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Rare semileptonic decays of heavy mesons in the heavy quark limit*

LI Hai-Bo(李海波)[2;1)] YANG Mao-Zhi(杨茂志)[1;2)]

[1] School of Physics, Nankai University, Tianjin 300071, China
[2] Institute of High Energy Physics, CAS, P.O.Box 918, Beijing 100049, China

Abstract We study the branching ratios of $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$ rare semileptonic decay processes, which are induced by decays of light quarks, the heavy quarks remain unchanged. The branching ratios of these decay processes are estimated in the heavy quark limit and with $SU(3)$ flavor symmetry. We find that the decay rates are very tiny in the framework of the Standard Model. We also estimate the sensitivities of the measurements of these rare decays at the future experiments, such as BES-III, super-B and LHC-b. Observations of these decays may shed some light on new physics beyond the standard model.

Key words weak decay, heavy quark, flavor symmetry

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1 Introduction

The weak decays of mesons containing a light and heavy quark (b or c) are a good place for studying the mechanism of weak and strong interactions. The decays induced by the heavy quark decay in the meson have been extensively studied in the literature. While in the rare decay processes of B and D mesons $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$, the heavy quark flavors (c or b) remain unchanged, the weak decays are managed by the light quark sectors. In the heavy quark limit [1–4] and applying the flavor $SU(3)$ symmetry of the light quarks, the matrix elements of the weak current can be constrained and the uncertainty can be estimated. In this work we study the rare decay processes $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$. Based on the spirit of heavy quark limit and applying the $SU(3)$ symmetry for the light quarks, the form factors describing the strong interaction in these decays can be estimated effectively. Therefore, these rare decays can be predicted with uncertainties of about 10% to 30% caused by the nonperturbative corrections of $1/m_c$ or $1/m_b$.

2 Decay amplitudes and widths

For the semileptonic decays $D^+ \to D^0 e^+ \nu$ and $D_S^+ \to D^0 e^+ \nu$, the decay amplitude is

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ij} \Big[\bar{u}(k_1) \gamma^\mu (1-\gamma_5) v(k_2) \times \langle D^0(p_2) | \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 | D^+_{(S)}(p_1) \rangle \Big], \qquad (1)$$

where $G_F = 1.16639 \times 10^{-5}$ GeV^{-2} is the Fermi constant, and V_{ij} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, the functions $\bar{u}(k_1)$ and $v(k_2)$ are Dirac spinors, which describe the productions of the neutrino with momentum k_1 and the anti-lepton with momentum k_2, respectively.

According to its Lorentz structure, the hadronic matrix element in Eq. (1) can be decomposed as

$$\langle D^0(p_2) | \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 | D^+_{(S)}(p_1) \rangle = f_+(q^2)(p_1+p_2)_\mu + f_-(q^2)(p_1-p_2)_\mu, \qquad (2)$$

where $f_\pm(q^2)$ are the form factors including all the

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1) E-mail: lihb@ihep.ac.cn

2) E-mail: yangmz@nankai.edu.cn

dynamics of the strong interaction. Considering the decomposition of the hadronic matrix element in Eq. (2), and neglecting the lepton mass, we get the amplitude as

$$\mathcal{A}=\frac{G_{\rm F}}{\sqrt{2}}V_{ij}f_+(q^2)(p_1+p_2)_\mu\bar{u}(k_1)\gamma^\mu(1-\gamma_5)v(k_2). \quad (3)$$

Then the decay width can be obtained as

$$\begin{aligned}\Gamma \ &= \ \frac{1}{192\pi^3m_1^3}G_{\rm F}^2|V_{ij}|^2\int{\rm d}q^2f_+^2(q^2)\times \\ &\quad \left[(m_1^2+m_2^2-q^2)^2-4m_1^2m_2^2\right]^{3/2}, \qquad (4)\end{aligned}$$

where m_1 and m_2 are the masses of the initial and final heavy hadron involved in these rare decays.

