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A new method to determine the projected coordinate origin of a cone-beam CT system using elliptical projection*

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Abstract In order to determine the projected coordinate origin in the cone-beam CT scanning system with respect to the Feldkamp-Davis-Kress (FDK) algorithm, we propose a simple yet feasible method to accurately measure the projected coordinate origin. This method was established on the basis of the theory that the projection of a spherical object in the cone-beam field is an ellipse. We first utilized image processing and the least square estimation method to get each major axis of the elliptical Digital Radiography (DR) projections of a group of spherical objects. Then we determined the intersection point of the group of major axis by solving an over-determined equation set that was composed by the major axis equations of all the elliptical projections. Based on the experimental results, this new method was proved to be easy to implement in practical scanning systems with high accuracy and anti-noise capability.

Key words cone-beam CT, projected coordinate origin, elliptical projection, geometrical calibration

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1 Introduction

With the rapid development of computer technology and the wide use of flat panel detectors, three-dimensional (3-D) CT has recently generated intense interest from both scientific studies and practical applications in the non-destructive testing (NDT) field [1, 2]. Among various 3-D reconstruction algorithms, the Feldkamp-Davis-Kress (FDK) algorithm [3, 4] has been demonstrated to be the most effective algorithm in practical engineering applications from the point of view of utilizing computing efficiency and complexity of practical realization. Basically, precise determination of the projected coordinate origin in the three-dimensional computed tomography scanning system is very important in the FDK algorithm. However, direct measurement of the projected position of the X-ray focus on the imaging plane of the detector, namely, the projected coordinate origin of 3D-CT system, is difficult to obtain. The measurement error will decrease the accuracy of the image reconstruction and cause artifacts in the reconstructed image. Previous studies [5, 6] have shown that the measurement method proposed on the basis of the nonlinear least square estimation was capable of determining projection center coordinates of small spherical objects, but the method had serious limitations because of its significant dependence on the initial value of estimated parameters, in which a small variance in the initial value would cause a significant change in the results estimated, and sometimes even abrupt changes were observed. In subsequent studies, Frederic et al. [7] proposed an analytical measurement method based on the assumption that two small spherical objects moving around in the cone-beam will have their projection centers tracing two ellipses on the detector. K. Yang et al. [8] put forward another method, which used multiple projection images acquired from rotating point-like objects to derive the geometrical parameters of a 3D-CT system. Beque [9] used the projection locations of tomographic acquisition of three point sources located at known distances from each

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other to estimate the geometrical parameters. Y. Sun et al proposed an improved method that used one projection of four phantoms to derive the geometrical parameters [10]. Summarizing, these methods will result in complex calculations and difficulties in practical applications, although their mathematical expression and simulation results are ideal. In addition to the above analytical methods, an easier method involving a grid collimator was used to estimate the projected coordinate origin [11]. The gray level distribution of the Digital Radiography (DR) image of the grid collimator was calculated by using the method of Gauss fitting. However, this method will result in low accuracy because of the non-ideal DR image of the grid collimator.

In this paper, we propose a simple, feasible and accurate method to determine the projected coordinate origin based on the theory that the projection of a spherical object in the cone-beam field is an ellipse. We first utilized image processing and the least square estimation method to get each major axis of the elliptical DR projections of a group of spherical objects. Then we determined the intersection point of the group of major axis by solving an over-determined equation set that was composed by the major axis equations of all the elliptical projections. The intersection point is the X-ray focus projection on the imaging plane of the detector. Based on the experimental results, this new method was proved to be easy to implement in practical scanning systems with high accuracy and anti-noise capability.

2 Methods

Figure 1 is an illustration of FDK scanning. Let's assume that the central ray (PO) of the cone-beam is perpendicular to the imaging plane of the flat panel detector, and the projection of the X-ray focus (point P) is point O, where the coordinates in the $x_\mathrm{d}O_\mathrm{d}z_\mathrm{d}$ system are $(\lambda_x,\ \lambda_z)$. $x_\mathrm{d}O_\mathrm{d}z_\mathrm{d}$ is the imaging plane coordinate system, which is fixed when the cone-beam scanning system is assembled. According to the FDK algorithm, point O is considered to be the coordinate origin of 3D-CT reconstruction, and its coordinates $(\lambda_x,\ \lambda_z)$ should be measured precisely. Our measuring method makes the following assumptions:

1) The cone-beam imaging system is defined in the xyz coordinate system.

