An analysis of nuclear charge radii based on the empirical formula

LI Ru-Heng(李汝恒)[1)] HU Yong-Mao(胡永茂) LI Mao-Cai(李茂材)

(College of Physics and Electron information, Dali University, Dali 671003, China)

Abstract Several nuclear charge radii had been calculated based on the law of $A^{1/3}$ and isospin dependence $Z^{1/3}$ formula which had been used to describe the charge radii data. It is achieved that the isospin dependence $Z^{1/3}$ formula is superior to the generally accepted $A^{1/3}$ law through mean root square deviation analysis, that is, the $Z^{1/3}$ formula is more effective to describe the charge radii data.

Key words charge radius, mean root square deviation, isospin dependence

PACS 25.40.Cm, 28.75.Gz, 21.60.-n

1 Introduction

Atomic nucleus radius is one of the basic parameters for describing the characters of nucleus. The measurement of nucleus radius is very accurate because of the developments in the examination techniques and measurement. Nucleus radius has been a key parameter for studying the whole characters of nucleus, exploring the strange nucleus phenomenon, and testing the micro theoretical model of atoms. Therefore, the deep studies about nucleus radius is important for nuclear physics. It is also important to astrophysics and atomic physics. In a whole, to study nucleus radius is of great scientific importance.

Nucleus radius is related to the distribution of nucleus density. The charge distributions in photons and neutrons are different, which can make the charge density distribution of nucleus different from those of proton density and matter density. In fact, the charge density distribution decides the radii of atomic nucleus. Nucleus radius can be measured via the experimental methods of electromagnetic interactions and strong interaction. The detailed methods are high energy electron dispersion[1], μ atom excited spectroscopy[2], isotope movement of atom spectra, and isotope movement of atom KX ray[3—7]. The main specialties of the above methods are as follows. First, the data of high energy electron dispersion experiments focus on light nuclei area. Second, the charge radii obtained via μ atom excited spectroscopy can be as accurate as 10^{-3} fm, but the method is only useful for long-age nuclei. In fact, almost all charge radii of stable nuclei have been obtained by this method. Third, via isotope movement methods, the charge radii of unstable nuclei that have age shorter than $1S$ can be measured. Fourth, the radii of some shorter-age nuclei can be obtained via strong interaction method. Following the developments of experimental techniques, the studies about atomic nucleus will be a very important subject in nuclear physics[8].

The structure of the paper is as follows. In Sect. 2, we briefly introduce the empirical formulae for description of nuclear charge radii. In Sect. 3, we compare the results determined by various formulae. Finally, in Sect. 4., we give our summary.

2 Empirical formulae for nuclear charge radii

Because the nuclear matter density is constant for weight and medium nucleus, the volume of nucleus increases with mass number:

$$R_{\rm c} = r_{\rm a} A^{1/3}, \qquad r_{\rm a} \approx 1.2\ {\rm fm}, \tag{1}$$

where r_0 is nuclear charge radii constant. Function (1) is called the $A^{1/3}$ law of nuclear charge radius[9].

Received 3 September 2008

1) E-mail: Ruheng_li@126.com

A lot of experimental analysis show that r_0 is not a constant, but a value decreasing with increasing mass number, when Eq. (1) is used for all nucleus. The deviation suggests that some important factors are ignored for the nuclear charge radii described by $A^{1/3}$ law. In order to solve this problem, the following empirical formulae are brought forward, basing on $A^{1/3}$ law:

$$r_0 = 1.25\left(1 - 0.2\frac{N-Z}{A}\right)\ \text{fm}. \quad (2)$$

Eq. (2) is taken from reference [10]. The isospin correction for describing the surplus of proton or neutron is added into the formula.

$$r_0 = 1.240\left(1 - 0.191\frac{N-Z}{A} + 1.646\frac{1}{A}\right)\ \text{fm}. \quad (3)$$

Eq. (3) is taken from Ref. [11]. The formula adds an additional term that can better describe the nuclear charge radii of all nucleus from the light ones to actinium ones.