For the flavor-changing neutral current (FCNC) processes $\mathrm{D_S^+}\to\mathrm{D^+e^-e^+}$ and $\mathrm{B_S^0}\to\mathrm{B^0e^-e^+}$, they can be described by the $\Delta S=1$ effective Hamiltonian at the quark level at scales $\mu<m_{\rm c}$ [5]:

$$\begin{aligned}\mathcal{H}_{\rm eff}^{\Delta S=1} \ &= \ \frac{G_{\rm F}}{\sqrt{2}}\Big[\sum_{i=1}^{6,7V}(V_{\rm us}^*V_{\rm ud}Z_i(\mu)-V_{\rm ts}^*V_{\rm td}Y_i(\mu))Q_i(\mu)- \\ &\quad V_{\rm ts}^*V_{\rm td}Y_{7A}(m_{\rm W})Q_{7A}(m_{\rm W})\Big], \qquad (5)\end{aligned}$$

where the operators $Q_i(\mu)$'s are defined as

$$\begin{aligned}Q_1 \ &= \ (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A} \\ Q_2 \ &= \ (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \\ Q_3 \ &= \ (\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A} \\ Q_4 \ &= \ (\bar{s}_\alpha d_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A} \\ Q_5 \ &= \ (\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A} \\ Q_6 \ &= \ (\bar{s}_\alpha d_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A} \\ Q_7 \ &= \ (\bar{s}d)_{V-A}(\bar{e}e)_V \\ Q_{7A} \ &= \ (\bar{s}d)_{V-A}(\bar{e}e)_A. \qquad (6)\end{aligned}$$

Here the indexes α and β are color numbers, the summation $\sum$ runs over all the quark flavors which are active at the scale μ, and the $V\pm A$ denotes $\gamma_\mu(1\pm\gamma_5)$.

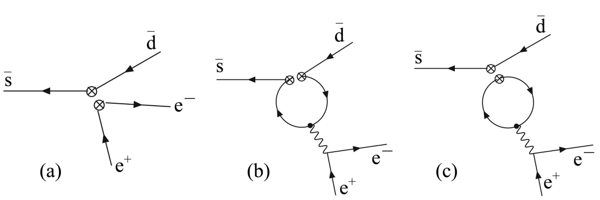

Fig. 1. Diagrams for the rare decays of $\mathrm{D_S^+}\to\mathrm{D^+e^+e^-}$ and $\mathrm{\bar{B}_S^0}\to\mathrm{\bar{B}^0e^+e^-}$ up to the one-loop level, where the cross circles denote the operator insertion. (a) the tree level diagram; (b) and (c) the one-loop level diagrams with two different ways of operator insertions.

At $\mu=1$ GeV with $\Lambda_{\overline{\rm MS}}^{(4)}=215$ MeV and the NDR scheme, the Wilson coefficients Z_i and Y_i's are [5] $Z_1=-0.409$, $Z_2=1.212$, $Z_3=0.008$, $Z_4=-0.022$, $Z_5=0.006$, $Z_6=-0.022$, $Z_{7V}=-0.015\alpha_{\rm QED}$, and $Y_1=Y_2=0$, $Y_3=0.025$, $Y_4=-0.048$, $Y_5=0.005$, $Y_6=-0.078$, $Y_{7V}=0.747\alpha_{\rm QED}$, $Y_{7A}=-0.700\alpha_{\rm QED}$, where $\alpha_{\rm QED}$ is the fine structure constant.

We calculate the FCNC processes $\mathrm{D_S^+}\to\mathrm{D^+e^+e^-}$ and $\mathrm{\bar{B}_S^0}\to\mathrm{B^0e^+e^-}$ up to the one-loop level based on the effective Hamiltonian. The diagrams we considered are depicted in Fig. 1. The amplitude for the FCNC processes is calculated to be

$$\begin{aligned}\mathcal{A} \ &= \ \left[a_1\bar{u}(k_1)\gamma^\mu v(k_2)+a_2\bar{u}(k_1)\gamma^\mu\gamma_5v(k_2)\right]\times \\ &\quad (p_1+p_2)_\mu f_+(q^2), \qquad (7)\end{aligned}$$

with

$$\begin{aligned}a_1 \ &= \ \frac{G_{\rm F}}{\sqrt{2}}V_{\rm us}^*V_{\rm ud}\Big\{Z_{7V}+\tau Y_{7V}+ \\ &\quad \frac{N_{\rm c}\alpha_{\rm QED}}{2\pi}\Big[\Big(\Big(\Big((Z_1+\frac{Z_2}{N_{\rm c}}\Big)Q_u+\Big(\frac{Z_3}{N_{\rm c}}+Z_4\Big)Q_{\rm d}+ \\ &\quad \tau\Big(\frac{Y_3}{N_{\rm c}}+Y_4\Big)Q_{\rm d}\Big)\Big(-\frac{2}{3}+G(0,q^2,\mu)\Big)+\Big(Z_3+ \\ &\quad \frac{Z_4}{N_{\rm c}}+Z_5+\frac{Z_6}{N_{\rm c}}+\tau\Big(Y_3+\frac{Y_4}{N_{\rm c}}+Y_5+\frac{Y_6}{N_{\rm c}}\Big)\Big)\times \\ &\quad \Big((Q_{\rm u}+Q_{\rm d})G(0,q^2,\mu)+Q_{\rm s}G(m_{\rm s}^2,q^2,\mu)\Big)+ \\ &\quad (Z_{7V}+\tau Y_{7V})\frac{Q_{\rm e}}{N_{\rm c}}G(0,q^2,\mu)\Big]\Big\}, \qquad (8)\end{aligned}$$