2) A spherical object with radius r in the cone-beam, as shown in Fig. 2.

To simplify the derivation, the X-ray focus F is assumed on the z-axis, and the sphere center C on the yOz-plane. Thus, the coordinates of points F and C are $(0,\ 0,\ D)$ and $(0,\ b_y,\ b_z)$, respectively. According to the actual scanning system, the following formula is obtained,

$$|D-b_z|>r>0. \tag{1}$$

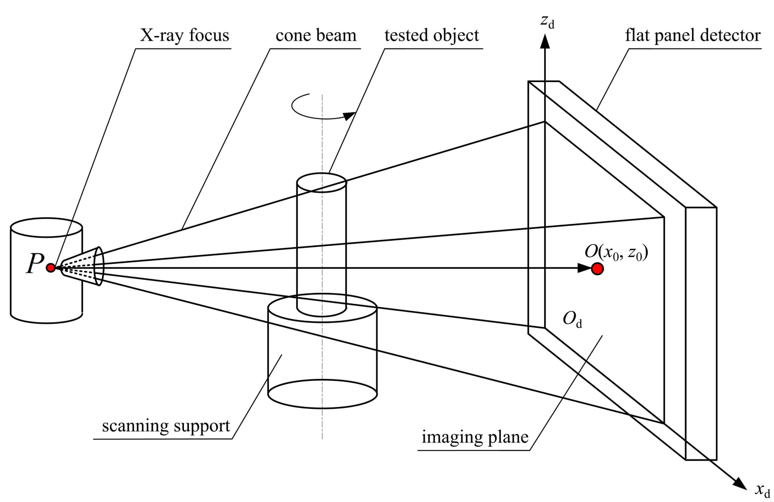

Fig. 1. A schematic of the FDK scanning method.

Figure 2 shows that by starting from the focus F and enveloping the sphere, the surface exhibits a cone shape, which we defined as Ω with semi-cone angle α. Intersection between Ω and the spherical surface is a space curve (defined as η), which is proved to be actually a circle centered on FC with a radius of $r\ \cos\ \alpha$. By taking any point $M(x,y,z)$ on surface Ω, the angle produced between vectors of $\overrightarrow{FM}$ and $\overrightarrow{FC}$ is α. Thus, we have the following equation of surface Ω,

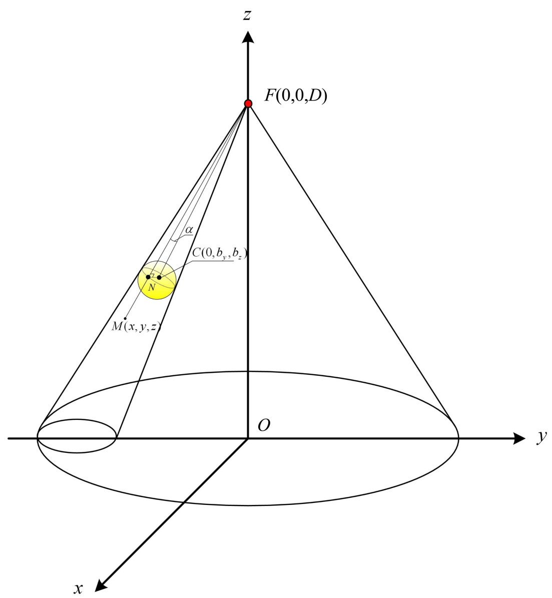

Fig. 2. A schematic of sphere projection in the cone beam.

$$\Omega:\frac{\overrightarrow{FM}\cdot\overrightarrow{FC}}{\left|\overrightarrow{FM}\right|\left|\overrightarrow{FC}\right|}=\cos\alpha,$$

i.e.,

$$\frac{yb_y+(z-D)(b_z-D)}{\sqrt{x^2+y^2+(z-D)^2}\sqrt{b_y^2+(b_z-D)^2}}=\cos\alpha. \quad (2)$$

