99.86%	17.0±0.4
			1332.50	99.98%	
^{137}Cs	11020±60	1	661.66	85.12%	31.7±0.8
^{152}Eu	4933±11	8	121.78	28.37%	27.5±0.8
			244.70	7.53%	
			344.28	26.57%	
			443.97	3.13%	
			778.90	12.97%	
			867.39	4.21%	
			964.06	14.63%	
			1408.02	20.85%	

3 Results and discussion

3.1 Experimental efficiency

Full-absorption peak efficiencies were obtained by measuring a set of 3 sources at 11 cm distance from the detector window. The efficiencies can be derived from the simple formula:

$$\varepsilon = \frac{C}{N \times P_\gamma}. \quad (1)$$

In Eq. (1), C is the net area of an energy peak, which is automatically calculated by the analysis software Genie 2000. In this process a Sum/Non-Linear Least Squares Fit Peak Area algorithm, including a calculation for defining the limits of the peak region also a calculation of the peak areas and their uncertainties for either singlets or multiplets were included. The peak areas are also corrected automatically by the software for random summing and subtraction of the

environmental background. N is the number of decays; P_γ is the emission probability of the considered γ-ray transition. In order to decrease the statistical uncertainty induced by the measurement, the measurement time has to be long enough to ensure that the net areas of each photo peak consist of over 20000 counts. The efficiency variance is denoted by σ_ε^2, and is obtained from the following equation:

$$\sigma_\varepsilon^2 = \varepsilon^2 \times \left[\left(\frac{\sigma_S}{S}\right)^2 + \left(\frac{\sigma_A}{A}\right)^2\right], \quad (2)$$

where σ_ε is the standard deviation of ε, S is the net peak area of the calibration peak, σ_S is the standard deviation of the peak area, A is the known activity of the calibration nuclide, and σ_A is the standard deviation of the source activity.

The efficiency calibration results are shown in Table 3. In Table 3, the efficiency is the absolute efficiency, which is derived from Eq. (2). The detecting efficiency decreases gradually as the energy increases. The uncertainty of the efficiency is about 3%, and this is mainly due to the calibration sources.

Table 3. Results of the efficiency calibration with radiation isotopes.

energy/keV	net peak area	peak area statistic uncertainty (%)	live time/s	absolute efficiency (%)	efficiency uncertainty(%)
121.78	2815759	0.06	16218.57	2.23	3.00
244.70	466667	0.160	16218.57	1.39	3.00
344.28	1201363	0.090	16218.57	1.01	3.00
443.97	103725	0.370	16218.57	0.75	3.02
661.66	212161	0.180	2207.62	0.52	2.51
778.90	253696	0.210	16218.57	0.44	3.01
867.39	71817	0.460	16218.57	0.38	3.04
964.06	230929	0.220	16218.57	0.35	3.01
1173.24	35259	0.540	676.43	0.31	2.56
1332.50	31596	0.570	676.43	0.28	2.56
1408.02	231412	0.210	16218.57	0.25	3.01

Table 4. R^2 of all three functions for different orders.

	order 1	order 2	order 3	order 4	order 5	order 6	order 7
function Ⅰ	0.99507	0.99675	0.99898	0.99916	0.99920	0.99928	0.99993
function Ⅱ	0.99158	0.99801	0.99971	0.99972	0.99972	0.99976	0.99976
function Ⅲ	0.91393	0.98520	0.99893	0.99914	0.99916	0.99918	0.99987

After the peak detection efficiency has been established for each calibration peak, a least-square procedure is used to fit all these data with the three functions of Table 1 with orders from 1 to 7. In order to compare how each function at a different order fits the energy peaks, the fit goodness is calculated at each order. The square of efficiency follows a χ^2 distribution if the efficiency follows a Gaussian distribution. Therefore, in order to compare the fitted functions, R^2 is introduced to denote the goodness of the fit, which is given in Eq. (3). In Eq. (3), if the function fits the measured values very well, the second item will be small or even zero, and R^2 will be close to 1.

$$R^2 = 1 - \frac{\sum(y-y')^2}{\sum(y-\overline{y})^2}, \quad (3)$$

here y is the measured value, y' is calculated from the fitted function and $\overline{y}$ is the average value of the measurements. Table 4 shows the goodness of the fit for different orders for each function. From the results in Table 4, it is obvious that R^2 of all the functions is bigger than 0.9, and R^2 of both functions Ⅰ and Ⅱ is even bigger than 0.99. R^2 gets closer to 1 as the orders increase. It seems that with a higher order one achieves a better performance in the calibration.

3.2 The optimum condition for the efficiency functions

In order to compare the performances better, all the functions with different orders are plotted in Fig. 1. For each function, the orders 2–5 and the orders 1, 6, 7 are plotted separately according to their different performance. It is obvious to see the fol-

lowing behavior of all the three kinds of functions. As the orders increase, functions with orders 2–5 perform well and orders 3–5 perform best. However, for orders larger than 5, although R^2 approaches values very near to 1, the performance gets worse. A linear fit leads to a bad performance because the efficiency decreases gradually as the energy increases, however not linearly, especially with function III. Higher orders give in principle a better fit, but the performance is also determined by the numbers of data points. For a sufficiently high number of data points, higher orders can indeed give a good performance, but a combination of only few data points together with high orders will lead to non-physical oscillations (see Fig. 1). From the comparison above it is obvious to see that the optimum orders for the efficiency functions in the energy range between 121–1408 keV are between 3–5; both lower and higher orders performs not very well.

