η-nucleus bound states in nuclei *

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Abstract The binding energies ϵ_η and widths Γ_η of η-mesic nuclei are calculated. We parameterize the η self-energy in the nuclear medium as a function of energy and density. We find that the single-particle energies are sensitive to the scattering length, and increase monotonically with the nucleus. The key point for the study of η-nucleus bound states is the η-nuclear optical potential. We study the *s*-wave interactions of η mesons in a nuclear medium and obtain the optical potential $U_\eta \approx -72$ MeV. Comparing our results with the previous results, we find that the ηN scattering length $a_{\eta N}$ is indeed important to the calculations. With increasing nuclear density the effective mass of the η meson decreases.

Key words mesic nuclei, binding energy, self-energy, optical potential

PACS 12.39.Fe, 12.40.Yx, 13.75.-n

1 Introduction

The nuclear medium effects on meson properties are interesting and have been investigated extensively in nuclear physics. The pion-nucleon and kaon-nucleon interaction have been studied much both theoretically and experimentally. Haider and Liu [1, 2] first predicted the existence of the η-mesic nucleus which is a bound state of an η meson in a nucleus. The formation of such bound systems is caused by the attractive interaction between the η meson and all the nucleons in the nucleus. According to many theoretical models which were proposed for the ηN interaction [3–6], the attractive interaction has a correlation with the η self-energy.

The existence of these η-mesic nuclear states depends on the value of the ηN scattering length $a_{\eta N}$. Fits to various data have yielded very different $a_{\eta N}$ [7–11]. These very different values arise because $a_{\eta N}$ is not directly measurable and must be inferred indirectly from other observations, such as the πN phase shifts.

In this paper, we study the properties of the η in the nuclear medium from a theoretical point of view. In order to produce an accurate as possible result for the η bound states in nuclei, we parameterize the η self-energy as a function of energy and density. The present work follows closely the method used in Ref. [6] to obtain the η self-energy by summing up the ηN interaction in the medium of all the nucleons in the Fermi sea. Combining with the η self-energy we obtain the binding energies and widths of the η states in different nuclei. We believe that it is valuable to understand the current experimental situation by analyzing in detail the dynamics pertinent to the formation of η-mesic nuclei.

The key point for the study of η-nucleus bound states is the η-nuclear optical potential. According to Ref. [6], we parameterize the η self-energy as a function of energy and density and our result is $U_\eta \approx -72$ MeV. There have several investigations been done in this field. Waas and Weise studied the *s*-wave interactions of η mesons in the nuclear medium and obtained a potential $U_\eta \approx -20$ MeV [3]. Chiang et al. obtained $U_\eta \approx -34$ MeV by assuming that the mass of the $N^*(1535)$ does not change in the medium [4]. Tsushima et al. predicted that the η-meson potential was typically −60 MeV using the quark-meson coupling (QMC) model [5]. Inoue and Oset also obtain $U_\eta \approx -54$ MeV with their model [6]. Zhong et al. obtained $U_\eta \approx -83$ MeV by deducing the η-nucleon interaction from the heavy-baryon chiral perturbation theory in the next-to-leading-order terms [12]. Obviously, there are model dependencies in describing the

Received 12 May 2009

∗ Supported by NSFC (10775012)

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in-medium properties of the η meson.

Finally, we mention that as the existence of η-mesic nuclei has not yet been experimentally confirmed with certainty, there are no data to be fitted. Hence, all of our results as well as those of others are pure predictions. If the existence of the η-mesic nucleus is experimentally confirmed, many new studies of nuclear and particle physics will become possible [13].

The paper is organized as follows. In the next section, we explain our approach to parameterize the η self-energy as a function of energy and density. Results and discussions are presented in Sec. 3. We calculate the binding energies and widths of the η state in different nuclei. Comparing our results with the previous results, we find that the ηN scattering length $a_{\eta N}$ is indeed important to the calculations. The conclusions of our study are summarized in Sec. 4.