The intersecting curve of surface Ω and the xOy-plane is the contour of the sphere's projection on the xOy- plane, which is defined as ξ. According to Eq. (2), the following equation of ξ is obtained,

$$\frac{yb_y+(z-D)(b_z-D)}{\sqrt{x^2+y^2+(z-D)^2}\sqrt{b_y^2+(b_z-D)^2}}=\cos\alpha\ |_{z=0},$$

i.e.,

$$\xi:\ \frac{[yb_y-D(b_z-D)]^2}{(x^2+y^2+D^2)[b_y^2+(b_z-D)^2]}=\cos\alpha^2. \quad (3)$$

Eq. (3) can be simplified as

$$k^2(x^2+y^2+D^2)=(yb_y+p)^2.$$

In the case of $k^2=[b_y^2+(b_z-D)^2]\cos\alpha^2$ and $p=-D(b_z-D)$, the above equation can further be transformed to

$$k^2x^2+(k^2-b_y^2)\left(y-\frac{b_yp}{k^2-b_y^2}\right)^2$$

$$=p^2-k^2D^2+\frac{b_y^2p^2}{k^2-b_y^2}. \quad (4)$$

For any point of N ($N\in\eta$), the following geometrical relationship is satisfied,

$$\left|\overrightarrow{FN}\right|^2=\left|\overrightarrow{FC}\right|^2\cos\alpha^2$$
$$=\left[b_y^2+(b_z-D)^2\right]\cos\alpha^2=k^2, \quad (5)$$

$$\left|\overrightarrow{FN}\right|^2=\left|\overrightarrow{FC}\right|^2-r^2=b_y^2+(b_z-D)^2-r^2. \quad (6)$$

Eqs. (5), (6) and (4) can be integrated as

$$k^2x^2+[(b_z-D)^2-r^2]\left(y-\frac{b_yp}{(b_z-D)^2-r^2}\right)$$
$$=\frac{D^2r^2k^2}{(b_z-D)^2-r^2},$$

i.e.,

$$\xi:\frac{x^2}{\dfrac{D^2r^2}{(b_z-D)^2-r^2}}+\frac{\left(y-\dfrac{b_yp}{(b_z-D)^2-r^2}\right)^2}{\dfrac{D^2r^2k^2}{[(b_z-D)^2-r^2]^2}}=1. \quad (7)$$

In accordance with $(b_z-D)^2-r^2>0$ shown in (1), formula (7) is definitely an elliptic equation. That means contour ξ of projection on the xOy-plane is actually the ellipse with its center on the y-axis. Thus, we can derive to the following formula:

$$\frac{D^2r^2k^2}{[(b_z-D)^2-r^2]^2}>\frac{D^2r^2}{(b_z-D)^2-r^2}.$$

This means that the major axis of the ellipse ξ is coincident with the y-axis, and also passes through the coordinate origin. Because the location of the projected X-ray focus spot is at the coordinate origin, the projection of the sphere can be concluded to be an ellipse in the field of the cone-beam X-ray, with its major axis passing through the projection of the X-ray focus.

This conclusion can be further expanded. When two spherical objects are placed in the cone-beam, two elliptical projections will exist on the projected plane, and the intersection point of their major axis must be the projected point of the X-ray focus, as shown in Fig. 3. These two conclusions are the fundamental principles of the measuring method proposed in this study.

To get the accurate equation of the major axis, some image and graphic processing methods are necessary. Fig. 4 shows the process of how to fit the major axis and the determination of the X-ray focus projection coordinates. Two elliptical projections of two spherical objects are captured firstly using a detector, followed by a series of edge detection, threshold segmentation, contour thinning and tracing to obtain the contour point coordinates of the ellipses. The least square fitting is then executed to get the equation of the major axis of each ellipse. The coordinates of the intersecting point, namely, the projection of the X-ray focus, are eventually obtained by solving the equations of the two major axes.

