Relativistic studies of the decay constants of S wave and P wave mesons*

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Abstract Relativistic corrections are important in hadronic physics since even for the heavy hadrons there are sizable relativistic corrections. Therefore one should use a relativistic model to describe the higher excited states. This note summarizes our predictions for the decay constants of the S wave and P wave heavy mesons by means of the instantaneous relativistic Bethe Salpeter equation (Salpeter equation).

Key words decay constant, Bethe-Salpeter, relativistic, heavy meson

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1 Introduction

The numerical values of meson decay constants are important, and its investigation has become a hot topic in recent years [1–13]. The reason is that the decay constants provide a direct source of information on the Cabibbo-Kobayashi-Maskawa matrix elements. They test the interaction between quarks and antiquarks since the values of the decay constants are sensitive to the wave functions of the corresponding mesons. In this paper we show our estimations of the decay constants of S-wave and P-wave mesons in the framework of the relativistic (instantaneous) Bethe-Salpeter method (which is also called Salpeter method). By analyzing the parity and possible charge conjugation parity, we give the relativistic configurations of the wave functions with definite parity and possible charge conjugation parity. The corresponding full Salpeter equations are solved. Finally, we present a careful study of the decay constants for the heavy $^1S_0$ pseudoscalar and $^3S_1$ vector states; for the P waves we calculated the decay constants of the $^3P_0$, $^3P_1$ and $^1P_1$ mesons. We should note that compared with the heave S wave states, the relativistic corrections are very large for the heavy P wave states, and one can not use a nonrelativistic model to describe the heavy P wave hadrons, though the nonrelativistic model has been very successful for heavy S wave hadrons.

2 Wave functions and decay constants

2.1 The $^1S_0$ state

Since the pseudoscalar (or $^1S_0$ state) has the parity or possible charge conjugation of $J^P=0^-$ (or $J^{PC}=0^{-+}$ for equal mass systems), the general form for the relativistic Salpeter wave function with the same quantum number can be written as (in the center of mass system) [14]:

$$\varphi_{0^-}(\vec{q}) = M\left[\gamma_0\varphi_1(\vec{q})+\varphi_2(\vec{q})+\frac{\not{q}_\perp}{M}\varphi_3(\vec{q})+\frac{\gamma_0\not{q}_\perp}{M}\varphi_4(\vec{q})\right]\gamma_5\,, \tag{1}$$

where $q_\perp=(0,\vec{q})$ is the relative momentum and M is the mass of the corresponding meson. The Salpeter equation has constraining relations between the wave functions:

$$\varphi_3(\vec{q}) = \frac{\varphi_2(\vec{q})M(-\omega_1+\omega_2)}{m_2\omega_1+m_1\omega_2}\,,$$

$$\varphi_4(\vec{q}) = -\frac{\varphi_1(\vec{q})M(\omega_1+\omega_2)}{m_2\omega_1+m_1\omega_2}\,,$$

where $\omega_1=(m_1^2+\vec{q}^2)^{1/2}$ and $\omega_2=(m_2^2+\vec{q}^2)^{1/2}$. The normalization condition is

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$$\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}4\varphi_1(\vec{q})\varphi_2(\vec{q})M^2\left\{\frac{\omega_1+\omega_2}{m_1+m_2}+\frac{m_1+m_2}{\omega_1+\omega_2}+\frac{2\vec{q}^{\,2}(\omega_1 m_1+\omega_2 m_2)}{(\omega_1 m_2+\omega_2 m_1)^2}\right\}=2M\,.$$

The decay constant of a pseudoscalar meson is defined as

$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P\rangle \equiv \mathrm{i}F_P P_\mu, \tag{2}$$

which can be written in the language of the Salpeter wave functions as:

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P\rangle = \mathrm{i}\sqrt{N_\mathrm{c}}\int\mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\varphi_{^1S_0}(\vec{q})\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\right]. \tag{3}$$

Therefore, we have

$$F_\mathrm{P}=4\sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\varphi_1(\vec{q}). \tag{4}$$