$$a_2 \ = \ (G_{\rm F}/\sqrt{2})V_{\rm us}^*V_{\rm ud}\tau Y_{7A}, \qquad (9)$$

where $N_{\rm c}$ is the color number, which is taken to be $N_{\rm c}=3$, $Q_{\rm u}$, $Q_{\rm d}$, $Q_{\rm s}$ and $Q_{\rm e}$ are the charges of the relevant quarks and electrons, $m_{\rm s}$ is the mass of the strange quark, and the function G is defined as

$$\begin{aligned}G(m^2,q^2,\mu) \ &= \ -\int_0^1{\rm d}x4x(1-x)\times \\ &\quad \ln\left[m^2-x(1-x)q^2-{\rm i}\epsilon\right]/\mu^2, \quad (10)\end{aligned}$$

which originates from the loop calculation. The parameter τ is defined to be

$$\tau=-V_{\rm ts}^*V_{\rm td}/V_{\rm us}^*V_{\rm ud}\,. \qquad (11)$$

With the amplitude in Eq. (7), the decay width can be obtained as

$$\begin{aligned}\Gamma \ &= \ \frac{1}{192\pi^3m_1^3}\int{\rm d}q^2f_+(q^2)^2(|a_1|^2+|a_2|^2)\times \\ &\quad \left[(m_1^2+m_2^2-q^2)^2-4m_1^2m_2^2\right]^{3/2}. \qquad (12)\end{aligned}$$

3 Estimation of the form factor $f_+(q^2)$

In the heavy quark limit $m_{c,b}\rightarrow\infty$, the energy released in the rare decays considered in this work is small compared to the mass of the heavy quark. The velocity of the heavy quark is almost unchanged in the decay processes. Therefore the light quark in the heavy meson only feels a static color field of the heavy quark. In the zero recoil limit $q^2=q^2_{\max}$, the momentum of the light quark also remains unchanged after the flavor transition of the light quark occurs. So the wave function of the light quarks in the initial and final mesons also remains the same if the flavor $SU(3)$ symmetry between the light quarks u, d and s holds [6]. Thus, in the heavy quark limit and with the flavor $SU(3)$ symmetry holding, the form factor $f_+(q^2)$ can be normalized to unity at the point of zero recoil $q^2=q^2_{\max}$,

$$f_+(q^2_{\max})=1. \tag{13}$$

The spirit of the above reasoning is the same as that in HQET [4], but with a little difference. In the theory of heavy quark symmetry, one considers the heavy quark transitions. Here the heavy quark remains unchanged in the processes we consider. But the same normalization result for the form factor can be obtained. This is an extension of the idea of heavy quark limit.

However, with the $SU(3)$ flavor symmetry broken, i.e., the mass of s quark m_s being larger than that of u and d quarks, the value of the form factors must deviates from unity even in the heavy quark limit $m_Q\rightarrow\infty$. The larger the s quark mass, the larger the deviation. Including this $SU(3)$ breaking effect, Eq. (13) is extended to be

$$f_+(q^2_{\max})=1+\lambda_{SU(3)}, \tag{14}$$

where the parameter $\lambda_{SU(3)}$ describes the correction due to the $SU(3)$ breaking effect in the heavy quark limit $m_Q\rightarrow\infty$.

For the real heavy quark mass $m_Q=m_b$ or m_c, there should be further nonperturbative corrections to the form factor $f_+(q^2_{\max})$, because the final meson will move with the momentum released in the light quark flavor transition process in this case. The larger the momentum of the final meson, the larger the difference of the wave functions of the light quarks in the initial and final mesons, therefore the larger the deviation of the form factor from unity. This correction to the form factor can be expanded with the parameter Λ_{QCD}/m_Q. Including the corrections of the heavy quark expansion, Eq. (14) is extended to

$$f_+(q^2_{\max})=1+\lambda_{SU(3)}+f_1\frac{\Lambda_{QCD}}{m_Q}+\cdots, \tag{15}$$

where $Q=$ b or c, and f_1 is the first order $1/m_Q$ correction to the form factor. It is usually assumed that the expansion parameter $f_1<1$, otherwise the heavy quark expansion would be meaningless. The $1/m_Q$ correction shall be treated as the uncertainty of the form factor in this work. Whether the heavy quark expansion for the form factor works, should be tested by future experiments.