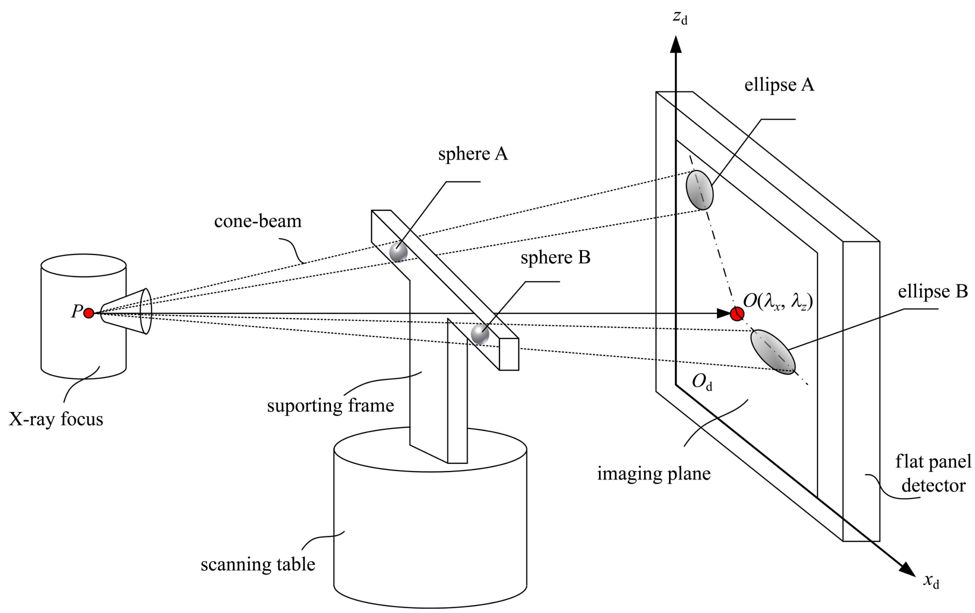

Fig. 3. The principle of measuring the focus coordinates based on double-sphere projection.

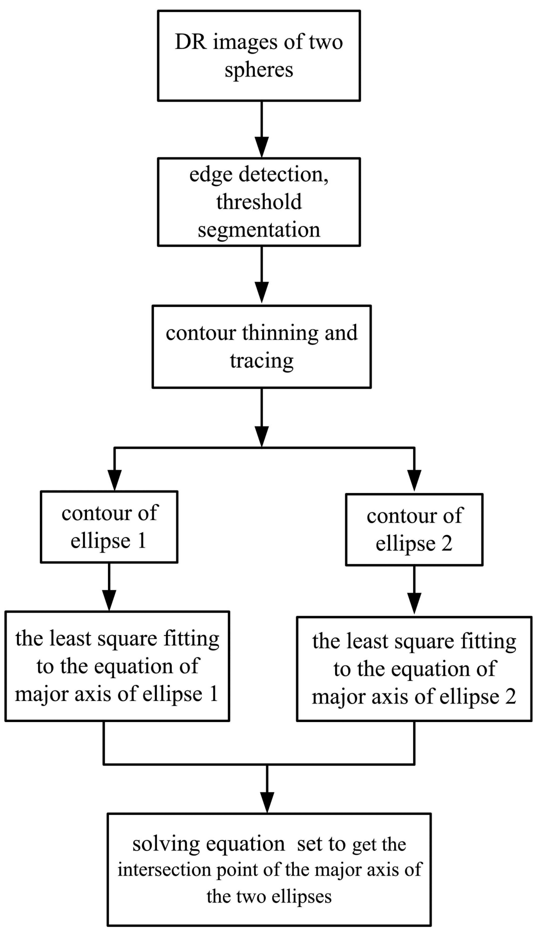

Fig. 4. The flow process of ellipses major axis fitting and the determination of the X-ray focus projection coordinates.