2.2 The $^3S_1$ state

The relativistic wave function for the vector 1^- state (1^{--} for equal mass systems) can be written as [15]:

$$\begin{aligned}\varphi^\lambda_{1^-}(q_\perp) = {}& q_\perp\cdot\epsilon^\lambda_\perp\left[f_1(q_\perp)+\frac{\not{P}}{M}f_2(q_\perp)+\right.\\ &\left.\frac{\not{q}_\perp}{M}f_3(q_\perp)+\frac{\not{P}\not{q}_\perp}{M^2}f_4(q_\perp)\right]+M\not{\epsilon}^\lambda_\perp f_5(q_\perp)+\\ &\not{\epsilon}^\lambda_\perp\not{P}f_6(q_\perp)+(\not{q}_\perp\not{\epsilon}^\lambda_\perp-q_\perp\cdot\epsilon^\lambda_\perp)f_7(q_\perp)+\\ &\frac{1}{M}(\not{P}\not{\epsilon}^\lambda_\perp\not{q}_\perp-\not{P}q_\perp\cdot\epsilon^\lambda_\perp)f_8(q_\perp),\end{aligned} \tag{5}$$

where P and $\epsilon^\lambda_\perp$ are the momentum and polarization vector of the vector meson. Only the four wave functions f_3, f_4, f_5 and f_6 are independent [15]. The normalization condition is:

$$\begin{aligned}&\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{16\omega_1\omega_2}{3}\left\{3f_5f_6\frac{M^2}{m_1\omega_2+m_2\omega_1}+\right.\\ &\left.\frac{\omega_1\omega_2-m_1m_2+\vec{q}^2}{(m_1+m_2)(\omega_1+\omega_2)}\left[f_4f_5-f_3\left(f_4\frac{\vec{q}^2}{M^2}+f_6\right)\right]\right\}=2M.\end{aligned} \tag{6}$$

The decay constant F_V of vector meson can be defined as:

$$\langle 0|\bar{q}_1\gamma_\mu q_2|V,\epsilon\rangle \equiv F_V M\epsilon^\lambda_\mu\,. \tag{7}$$

In Ref. [15] only the leading order calculation for the decay constant

$$F_V=4\sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}f_5(\vec{q})$$

has been given, whereas the complete expression is given by

$$F_V=4\sqrt{N_\mathrm{c}}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\left[f_5(\vec{q})-\frac{\vec{q}^2}{3M^2}\right], \tag{8}$$

however, the numerical results are similar to those of Ref. [15].

2.3 The $^3P_0$ state

The general form of the relativistic Salpeter wave function of the $^3P_0$ state, $J^P=0^+$ (0^{++} for equal mass systems), can be written as [16]:

$$\varphi_{0^+}(q_\perp)=f_1(q_\perp)\not{q}+f_2(q_\perp)\frac{\not{P}\not{q}_\perp}{M}+f_3(q_\perp)M+f_4(q_\perp)\not{P}. \tag{9}$$

The normalization equation is given by (all the wave functions f_i are independent):

$$\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{16f_1f_2\omega_1\omega_2\vec{q}^2}{m_1\omega_2+m_2\omega_1}=2M. \tag{10}$$

The decay constant $F_{^3P_0}$ of the scalar $^3P_0$ meson is obtained as

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^3P_0\rangle\equiv F_{^3P_0}P_\mu,$$

$$F_{^3P_0}=\frac{4\sqrt{N_\mathrm{c}}}{M}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{f_2\vec{q}^2(\omega_2-\omega_1)}{(m_1\omega_2+m_2\omega_1)}. \tag{11}$$

2.4 The $^3P_1$ state

The relativistic wave function of the $^3P_1$ state, $J^P=1^+$ (1^{++} for equal mass systems), can be written as [16]:

$$\begin{aligned}\varphi_{1^+}(q_\perp) = {}&\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}P^\nu q^\alpha_\perp\epsilon^\beta\left[f_1M\gamma^\mu+f_2\not{P}\gamma^\mu+\right.\\ &\left.f_3\not{q}_\perp\gamma^\mu+\mathrm{i}f_4\varepsilon^{\mu\rho\sigma\delta}q_{\perp\rho}P_\sigma\gamma_\delta\gamma_5/M\right]/M^2.\end{aligned}$$

The normalization condition is:

$$\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{32f_1f_2\omega_1\omega_2(\omega_1\omega_2-m_1m_2+\vec{q}^2)}{3(m_1+m_2)(\omega_1+\omega_2)}=2M. \tag{12}$$

The decay constant $F_{^3P_1}$

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^3P_1,\epsilon\rangle\equiv F_{^3P_1}M\epsilon^\lambda_\mu,$$

$$F_{^3P_1}=\frac{8\sqrt{N_\mathrm{c}}}{3M}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{f_2(\omega_1\omega_2-m_1m_2+\vec{q}^2)}{(m_1+m_2)}. \tag{13}$$

