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Control of entanglement sudden death induced by Dzyaloshinskii-Moriya interaction*

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Abstract We investigate the entanglement dynamics of a system composed of two non-interacting qubits, A and B. A third qubit, C, only has the Dzyaloshinskii-Moriya (DM) spin-orbit interaction with qubit B. We find that the DM interaction can induce the entanglement sudden death (ESD) of the system qubits A and B, and properly mixing the initial state of the system and adjusting the state of qubit C are two effective methods of controlling ESD.

Key words entanglement sudden death, Dzyaloshinskii-Moriya interaction, control

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1 Introduction

Entanglement is a key resource in quantum information processing [1]; for example, quantum teleportation [2, 3], quantum dense coding [4] and quantum cryptography [5, 6]. Entanglement dynamics is an emerging field. One interesting phenomenon, the so-called entanglement sudden death (ESD), was reported by Yu and Eberly [7]. The special property of ESD is that entanglement disappears nonsmoothly for a finite time, out of the expectation that it should disappear smoothly. ESD also occurrs in the two qubits with Ising type interaction [8], the non-degenerate two photon Tavis-Cummings model [9], etc. Moreover, ESD has been observed in an optical system [10].

Recently, the spin-chain has been an active subject in terms of entanglement [11–18], quantum phase transition [19] and quantum simulation [20]. Moreover, the effect of the spin-orbit Dzyaloshinskii-Moriya (DM) interaction [21, 22] on the quantum information processing (QIP) of the spin-chain has generated wide interest [23], as it provides one with a new parameter to control entanglement. The anisotropic and antisymmetric DM interaction was introduced about fifty years ago to explain the weak ferromagnetism of antiferromagnetic crystals.

However, these research focuses on the thermal entanglement of the spin-chain, and there are few papers studying the effect of DM interaction on entanglement dynamics. That is the aim of this paper. Here, similar to Ref. [24], we consider the entanglement dynamics of a system composed of two non-interacting qubits A and B. A third controller qubit C is introduced, which only has a DM interaction with the qubit B. We find that the DM interaction can induce the ESD of the system qubits A and B, and properly mixing the initial state of the system and adjusting the state of qubit C are two effective ways to control ESD.

2 Measure of entanglement

For a bipartite state, there are some equivalent measures of the degree of entanglement, such as concurrence, entanglement entropy, negativity and tangle. In this paper, we adopt the Wootters concurrence [25]. For two qubits, the concurrence can be calculated by

$$C(\rho)=\max(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}), \qquad (1)$$

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where λ_i $(i=1,2,3,4)$ are the eigenvalues in decreasing order of the matrix

$$R=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y). \tag{2}$$

Here, ρ^* are the complex conjugation of ρ in the standard basis and σ_y is the Pauli matrix,

$$\sigma_y=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}. \tag{3}$$

The concurrence is monotone for the bipartite entanglement, $C=0$ corresponds to the separable state and $C=1$ to maximal entanglement one. For the density matrix

$$\rho=\begin{pmatrix}a & 0 & 0 & w\\ 0 & b & z & 0\\ 0 & z^* & c & 0\\ w^* & 0 & 0 & d\end{pmatrix}, \tag{4}$$

the concurrence can be easily obtained [26],

$$C(\rho)=2\max\{0,|z|-\sqrt{ad},|w|-\sqrt{bc}\}. \tag{5}$$

3 Entanglement dynamics of the system

Similar to Ref. [24], we consider the entanglement dynamics of the system composed of two non-interacting qubits A and B. A third controller qubit C is introduced, which only has a DM interaction with qubit B. The Hamiltonian of the whole system is

$$H=H^{\mathrm{AB}}+H^{\mathrm{BC}}_{\mathrm{DM}}, \tag{6}$$

with

$$H^{\mathrm{AB}}=0,\quad H^{\mathrm{BC}}_{\mathrm{DM}}=\vec{D}\cdot(\vec{\sigma}_{\mathrm{B}}\times\vec{\sigma}_{\mathrm{C}}). \tag{7}$$

Choosing $\vec{D}=D\vec{x}$, $H^{\mathrm{BC}}_{\mathrm{DM}}$ reduces to

$$H^{\mathrm{BC}}_{\mathrm{DM}}=D(\sigma^y_{\mathrm{B}}\sigma^z_{\mathrm{C}}-\sigma^z_{\mathrm{B}}\sigma^y_{\mathrm{C}}). \tag{8}$$

Here, D is dimensionless.