2 Formulation

The effective mass of the η is defined as the pole of the full propagator with self-energy corrections of the η in the limit $\vec{k}\to 0$. From the full propagator of the η we find that the effective mass m_η^* of the η is determined by the equation

$$(k^0)^2-m_\eta^2-\mathrm{Re}[\Pi_\eta(k^0,\vec{0};\rho)]=0, \tag{1}$$

and

$$m_\eta^*=\left\{m_\eta^2+\mathrm{Re}[\Pi_\eta(k^0=m_\eta,\vec{0};\rho)]\right\}^{1/2}, \tag{2}$$

where $\Pi_\eta(k^0,\vec{0};\rho)$ is the η self-energy in the nuclear medium. m_η and k^0 denote the mass and energy of the η meson respectively. $\rho=\rho_s$ is the scalar density of nucleon. The optical potential $U_\eta(\rho)$ and the effective η-neutron scattering length $a_{\eta N}(\rho)$ are defined as

$$U_\eta(\rho)=\frac{\Pi_\eta(m_\eta,\vec{0};\rho)}{2m_\eta}, \tag{3}$$

$$a_{\eta N}(\rho)=-\frac{1}{4\pi}\frac{M_N}{M_N+m_\eta}\frac{\Pi_\eta(m_\eta,\vec{0};\rho)}{\rho}, \tag{4}$$

with M_N the nucleon mass. We evaluate the η self-energy for $k^0=m_\eta$ and $\vec{k}=0$ as a good approximation. These values are useful when we study the bound states of an η in nuclei using the local density approximation.

The η meson in the nuclear medium satisfies static Klein-Gordon equation [14]:

$$[\partial_\mu\partial^\mu+m_\eta^2+\Pi_\eta(k^0,\vec{0};\rho)]\eta=0. \tag{5}$$

According to Eq. (5), with $\eta=R_l(r)Y_l(\theta,\varphi)$, this leads to

$$\left[\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}+(E^2-m_\eta^2)-\Pi_\eta-\frac{l(l+1)}{r^2}\right]R_l(r)=0, \tag{6}$$

with $\chi(r)=rR_l(r)$, therefore the above equation can be written as

$$\frac{\mathrm{d}^2\chi(r)}{\mathrm{d}r^2}-\left[(E^2-m_\eta^2)-\Pi_\eta-\frac{l(l+1)}{r^2}\right]\chi(r)=0. \tag{7}$$

As expressed in Appendix A, Eq. (7) can be solved by the Gowell central differential numerical method which has been studied in Ref. [15].

We parameterize the η self-energy $\Pi_\eta(k^0,\vec{0};\rho)$ as a function of energy and density [6]. We can parameterize it in the region -50 MeV $<k^0-m_\eta<0$, as

$$\mathrm{Re}[\Pi_\eta(k^0,\vec{0};\rho)]=a(\rho)+b(\rho)(k^0-m_\eta)+ c(\rho)(k^0-m_\eta)^2+d(\rho)(k^0-m_\eta)^3, \tag{8}$$

$$\mathrm{Im}[\Pi_\eta(k^0,\vec{0};\rho)]=e(\rho)+f(\rho)(k^0-m_\eta)+ g(\rho)(k^0-m_\eta)^2+h(\rho)(k^0-m_\eta)^3, \tag{9}$$

with

$$\begin{aligned}
a(\rho)&=-36200.3\rho/\rho_0-24166.6\rho^2/\rho_0^2\ \mathrm{MeV}^2,\\
b(\rho)&=-1060.05\rho/\rho_0-326.803\rho^2/\rho_0^2\ \mathrm{MeV},\\
c(\rho)&=-13.2403\rho/\rho_0-0.154177\rho^2/\rho_0^2,\\
d(\rho)&=-0.0701901\rho/\rho_0+0.0173533\rho^2/\rho_0^2\ \mathrm{MeV}^{-1},\\
e(\rho)&=-43620.9\rho/\rho_0+11408.4\rho^2/\rho_0^2\ \mathrm{MeV}^2,\\
f(\rho)&=-1441.14\rho/\rho_0+511.247\rho^2/\rho_0^2\ \mathrm{MeV},\\
g(\rho)&=-27.6865\rho/\rho_0+10.0433\rho^2/\rho_0^2,\\
h(\rho)&=-0.221282\rho/\rho_0+0.0840531\rho^2/\rho_0^2\ \mathrm{MeV}^{-1}.
\end{aligned}$$

By using the Klein-Gordon equation and substituting $\rho\to\rho(r)$ in the spirit of the local density approximation, we can then obtain the binding energies and widths of the η state in different nuclei.