2.5 The $^1P_1$ state

The relativistic wave function of the $^1P_1$ state, $J^P=1^+$ (1^{+-} for equal mass systems), can be written as [16]:

$$\begin{aligned}\varphi_{1^+}(q_\perp) = {}& q_\perp\cdot\epsilon^\lambda_\perp\left[f_1(q_\perp)+f_2(q_\perp)\frac{\not{P}}{M}+\right.\\ &\left.f_3(q_\perp)\frac{\not{q}_\perp}{M}+f_4(q_\perp)\frac{\not{P}\not{q}}{M^2}\right]\gamma_5\,.\end{aligned} \tag{14}$$

The normalization condition:

$$\int \frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{16 f_1 f_2 \omega_1 \omega_2 \vec{q}^2}{3(m_1\omega_2+m_2\omega_1)} = 2M\,. \tag{15}$$

The decay constant $F_{^1P_1}$

$$\langle 0|\bar{q}_1\gamma_\mu(1-\gamma_5)q_2|^1P_1,\epsilon\rangle \equiv F_{^1P_1} M\epsilon^\lambda_\mu\,,$$

$$F_{^1P_1} = \frac{4\sqrt{N_c}}{3M}\int\frac{\mathrm{d}\vec{q}}{(2\pi)^3}\frac{f_1(m_1-m_2)\vec{q}^2}{(\omega_1\omega_2+m_1m_2+\vec{q}^2)}. \tag{16}$$

2.6 Mixing of $^1P_1$ and $^3P_1$ states in heavy-light 1^+ mesons

For charmonium and bottomonium, because of the quantum number of charge conjugation, we can identify the states 1^{+-} ($^1P_1$) and 1^{++} ($^3P_1$), but we can not distinguish the two unequal mass system 1^+ states in S-L coupling, so we have to change our notation of S-L coupling to j-j coupling and use the notation of $P_1^{1/2}$, $P_1^{3/2}$. The transition equations are

$$|P_1^{1/2}\rangle = \sqrt{\frac{2}{3}}|^1P_1\rangle - \frac{1}{\sqrt{3}}|^3P_1\rangle,$$

$$|P_1^{3/2}\rangle = \frac{1}{\sqrt{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle. \tag{17}$$

3 Numerical results

The numerical predictions are shown in Tables 1—4. We want to point out that the relativistic corrections are very important, even for charmonium and bottomonium there are large relativistic corrections for the P wave states and the higher S wave excited states, and in these cases the nonrelativistic model cannot work.

Table 1. Decay constants of 0^- mesons (in MeV).

F_{B_c}	F_{B_s}	F_{B_d}	F_{B_u}	F_{η_c}	F_{D_s}	F_{D_d}	F_{D_u}
322	216	197	196	292	248	230	230

Table 2. Decay constants of 1^- mesons (in MeV).

	F_Υ	$F_{B_c^*}$	$F_{B_s^*}$	$F_{B_d^*}$	$F_{B_u^*}$	J/ψ	$F_{D_s^*}$	$F_{D_d^*}$	$F_{D_u^*}$
$1S$	471	395	256	226	224	432	353	321	319
$2S$	328	306	231	209	209	343	295	274	273

Table 3. Decay constants for P wave states (in MeV).

	$1\,^3P_0$	$2\,^3P_0$	$1\,^3P_1$	$2\,^3P_1$	$1\,^1P_1$	$2\,^1P_1$
$b\bar{b}$	0	0	129	−131	0	0
$c\bar{b}$	88	−85	160	−165	50	−49
$s\bar{b}$	140	−130	157	−156	76	−71
$d\bar{b}$	145	−129	150	−144	76	−70
$u\bar{b}$	145	−128	150	−143	76	−70
$c\bar{c}$	0	0	206	−207	0	0
$s\bar{c}$	112	−91	219	−204	62	−50
$d\bar{c}$	132	−102	212	−190	72	−56
$u\bar{c}$	133	−102	211	−189	72	−56

Table 4. Decay constants for 1^+ states (in MeV).

	$1\,P_1^{1/2}$	$2\,P_1^{1/2}$	$1\,P_1^{3/2}$	$2\,P_1^{3/2}$
$c\bar{b}$	160	−163	−52	55
$s\bar{b}$	172	−168	−29	32
$d\bar{b}$	166	−160	−25	26
$u\bar{b}$	166	−157	−25	25
$s\bar{c}$	215	−195	−76	77
$d\bar{c}$	215	−187	−64	64
$u\bar{c}$	214	−187	−63	63

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