Note that the equation of the major axis is obtained via the method of nonlinear least square fitting. The elliptic equation is [12]

$$x^2+az^2+bxz+cx+dz+e=0, \tag{8}$$

where a, b, c, d and e are the elliptic equation parameters. Based on formula (7), the following function can be derived,

$$z = \frac{1}{2a}\Big[-bx-d \pm\sqrt{(b^2-4a)x^2+(2bd-4ac)x-d^2-4ae}\Big].$$

By defining the following error function,

$$Q=\sum_{i=1}^{n}[z_i-z(x_i,a,b,c,d,e)]^2, \tag{9}$$

where $(x_i,\ z_i)$ represents the contour points of the elliptical projection. And from the non-linear least square solution of Eq. (8), we can indeed obtain the values of the parameters of a, b, c, d and e. Thus, the elliptic geometric parameters can be presented in the following equations,

$$x_c = \frac{bd-2ac}{4a-b^2},\ z_c=\frac{bc-2d}{4a-b^2},$$

$$\mathrm{tg}\theta = \frac{a-1-\sqrt{(1-a)^2+b^2}}{b},$$

where $(x_c,\ z_c)$ is the ellipse center coordinates and $\mathrm{tg}\theta$ is the slope of the major axis of ellipse.

The high accuracy of this method has been demonstrated by results from the computer simulation. However, for the actual CT system, the X-ray focus is not an ideal point source. Due to the influences of scatter, dark current and non-uniformity in pixel response, the DR image edge was observed to be blurry and noisy [13–15], thus resulting in increased error of fitting the major axis. We indeed find that the final results are very sensitive to the error of fitting the major axis. When two ellipse projections are used to calculate the projected coordinate origin, even in the case of a tiny fitting error from either of the two major axes, a remarkable deviation could be observed compared with their true values that come from the coordinates of the intersecting point. From a practical application point of view, we found that it is very difficult to get the high reproducibility of calculating results from only two elliptical projections.

To resolve this problem, we further propose an improved four- step method, as follows.

Step1: A supporting frame with one spherical object is fixed on the scanning stage. Then the scanning stage is adjusted to a suitable location, and the distance between the X-ray source and the detector is adjusted to a suitable value, which can ensure that the projection of the spherical object is an ideal ellipse.

Step2: Move scanning stage along the x_d direction (see Fig. 3) and DR projections are captured.

Step3: By repeating step 2, a series of elliptical projections of the spherical object along the x_d direction at different positions are collected. All of the projections are composed of one image.

Step4: Follow the process of Fig. 4 until each major axis equation is obtained. All of the equations compose one equation set,

$$\begin{cases} a_1x+z=b_1 \\ a_2x+z=b_2 \\ \cdots\cdots \\ a_Nx+z=b_N \end{cases} \text{,namely, } Ap=b, \tag{10}$$

where

$$A=\begin{bmatrix} a_1 & 1 \\ a_2 & 1 \\ \cdots\cdots & \\ a_N & 1 \end{bmatrix},\quad p=\begin{bmatrix} x \\ z \end{bmatrix},\quad b=\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_N \end{bmatrix}.$$

The solution of the equation set is the final result we needed. Obviously, Eq. (10) shows that the number of equations is bigger than the number of unknown parameters. This means that we have an over-determined linear equation set. Typically, the over-determined equation set has no unique solution, meaning that we can only find the most approximate solution. The error function is defined as

$$r=b-Ap.$$

The solution p^*, allowing $\|r\|_2^2$ to reach its minimal value, is the least square solution of the over-determined equation set. Thus the method to find the intersecting point of two lines is transferred to a group of lines, which can effectively guarantee the high accuracy of the solution.

3 Experimental results

To verify the feasibility and accuracy of the proposed method, we performed experiments in a 225 kV Micro-CT system. The system is equipped with a micro-focus X-ray tube, and a Flat panel detector (PaxScan4030CB) with a image area of 400 mm×300 mm and a pixel size of 0.194 mm. The spherical object is a steel ball with an outer diameter of 10 mm, which is fixed on the supporting frame and projected onto the imaging plane with a good elliptical shape. The supporting frame is made of plastic so as to make its projection onto the DR image almost invisible at a suitable X-ray energy. The intersection point of all the major axes is the projection of the X-ray focus. Fig. 5 shows an image composed of six projections of the steel ball captured along the x_d direction with an interval distance of 11 mm. In order to get ideal elliptical projections, we ajust the detector close to the X-ray focus, which can enlarge the cone-beam angle. We also adjusted the steel ball far from the central X-ray beam and the imaging plane. The distance between the X-ray focus and the imaging plane was 420 mm, and the distance between the steel ball and the imaging plane was 340 mm, which meant that the imaging magnification ratio was 5.25.