The spatial distribution of the nucleon density in nuclei is best described by the Fermi function [16]

$$\rho(r)=\frac{\rho_0}{1+\mathrm{e}^{(r-R)/a}}\ , \tag{10}$$

where the quantity ρ_0 is the nucleon density at the center of the nucleus and R corresponds to the distance from the center at which the density has decreased to half its maximum value. The quantity a characterizing the thickness of the surface layer in which the rapid decrease of density occurs is usually called the diffuseness of the nuclear surface.

The complex energy E is given by

$$E=-\epsilon_\eta+m_\eta-\mathrm{i}\Gamma/2\ , \tag{11}$$

where the real part corresponds to the η single particle binding energy ϵ_η, it is defined as

$$\epsilon_\eta = m_\eta - \mathrm{Re}E \ . \qquad (12)$$

The imaginary part of the complex energy corresponds to the widths of the η state in the nuclear medium,

$$\Gamma = -2\mathrm{Im}E \ . \qquad (13)$$

3 Results and discussion

The binding energies and widths of the η-mesic nuclei given by the off-shell microscopic calculation are presented in Table 1. As can be seen from the table, the binding energy increases as the nucleus becomes heavier. In addition, the number of bound states of the η increases with increasing mass number A. The *p*-wave and *d*-wave interaction are also attractive at the threshold but their magnitudes are very small and have a negligible effect on ϵ_η and Γ_η [17, 18]. But if the nucleus is heavy enough, for example, ${}^{132}_{\eta}$Xe and ${}^{208}_{\eta}$Pb, the 1*p* orbit of the η meson bound states also exists. It would be interesting to compare these results with experimental data.

Table 1. Binding energies and widths (both in MeV) of η-mesic nuclei given by the full off-shell calculation.

nucleus	orbit	ϵ_η	Γ_η
${}^{28}_{\eta}$Si	1*s*	37.6	62.4
${}^{32}_{\eta}$S	1*s*	38.2	66.4
${}^{36}_{\eta}$Ar	1*s*	39.8	63.8
${}^{40}_{\eta}$Ca	1*s*	40.5	61.9
${}^{44}_{\eta}$Ti	1*s*	42.1	61.8
${}^{132}_{\eta}$Xe	1*s*	48.9	65.0
	1*p*	34.7	46.3
${}^{208}_{\eta}$Pb	1*s*	50.2	62.0
	1*p*	35.5	42.6

The optical potential $U_\eta(r)$ of η mesons in ^{40}Ca as a function of the nuclear radius r is plotted in Fig. 1. The solid line denotes the real part of the potential, and the dotted line denotes the imaginary part of the potential. From Fig. 1, we can see that the optical potential $U_\eta(r)$ decreases with increasing nuclear radius from about 72 MeV to zero. On the one hand, the optical potential increases with the nuclear density. On the other hand, the optical potential depends strongly on the value of the scattering length $a_{\eta N}$. In our calculations, the optical potential can change from −65 to −88 MeV when we modify the scattering length $a_{\eta N}$ from 0.87 to 1.12 fm.

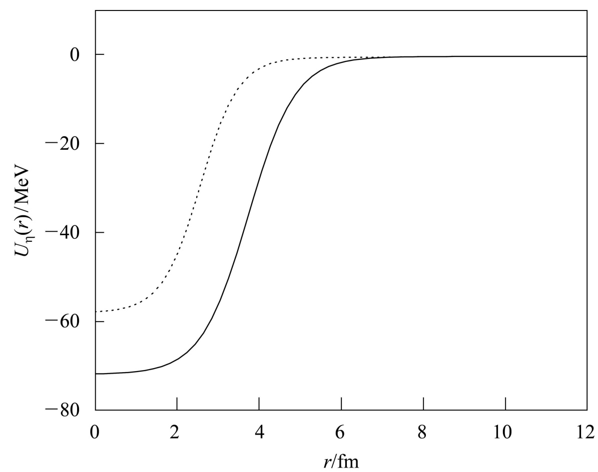

Fig. 1. The optical potential $U_\eta(r)$ of η mesons in ^{40}Ca as a function of the nuclear radius r. The solid line denotes the real part of the potential, and the dotted line denotes the imaginary part of the potential.

Figure 2 shows the radial wave function distribution of η mesons in ^{40}Ca as a function of the nuclear radius r. From Fig. 2, we can see that the radial wave function of the η mesons is a regular distribution for $r < 5$ fm. The radial wave function distribution reaches a maximum at $r = 2$ fm and almost vanishes in the $r > 5$ fm region. From this we can conclude that the η mesons in ^{40}Ca are mainly concentrated in the center of the nuclei.