Without the loss of generality, we define $|g\rangle$ ($|e\rangle$) as the ground (excited) state of a qubit. We choose the initial state of the whole system as $\rho_0=\rho^{\mathrm{AB}}(0)\bigotimes\rho^{\mathrm{C}}(0)$ with

$$\begin{aligned}\rho^{\mathrm{AB}}(0) &= r|\varphi^{\mathrm{AB}}\rangle\langle\varphi^{\mathrm{AB}}|+(1-r)\rho^{\mathrm{AB}}_1,\\ \rho^{\mathrm{C}}(0) &= |\varphi^{\mathrm{C}}\rangle\langle\varphi^{\mathrm{C}}|.\end{aligned} \tag{9}$$

Here,

$$\begin{aligned}|\varphi^{\mathrm{AB}}\rangle &= \cos(\alpha)|eg\rangle+\sin(\alpha)|ge\rangle,\\ |\varphi^{\mathrm{C}}\rangle &= \cos(\beta)|e\rangle+\sin(\beta)|g\rangle.\end{aligned} \tag{10}$$

The density matrix of the whole system at arbitrary time t is

$$\rho(t)=U(t)\rho_0U(t)^{\dagger}, \tag{11}$$

with the evolution operator $U(t)=\exp(-\mathrm{i}Ht)$ $(\hbar=1)$.

After some straightforward calculations, we obtain the unitary operator $U(t)$,

$$U(t)=\begin{pmatrix}M & 0\\ 0 & M\end{pmatrix}. \tag{12}$$

Here, M is the (4×4) square matrix with the elements

$$\begin{aligned}U_{11} &= U_{22}=U_{33}=U_{44}=\cos(Dt)^2,\\ U_{14} &= U_{23}=U_{32}=U_{41}=\sin(Dt)^2,\\ U_{12} &= U_{24}=U_{31}=U_{43}=\frac{1}{2}\sin(2Dt),\\ U_{13} &= U_{21}=U_{34}=U_{42}=-U_{12}.\end{aligned} \tag{13}$$

The reduced density matrix of qubits A and B is

$$\rho^{\mathrm{AB}}=\mathrm{Tr}_{\mathrm{C}}[\rho(t)], \tag{14}$$

where Tr_{C} denotes tracing over the state of qubit C.

For the general parameters, the analytical solution of the system concurrence is quite complicated and its

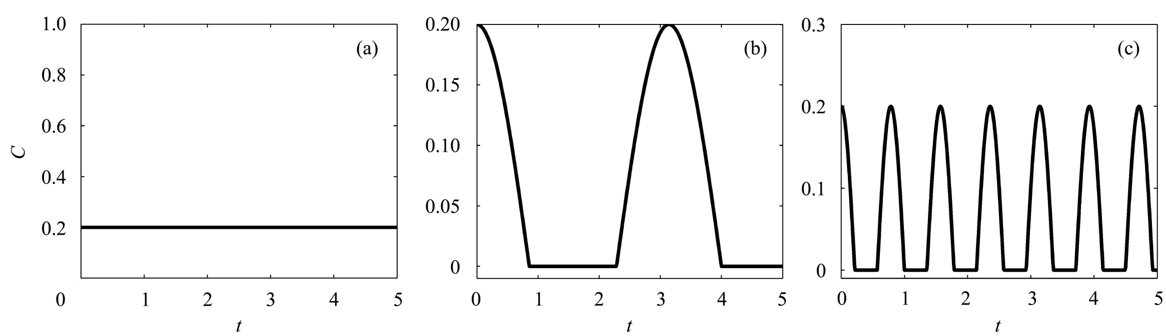

Fig. 1. The evolution of the concurrence of the system ρ^{AB} with the initial state $\rho^{\mathrm{AB}}_1=|ee\rangle\langle ee|$. (a), (b) and (c) correspond to $D=0,\ 0.5,\ 1.5$, respectively. The other parameters are $\alpha=\dfrac{\pi}{4}$, $\beta=\dfrac{\pi}{2}$, $r=0.2$.

form is not particularly instructive, hence it will not be reproduced here. We perform numerical simulation to get knowledge of the effect of the DM interaction and the initial state of the controller qubit on the entanglement dynamics of the system. Our main results are summarized in Figs. 1–4.