The parameters $\lambda_{SU(3)}$ and f_1 are not completely independent. With the exact $SU(3)$ symmetry, i.e., $m_s\rightarrow 0$ (here we neglect the masses of the u and d quarks), the energy release in the rare decay process is zero. So both the initial and final mesons are static in the center-of-mass-frame. In this case, the wave functions of the light quarks in the initial and final mesons are completely overlapping, and then the value of the form factors should be exactly 1. Thus, both the parameters $\lambda_{SU(3)}$ and f_1 become zero. Therefore , both the parameters $\lambda_{SU(3)}$ and f_1 are the result of $SU(3)$ breaking. The parameter $\lambda_{SU(3)}$ describes the $SU(3)$ breaking correction in the limit $m_Q\rightarrow\infty$, while f_1 denotes the correction which can be expanded by the inverse powers of the heavy quark mass.

It is difficult to estimate the parameter $\lambda_{SU(3)}$ directly. To estimate this parameter, one can consider the mass differences of the D_s and D system, and of the B_s and B system, respectively. The relative mass differences are $\frac{m_{D_s}-m_D}{m_{D_s}+m_D}=2.6\%$, and $\frac{m_{B_s}-m_{B^+}}{m_{B_s}+m_{B^+}}=0.8\%$. The mass differences of the D_s-D and B_s-B meson pairs are the result of the $SU(3)$ breaking effect. The relative mass-difference, normalized by the sum of these two mesons, can be viewed as the order of the $SU(3)$ breaking effect in the D and B systems, respectively. Here we estimate the corrections due to the parameter $\lambda_{SU(3)}$ to be about $\pm 2.6\%$ for $D_s\rightarrow D$ transitions and 0.8% for $B_s\rightarrow B$ transitions. This effect is also treated as uncertainty of the form factor.

The momentum transfer squared q^2 is in the range range $0<q^2<(m_{H1}-m_{H2})^2$, where m_{H1} and m_{H2} are the masses of the initial and final mesons containing the heavy quark, respectively. The smaller the final meson mass, the larger the possible range of q^2. For the rare decay processes considered in this work, the mass of the final meson is close to that of the initial meson, so the ranges of q^2 are all very small. For example, the relative value of $\frac{q^2_{\max}-q^2_{\min}}{m^2_{D_s}}$ is only 0.28% for $D_s^+\to D^0$ transition process, the relative range being even smaller for the other processes. Therefore we will neglect the q^2 dependence of the form factor by taking $f(q^2)\approx f(q^2_{\max})$ in the numerical estimation.

4 Numerical results and discussion

In the numerical calculations we use the following values for the CKM matrix elements in the Wolfenstein parameterization [7]:

$$\lambda=0.2272,\ A=0.818,\ \bar{\rho}=0.221,\ \bar{\eta}=0.340. \quad (16)$$

Taking the mass of the strange quark m_s = 100 MeV, and applying the heavy quark limit and $SU(3)$ flavor symmetry, we get the following branching fractions:

$$\begin{aligned}
\mathcal{B}(D^+\to D^0e^+\nu) &= 2.8^{+0.8}_{-0.7}\times10^{-13},\\
\mathcal{B}(D_S^+\to D^0e^+\nu) &= 3.1^{+0.9+0.2}_{-0.8-0.2}\times10^{-8},\\
\mathcal{B}(B_S^0\to B^+e^-\bar{\nu}) &= 4.5^{+0.4+0.1}_{-0.4-0.1}\times10^{-8},\\
\mathcal{B}(D_S^+\to D^+e^+e^-) &= 3.7^{+1.0+0.2}_{-0.9-0.2}\times10^{-17},\\
\mathcal{B}(B_S^0\to B^0e^+e^-) &= 6.5^{+0.6+0.1}_{-0.5-0.1}\times10^{-17}, \quad (17)
\end{aligned}$$

where the first uncertainties are caused by the $1/m_Q$ correction, which are estimated by varying the value of the form factor in the heavy quark limit with $\pm\Lambda_{QCD}/m_Q$, and the second ones caused by the $SU(3)$ breaking effect described by the parameter $\lambda_{SU(3)}$. Here we take $m_c=1.5$ GeV, $m_b=4.8$ GeV, and $\Lambda_{QCD}=200$ MeV.