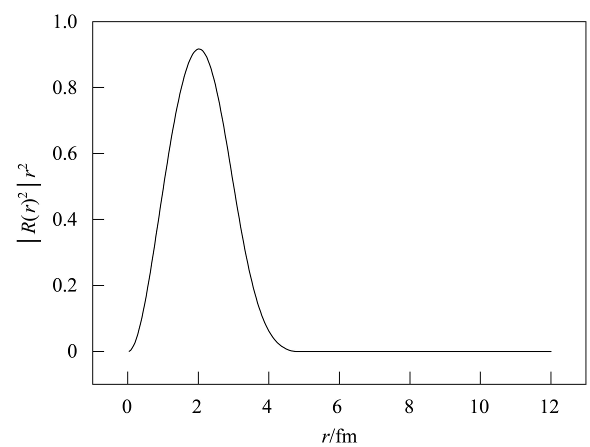

Fig. 2. The radial wave function of η mesons in ^{40}Ca as a function of the nuclear radius r.

With the normal nuclear density $\rho_0 = 0.15$ fm^3, the effective mass of the η meson m^*_η as a function of the nuclear density ρ/ρ_0 is shown in Fig. 3. We observe that with increasing nuclear density the effective mass of the η meson decreases from the value of its free mass, i.e., 547.3 MeV, to about 488 MeV. The decreasing of the effective mass of the η meson implies a lengthening of the interacting range between nucleons.

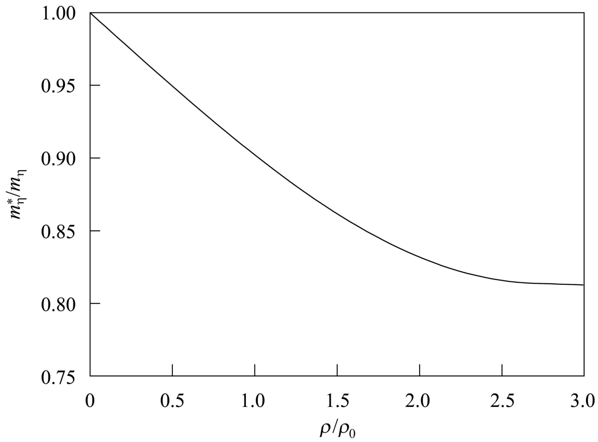

Fig. 3. The effective mass of the η meson m^*_η as a function of nuclear density ρ/ρ_0, with the normal nuclear density $\rho_0=0.15$ fm^3.

4 Conclusions

In summary, our study shows that the calculated binding energies and widths of the η-nucleus bound states strongly depend on the η-nucleus optical potential. In addition we have also illuminated the self-energy of the baryons and mesons in the intermediate states, including the η self-energy in a self-consistent way. We study the s-wave interactions of η mesons in the nuclear medium and obtain the optical potential $U_\eta\approx-72$ MeV. It should be interesting to test these predictions with experimental data of η mesic nuclei.

We would like to thank Bao-Xi Sun for the useful discussions.

Appendix A:

Gowell central differential method

Expanding $\chi(r_{i+1})$ and $\chi(r_{i-1})$ to a Taylor series with respect to r_i, we have

$$\chi(r_{i+1})=\chi(r_i)+h\chi'(r_i)+\frac{h^2}{2}\chi''(r_i)+\frac{h^3}{6}\chi(3)(r_i)+\cdots, \tag{A1}$$

$$\chi(r_{i-1})=\chi(r_i)-h\chi'(r_i)+\frac{h^2}{2}\chi''(r_i)-\frac{h^3}{6}\chi(3)(r_i)+\cdots, \tag{A2}$$

and then

$$\chi(r_{i+1})-2\chi(r_i)+\chi(r_{i-1})=h^2\chi''(r_i)+\frac{h^4}{12}\chi(4)(r_i)+\cdots. \tag{A3}$$

If h is small enough we obtain Eq. (A4) from Eq. (A3).