3.3 Verification with the radioactive source ^{133}Ba

After comparison of all the three functions, the ^{133}Ba radioactive source was used to verify the result. The information on the ^{133}Ba radioactive source is specified in Table 5. Because all the three functions perform well at orders 3–5, just one function at one order is chosen to be validated instead of all the three. Function III at order 5 is chosen in the experiment and its fitted efficiency formula is given in Eq. (4).

$$\lg\varepsilon = -\frac{1.323\times10^{-4}}{E^{-1}} - 2.59 + \frac{2.79\times10^{2}}{E} - \frac{1.128\times10^{4}}{E^{2}} - \frac{4.406\times10^{6}}{E^{3}} + \frac{4.098\times10^{8}}{E^{4}}. \quad (4)$$

The activity of the ^{133}Ba radioactive source can be calculated by Eq. (5) with the activities of all the three branches in Table 6. The activity is about 22.3 kBq. In Table 6, the detecting efficiency is derived from Eq. (4). Comparing with the activity of itself, the relative differences are below 1.5%. The result is in accordance with its standard value and this provides a powerful validation for the fit result.

$$N = \frac{\sum P_\gamma \times A_\gamma}{\sum P_\gamma}. \quad (5)$$

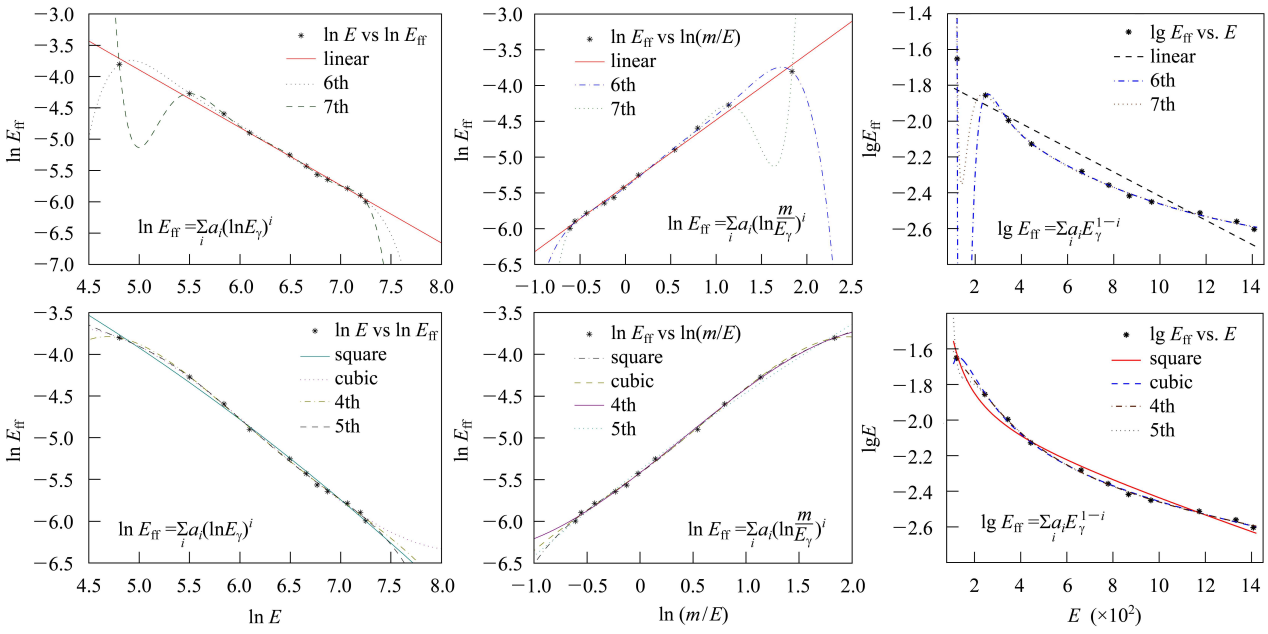

Fig. 1. Comparison of all the three functions for the orders 1–7.

Table 5. Nuclear data of the ^{133}Ba radioactive source.

sources	half-life/d	No. of peaks used	E_γ /keV	P_γ/per decay(%)	activity /kBq
^{133}Ba	3862±15	3	276.40	7.15	22.0±0.7
			302.85	18.31	
			356.02	61.94	

Table 6. Validation result with the ^{133}Ba radioactive source.

energy/keV	net peak area	peak area statistic uncertainty (%)	detecting efficiency (%)	live time/s	activity/kBq	activity uncertainty (%)
276.40	35652	0.55	1.25	1804.78	22.1	1.37
302.85	83822	0.35	1.14		22.2	1.19
356.01	242241	0.20	0.97		22.4	0.99

4 Conclusion

With the 11 energy peaks of three commonly used radioactive sources ^{152}Eu, ^{137}Cs and ^{60}Co, the optimum orders of the three functions in the Genie 2000 gamma spectra analysis software are 3–5 in the 121–1408 keV energy range. This optimum condition is obtained from calibrating the HPGe γ-ray detector at 11 cm by three radioactive sources of point form providing 11 energy peaks. From the validating result of ^{133}Ba it is obvious that the 11 energy peaks of the three sources are enough to do the calibration in many PGAA measurements with the optimum orders of the fit functions. Fitting results at different orders are affected greatly by the number of energy peaks used in the calibration. For other energy ranges,in order to obtain a better fitting efficiency, orders of the fitting functions must be chosen according to the number of energy peaks used in the calibration.

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