The mass of the strange quark is very close to the mass differences of the initial and final mesons in the decay processes involving a s quark, $D_S^+\to D^0e^+\nu$, $D_S^+\to D^+e^+e^-$, $B_S^0\to B^+e^-\bar{\nu}$, $B_S^0\to B^0e^+e^-$, which is about 100 MeV. It is interesting to study the role of m_s in these decay processes by slightly varying the mass of the strange quark. The results are given in Table 1. The decay rates are only slightly changed with m_s varying in the range 80 MeV $<m_s<$ 140 MeV.

Within the SM framework, we find that the branching fractions for these rare decays are tiny. However, in the coming experiments at BES-III[8], LHC-b [9] and super-B factory [10], it would be interesting to search for these semileptonic decays. Especially, the decays of $D_S^+\to D^0e^+\nu$ and $B_S^0\to B^+e^-\bar{\nu}$ may be reached at the super-B factory.

Table 1. The decay rates of the rare processes involving a strange quark. The second uncertainty is estimated from the $SU(3)$ breaking effect due to the mass of the strange quark.

	$m_s=80$ MeV	$m_s=100$ MeV	$m_s=140$ MeV
$D_S^+\to D^0e^+\nu$	$3.1^{+0.9+0.2}_{-0.8-0.2}\times10^{-8}$	$3.1^{+0.9+0.2}_{-0.8-0.2}\times10^{-8}$	$3.1^{+0.9+0.2}_{-0.8-0.2}\times10^{-8}$
$B_S^0\to B^+e^-\bar{\nu}$	$4.5^{+0.4+0.1}_{-0.4-0.1}\times10^{-8}$	$4.5^{+0.4+0.1}_{-0.4-0.1}\times10^{-8}$	$4.5^{+0.4+0.1}_{-0.4-0.1}\times10^{-8}$
$D_S^+\to D^+e^+e^-$	$3.7^{+1.0+0.2}_{-0.9-0.2}\times10^{-17}$	$3.7^{+1.0+0.2}_{-0.9-0.2}\times10^{-17}$	$3.8^{+1.0+0.2}_{-0.9-0.2}\times10^{-17}$
$B_S^0\to B^0e^+e^-$	$6.5^{+0.6+0.1}_{-0.5-0.1}\times10^{-17}$	$6.5^{+0.6+0.1}_{-0.5-0.1}\times10^{-17}$	$6.6^{+0.6+0.1}_{-0.5-0.1}\times10^{-17}$

Since the electron is very soft, one cannot reconstruct at e^+e^- colliders both the electron and neutrino in the experiment near the charm meson threshold. If searching at BES-III for the decay $D^+\to D^0e^+\nu$ on the $\psi(3770)$ peak, the charged D mesons will be produced in pairs, $e^+e^-\to\psi(3770)\to D^+D^-$. Thus, the following six tag modes, $D^-\to K^+\pi^-\pi^-$, $K^+\pi^-\pi^-\pi^0$, $K_S\pi^-$, $K_S\pi^-\pi^-\pi^+$, $K_S\pi^-\pi^0$ and $K^+K^-\pi^-$, can be used to fully reconstruct one of the charged D mesons. The summed branching fractions of the six tag modes are about 28% of all the charged D decays [7]. The tag efficiency for the charged D mesons are about 20%, which means that 20% of all the D^+D^- pairs can be tagged [8]. For this case, in order to detect the decay $D^+\to D^0e^+\nu$, one can reconstruct the neutral D meson decay by using 46% of all of the D^0 decays modes in the tagged charged D sample [8]. If we see any event of the production of the neutral D mesons against the charged D mesons, it indicates the observation of the rare semileptonic decay. For the decay $D_S^+\to D^0e^+\nu$, the same method can be applied by using the data collected at the center of mass $E_{CM}=4170$ MeV. With 20 fb^{-1} data on the $\psi(3770)$ peak, the sensitivity of the measurement of $D^+\to D^0e^+\nu$ can be 10^{-6} at the BES-III experiment, while, for the measurement of $D_S^+\to D^0e^+\nu$, it can reach the 10^{-5} level with 20 fb^{-1} data running at $E_{CM}=4170$ MeV. These estimations are listed in Table 2.