Figs. 1 and 2 are the evolutions of the concurrence of the system qubits ρ^{AB} with the initial state $\rho_1^{\mathrm{AB}} = |ee\rangle\langle ee|$. Fig. 1 (a)–(c) correspond to $D=0$, 0.5 and 1.5, respectively. The concurrence of the system is a constant when there is no DM interaction between qubits B and C, while ESD appears when there is DM interaction. And the frequency of the ESD increases with the strength of DM interaction. Fig. 2(a) and (b) correspond to $r=0.2$ and 1, respectively. It is easy to see that there is (no) ESD for $r=(1)0.2$.

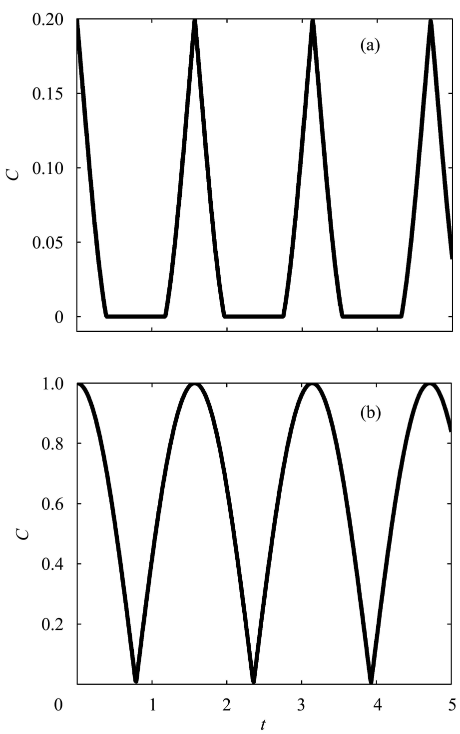

Fig. 2. The evolution of the concurrence of the system qubits ρ^{AB} with the initial state $\rho_1^{\mathrm{AB}} = |ee\rangle\langle ee|$. (a) and (b) correspond to $r=0.2$ and 1.0, respectively. The other parameters are $\alpha=\dfrac{\pi}{4}$, $\beta=\dfrac{\pi}{4}$ and $D=1$.

The results in Figs. 1 and 2 can be understood as follows. The DM interaction between qubits B and C can transfer the initial entanglement of the system qubits A and B to the other subsystems, which induce the ESD of the system. However, ESD disappears as a big enough initial entanglement in the system qubits can insist the entanglement transferring effect of DM interaction. The periods of the nonzero matrix elements of the unitary operator $U(t)$ are π/D, therefore the frequency of the ESD increases with the DM interaction strength D.

There is no doubt that ESD is very interesting. However, the abrupt disappearance of entanglement may be bad news, as most of the quantum information process is based on entanglement. For example, in the Bennent's quantum teleportation scheme, the quantum channel between Alice and Bob is one entanglement pair. When the entanglement between Alice and Bob disappears abruptly, the quantum channel between them is also cut off, which may spoil the quantum information transferring from Alice to Bob. Therefore, from the point of view that the entanglement is the resource, it should be better to find methods controlling the occurrence of ESD.