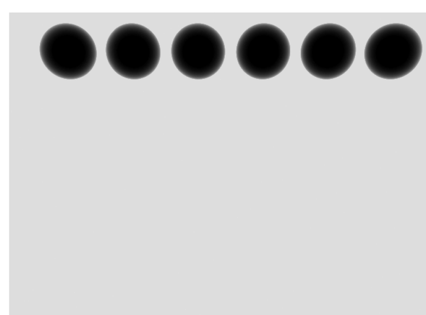

Fig. 5. The image composed of elliptical DR images at different positions.

$$\begin{cases} 1.449x+z=2141.436 \\ 0.903x+z=1722.373 \\ 0.358x+z=1303.371 \\ -0.185x+z=884.944 \\ -0.729x+z=466.107 \\ -1.268x+z=48.346 \end{cases}. \tag{11}$$

Following the process of Fig. 4, every equation of the ellipse's major axis was fitted, and that produced an equation set (11). By solving the equation set using the MATLAB function, the least square solution of the over-determined equation set was determined to be (1026.698, 770.071). The initial value of (λ_x, λ_z) is (1022, 777) when the 225 kV Micro-CT system was produced by the manufacturer. We used a different value to reconstruct the central slice of the object by the FDK algorithm. The reconstructed whole and Region of Interest (ROI) images when (λ_x, λ_z) are (1026.698, 770.071) and (1022, 777) are shown in Fig. 6 and Fig. 7, respectively. It is obvious that the reconstructed image based on the (λ_x, λ_z) value we calculated is of high contrast and has sharp edges.

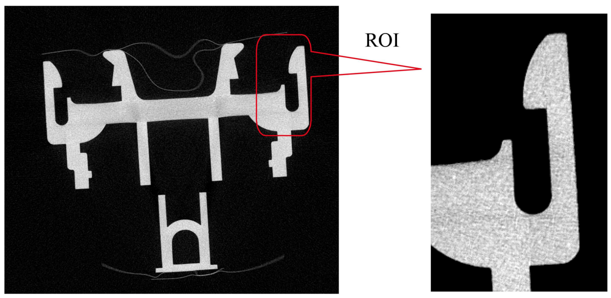

Fig. 6. Reconstructed image when (λ_x, λ_z)= (1026.698, 770.071). (a) Whole CT image; (b) ROI CT image.

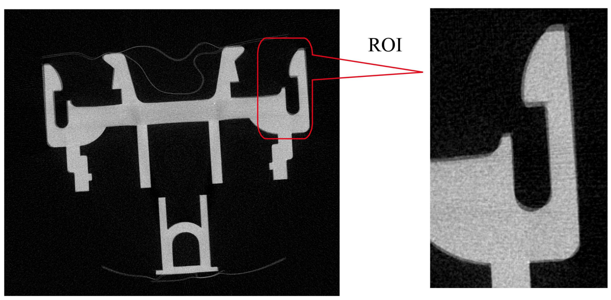

Fig. 7. Reconstructed image when (λ_x, λ_z)= (1022, 777). (a) Whole CT Image; (b) ROI CT Image.

4 Summary

We have successfully developed a method to determine the projected coordinate origin with high accuracy. Through computer simulation, we found that the errors of (λ_x, λ_z) created by this method were within 1 pixel, which were allowable in a practical cone-beam CT system. This method is feasible both practically and theoretically, simply by putting one spherical object in the cone-beam field and adjusting the magnification ratio to ensure an ideal elliptical projection. In reality, the practical way is to locate the spherical object far from the central X-ray beam, and to select a high magnification ratio. In addition, to make the detector close to the X-ray focus is also necessary, which can enlarge the cone-beam angle. The experimental results proved that this method can improve the reconstructed image quality effectively (see Fig. 6).

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