These semi-leptonic decays can also be searched in the B and super-B factories by using data at the $\Upsilon(4S)$ peak. One can reconstruct the following decay

Table 2. Experimental sensitivities at BES-III, B factory, Super-B and LHC-b for the rare decays. We assume the integrated luminosities are 20 fb^{-1} (BES-III at $\psi(3770)$ peak and 4170 MeV), 1 ab^{-1} and 50 ab^{-1} at B factory and Super-B (at $\Upsilon(4S)$ peak), 10 fb^{-1} at LHC-b, respectively.

decays	BES-III ($\times10^{-6}$)	B factory ($\times10^{-8}$)	Super-B ($\times10^{-10}$)	LHC-b ($\times10^{-9}$)
$D^+\to D^0e^+\nu$	1.0	1.1	2.3	3.8
$D_S^+\to D^0e^+\nu$	5.0	1.1	2.3	3.8
$D_S^+\to D^+e^+e^-$	11.5	2.0	4.6	5.0

chain to search for the rare decays $D^+\to D^0e^+\nu$:

$$D^{*+}\to D^+\pi^0_{\rm soft},\ D^+\to D^0e^+_{\rm soft}\nu, \quad (18)$$

where the D^{*+} is boosted, and both π^0 and electron could have momenta with a few hundred MeV, which can be detected and reconstructed in the detector. Since the missing neutrino has a very low momentum, one can partially reconstruct the decay of D^+, and looking at the mass difference $\Delta m=m((D^0e^+)\pi^0_{\rm soft})-m(D^0e^+)$. The signal events should peak around the mass difference of $m_{D^{*+}}-m_{D^+}=140$ MeV on the Δm distribution. In the mass difference, the uncertainty of the reconstruction of (D^0e^+) can cancel out. The resolution on the Δm will be dominated by the detection of the soft π^0. This is a powerful variable to separate the background from the signal events. For the decay $D_S^+\to D^0e^+\nu$ one can use the reaction:

$$D_S^{*+}\to D_S^+\gamma_{\rm soft},\ D_S^+\to D^0e^+_{\rm soft}\nu \quad (19)$$

to extract the rare decay signal by looking at the mass difference $\Delta m = m((D^0e^+)\gamma_{\rm soft})-m(D^0e^+)$. With 1 ab^{-1} and 50 ab^{-1} luminosity at B factories and super-B, the sensitivities could be 10^{-8} and 10^{-10} respectively. In Table 2, the sensitiveities of the measurements of the rare D^+ and D_S^+ decays are summarized for different experiments.

At the super-B factory, the data taken at $\Upsilon(5S)$ can be used to search for the rare decays $B_S^0\to B^+e^-\bar{\nu}$ and $B_S^0\to B^0e^-e^+$. The cross section of the $\Upsilon(5S)$ production at e^+e^- collisions is $\sigma(e^+e^-\to\Upsilon(5S))=0.301\pm0.002\pm0.039$ nb [11]. Unlike the $\Upsilon(4S)$ state, $\Upsilon(5S)$ is heavy enough to decay into several B meson states, in which the vector-vector ($B_S^*\bar{B}_S^*$) and vector-pseudoscalar ($B_S^*\bar{B}_S+B_S\bar{B}_S^*$) combinations are dominant [12]. About 30% of the $\Upsilon(5S)$ decays into B_S final states [10]. With the 30 ab^{-1} data at the $\Upsilon(5S)$ peak at super-B, the sensitivity of the measurements of the rare B_S decays can be 10^{-9} by assuming a 30% efficiency for the B_S reconstruction.

5 Summary

In summary, we calculated the decay rates of the rare D^+, D_S^+ and B_S decays, in which only the light quarks decay weakly, while the heavy flavors remain unchanged. Applying the heavy quark limit and the $SU(3)$ flavor symmetry, the form factors describing the strong interaction in these decays can be obtained. Considering $SU(3)$ symmetry breaking and the heavy quark expansion, the uncertainty for the form factors can be estimated. Therefore, these rare decays can be predicted with the uncertainty estimated by considering $SU(3)$ symmetry breaking and the heavy quark expansion. We also estimated the sensitivities of the measurements of these rare decays in future experiments, such as BES-III, super-B and LHC-b. Especially, the decays of $D_S^+\to D^0e^+\nu$ and $B_S^0\to B^+e^-\bar{\nu}$ may be reached at the super-B factory. Observations of these decays will be used to test the heavy quark expansion in the rare simleptonic decays. Furthermore, any indication of deviations from the SM prediction may also shed light on the search for New Physics.

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