$$h^2\chi''(r_i)\approx\chi(r_{i+1})-2\chi(r_i)+\chi(r_{i-1}). \tag{A4}$$

Differentiating Eq. (A4) twice we obtain

$$h^2\chi^{(4)}(r_i)=\chi''(r_{i+1})-2\chi''(r_i)+\chi''(r_{i-1}). \tag{A5}$$

Substituting Eq. (A5) into Eq. (A3) leads to

$$\chi(r_{i+1})-2\chi(r_i)+\chi(r_{i-1})= \frac{5h^2}{6}\chi''(r_i)+\frac{h^2}{12}[\chi''(r_{i+1})+\chi''(r_{i-1})]+\cdots. \tag{A6}$$

The iterative result of $\chi(r_i)$ follows upon substituting Eq. (7) into Eq. (A6), which produces

$$\chi(r_{i+1})=\frac{\left[2+\frac{5h^2}{6}T(r_i)\right]\chi(r_i)-\left[1-\frac{h^2}{12}T(r_{i-1})\right]\chi(r_{i-1})}{1-\frac{h^2}{12}T(r_{i+1})}, \tag{A7}$$

where $T(r)=-(E^2-m_\eta^2)+\Pi_\eta+\frac{l(l+1)}{r^2}, r_i=\mathrm{i}h$. The initial conditions of Eq. (A7) are given by the following:

$$\chi(0)=0,\quad \chi(h)=h^{l+1},\quad T(0)\chi(0)=2\delta_{l1}. \tag{A8}$$

For large r, $r\to+\infty$, the formal result of Eq. (7) can be written as

$$\chi^B=f(B)\exp(kr), \tag{A9}$$

where $k^2=E^2-m_\eta^2$, B is the η binding energy. If B_1 is an energy in the neighborhood of the binding energy, we can expand the function $f(B_1)$ near B into a Taylor series

$$f(B_1)=f(B)+f^{'}(B)(B_1-B)+\cdots. \tag{A10}$$

If (B_1-B) is small enough, then we can gain $f'(B)$ as

$$f'(B)=\frac{f(B_1)-f(B)}{B_1-B}= \frac{\chi^{B_1}(r)\exp(-k_1r)-\chi^B(r)\exp(-kr)}{B_1-B}. \tag{A11}$$

If B is the eigenenergy, $f(B)$ must be vanish, and then we have

$$f(B)=f(B_1)+f^{'}(B_1)(B-B_1)=0, \tag{A12}$$

and then

$$B=B_1-\frac{f(B_1)}{f'(B)}. \tag{A13}$$

According to the method, we solve Eq. (5) by an iterative procedure until B_1 tends to B.

References

1 Haider Q, LIU L C. Phys. Lett. B, 1986, **172**: 257
2 LIU L C, Haider Q. Phys. Rev. C, 1986, **34**: 1845
3 Waas T, Weise W. Nucl. Phys. A, 1997, **625**: 287
4 Chiang H C, Oset E, LIU L C. Phys. Rev. C, 1991, **44**: 738
5 Tsushima K, LU D H, Thomas A W et al. Phys. Lett. B, 1998, **443**: 26
6 Inoue T, Oset E. Nucl. Phys. A, 2002, **710**: 354
7 Bhalerao R S, LIU L C. Phys. Rev. Lett., 1985, **54**: 865
8 Green A M, Wycech S. Phys. Rev. C, 1997, **55**: R2167
9 Green A M, Wycech S. Phys. Rev. C, 1999, **60**: 035208
10 Bennhold C, Tanabe H. Nucl. Phys. A, 1991, **530**: 625
11 Arima M, Shimizu K, Yazaki K. Nucl. Phys. A, 1992, **543**: 613
12 ZHONG Xian-Hui, LI Lei, NING Ping-Zhi et al. Phys. Rev. C, 2006, **73**: 015205
13 Haider Q, LIU L C. Phys. Rev. C, 2002, **66**: 045208
14 SONG Chun-Yan, ZHONG Xian-Hui, LI Lei et al. EPL, 2008, **81**: 42002
15 CAI Chong-Hai, LI Lei. HEP & NP, 2003, **27**: 1005 (in Chinese)
16 Sitenko A G, Tartakovskii V K. Lectures on the Theory of the Nucleus. Oxford: Pergamon Press, 1975. 85
17 Green A M, Wycech S. Phys. Rev. C, 2005, **71**: 014001
18 Arndt R A, Briscoe W J, Morrison T W et al. Phys. Rev. C, 2005, **72**: 045202