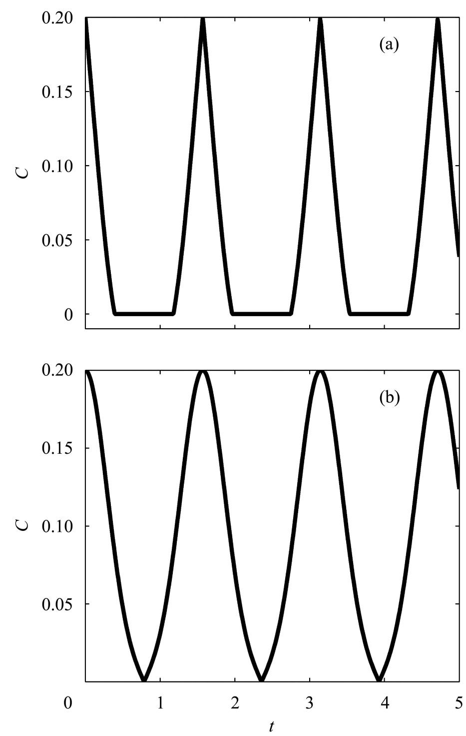

Fig. 3. The evolution of the concurrence of the system qubits ρ^{AB}. (a) and (b) correspond to the initial state $\rho_1^{\mathrm{AB}} = |ee\rangle\langle ee|$, $|eg\rangle\langle eg|$, respectively. The other parameters are $\alpha=\dfrac{\pi}{4}$, $\beta=\dfrac{\pi}{4}$, $D=1$ and $r=0.2$.

Figure 3 shows the evolution of the system entanglements with two different kinds of initial states $\rho_1^{\mathrm{AB}} = |ee\rangle\langle ee|$ and $|eg\rangle\langle eg|$. When $\rho_1^{\mathrm{AB}} = |ee\rangle\langle ee|$, there is ESD. However, when $\rho_1^{\mathrm{AB}} = |eg\rangle\langle eg|$, ESD disappears. The reason is as follows. One part of the

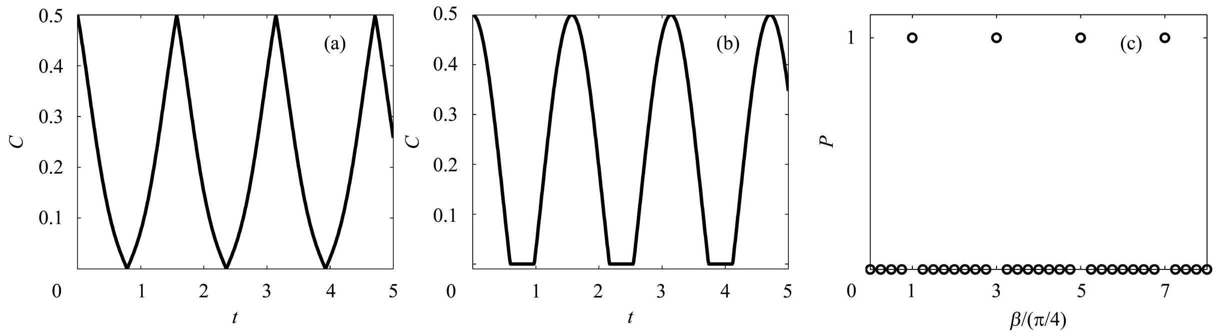

Fig. 4. (a) and (b) are the evolutions of the concurrence of the system with the initial states $\rho_1^{AB}=|ee\rangle\langle ee|$, corresponding to $\beta=\frac{3}{4}\pi,\pi$, respectively. (c) the relationship between ESD and the state of the controlling qubit C. P=0 (1) denotes ESD (no ESD). The other parameters are $\alpha=\frac{\pi}{4}$, $r=0.5$ and $D=1$.

system initial state is $|\varphi^{AB}\rangle=\cos(\alpha)|eg\rangle+\sin(\alpha)|ge\rangle$. Compared with the state $\rho_1^{AB}=|ee\rangle\langle ee|$, the state $\rho_1^{AB}=|eg\rangle\langle eg|$ is much more similar than $|\varphi^{AB}\rangle$. So, it is stronger to insist the entanglement transferring effect and prevent the ESD. Further study shows for the initial state $\rho_1^{AB}=|ee\rangle\langle ee|$, when $r\leqslant 0.3$, there is ESD. For $\rho_1^{AB}=|eg\rangle\langle eg|$, ESD occurs only when $r\leqslant 0.1$. So we can control the ESD by properly mixing the initial state of the system.

Figure 4(a) and (b) are the evolutions of the concurrence of the system with the initial states $\rho_1^{AB}=|ee\rangle\langle ee|$. (a) and (b) correspond to $\beta=\frac{3}{4}\pi$, π, i.e., the qubit C in different states. When $\beta=\pi$, there is ESD in the system. However when $\beta=\frac{3}{4}\pi$, ESD disappears. We further study the relationship between ESD and the state of the controlling qubit C, and find when $\beta=(2k+1)\frac{\pi}{4}$, $k=0,1,2...$, there is no ESD in the system [Fig. 4(c)]. Therefore, we can also control the ESD by adjusting the state of the qubit C.

4 Conclusions

Based on the concurrence, we consider the entanglement dynamics of the system composed of two non-interacting qubits A and B. There is a third controlling qubit C, which only has a DM interaction with qubit B. We find that the DM interaction can induce the ESD of the system qubits. Properly mixing the initial state of the system and adjusting the state of the qubit C are two effective methods of controlling ESD.

References

1 Nielsen M A, CHUANG I L. Quantum Computation and Quantum Information. Cambridge: Cambridge University Press, 2000

2 Bennett C H, Brassard G, Crepeau C et al. Phys. Rev. Lett., 1993, **70**: 1895

3 Bouwmeester D, PAN J W, Mattle K et al. Nature, 1997, **390**: 575

4 Bennett C H, Wiesner S J. Phys. Rev. Lett., 1992, **69**: 2881

5 Ekert A K. Phys. Rev. Lett., 1991, **67**: 661

6 XU F X, CHEN W, WANG S et al. Chin. Sci. Bull., 2009, **54**(17): 2991–2997

7 YU T, Eberly J H. Phys. Rev. Lett., 2004, **93**: 140404

8 CUI H T, LI K, YI X X. Phys. Lett. A, 2007, **365**: 44

9 ZHANG G F, CHEN Z Y. Opt. Commun., 2007, **275**: 274

10 Almeida M P, de Melo F, Hor-Meyll M et al. Science, 2007, **316**: 579

11 ZHANG J F, LONG G L, ZHANG W et al. Phys. Rev. A, 2005, **72**: 012331

12 ZHANG Y, LONG G L. Commun. Theor. Phys., 2007, **48**(2): 249–254

13 LIU D, ZHANG Y, LONG G L. Prog. Nat. Sci., 2007, **17**(10): 1147–1151

14 HU M L, TIAN D P. Chin. Phys. C, 2008, **32**(4): 303–307

15 QIN M, TIAN D P. Chin. Phys. C, 2009, **33**(4): 249–251

16 MA X S, ZHANG J Y, CONG H S et al. Sci. Chin. Ser. G, 2008, **51**(12): 1897–1904

17 WANG F, JIA H H, ZHANG H L et al. Sci. Chin. Ser. G, 2009, **52**(12): 1919–1924

18 WANG X G. Phys. Lett. A, 2001, **281**: 101

19 Iwashita T, Uragami K, Goto K et al. Physica B, 2003, **329–333**: 1284–1285; LOU P, WU W C, CHANG M C. Phys. Rev. B, 2004, **70**: 064405

20 LIU W Z, ZHANG J F, LONG G L. Chin. Sci. Bull., 2009, **54**(22): 4262–4265; LIU W Z, ZHANG J F, DENG Z W, LONG G L. Sci. Chin. Ser. G, 2008, **51**(8) 1089–1096

21 Dzyaloshinskii I. J. Phys. Chem. Solids, 1958, **4**: 241

22 Moriya T. Phys. Rev. Lett., 1960, **4**: 228

23 ZHANG G F. J. Phys.: Cond. Matt., 2007, **19**: 456205; ZHANG G F. Phys. Rev. A, 2007, **75**: 034304; ZHANG G F. Phys. Scr, 2009, **79**: 015001; ZHOU Y, ZHANG G F, LI S S et al. Eur. Phys. Lett., 2009, **86**: 50004

24 Jodarn T F, Shaji A, Sudarshan E C G. Phys. Rev. A, 2007, **76**: 022102

25 Wooters W K. Phys. Rev. Lett., 1998, **80**: 2245

26 O'connor K M, Wooters W K. Phys. Rev. A, 2001, **63**: 052302