In another view, some researchers presented a $Z^{1/3}$ law[12, 13].

$$R_c = r_z Z^{1/3}, \qquad r_z \approx 1.64\ \text{fm}. \quad (4)$$

Because $Z^{1/3}$ formula has significant betterment for describing nuclear radii, $Z^{1/3}$ formula can be used instead of $A^{1/3}$ formula, especially for nuclide near stable line. The $Z^{1/3}$ law predicts that the various isotopes of an element have the same charge mean square root radius. Comparing with the experimental data of the distribution of nuclear charge density, which are obtained via μ atom spectroscopy and the isotope movements of atomic spectra[3—7], it needs to study more on the experimental information of the increasing of the mean square root radius of isotopes with neutron number when using the description of $Z^{1/3}$ formula. Some scholars brought forward a new formula (5)[14] for the isospin dependent charge radii. The new formula is presented basing on a great of charge radii data of many nuclide and takes self-consistent relativity continuum spectra theory[15, 16].

$$r_z = 1.631[1 + 0.062(\eta - \eta^*)]\ \text{fm}, \quad (5)$$

where $\mu = N/Z$, $\mu^* = N^*/Z$. The parameter 1.631 is the charge radius constant of stable nucleus and it is related to the saturation property of nuclear force. The parameter 0.062 is derived from empirical data and it expresses the isospin dependence of nuclear charge radii[14].

3 Comparison of different empirical formulae

We use the data supplied by the Periodic Table worksheet, taking $|N^* - N| \leqslant 2$ as the criterion of whether the a nuclide is a β stable line nearby one or not, to calculate the radii of nuclide series. The standard deviation calculated via Eqs. (1) and (4) are denoted using $\sigma_{\rm RA}$ and $\sigma_{\rm RZ}$, respectively. Similar deviations calculated via Eqs. (2), (3) and (5) are expressed using $\sigma_{\rm RA1}$, $\sigma_{\rm RA2}$, and $\sigma^*_{\rm RZ}$, respectively. The results are listed in the Tables 1 and 2, for the cases of near and far from β stable line, respectively.

Table 1. Comparison of the standard deviations of nuclear charge radii for the neighborhood of β stable lines with different masses, when taking various formula of nuclear charge radii.

nuclide	number	$\sigma_{\rm RA}$	$\sigma_{\rm RZ}$	$1-\frac{\sigma_{\rm RZ}}{\sigma_{\rm RA}}$	$\sigma_{\rm RA1}$	$\sigma_{\rm RA2}$	$\sigma^*_{\rm RZ}$	$1-\frac{\sigma^*_{\rm RZ}}{\sigma_{\rm RA1}}$	$1-\frac{\sigma^*_{\rm RZ}}{\sigma_{\rm RA2}}$
A⩾10	283	1.203	1.129	6.2%	1.163	1.095	0.128	3.0%	−3.0%
A⩾30	261	0.987	0.906	8.3%	0.944	0.905	0.906	4.0%	−0.1%
A⩾50	236	0.830	0.754	9.2%	0.790	0.763	0.753	4.7%	1.2%
A⩾70	212	0.707	0.639	9.6%	0.671	0.651	0.636	5.2%	2.2%
A⩾90	182	0.585	0.521	10.9%	0.551	0.536	0.519	5.9%	3.2%
A⩾100	167	0.531	0.472	11.0%	0.500	0.487	0.470	6.1%	3.6%
A⩾150	99	0.326	0.289	11.4%	0.307	0.301	0.285	7.2%	5.1%

Table 2. Comparison of the standard deviations of nuclear charge radii for all nuclide (including the ones far from β stable line).

nuclide	number	$\sigma_{\rm RA}$	$\sigma_{\rm RZ}$	$1-\frac{\sigma_{\rm RZ}}{\sigma_{\rm RA}}$	$\sigma_{\rm RA1}$	$\sigma_{\rm RA2}$	$\sigma^*_{\rm RZ}$	$1-\frac{\sigma^*_{\rm RZ}}{\sigma_{\rm RA1}}$	$1-\frac{\sigma^*_{\rm RZ}}{\sigma_{\rm RA2}}$
A⩾10	311	1.216	1.140	6.2%	1.175	1.109	1.137	3.2%	−2.5%
A⩾30	290	1.007	0.925	8.1%	0.964	0.925	0.923	4.2%	0.2%
A⩾50	265	0.857	0.780	9.0%	0.816	0.788	0.776	4.9%	1.6%
A⩾70	241	0.740	0.671	9.3%	0.704	0.683	0.666	5.4%	2.5%
A⩾90	211	0.631	0.565	10.5%	0.597	0.581	0.561	6.0%	3.4%
A⩾100	194	0.579	0.520	10.2%	0.548	0.543	0.514	6.3%	3.8%
A⩾150	118	0.374	0.338	9.5%	0.355	0.348	0.329	7.3%	5.2%

It can be seen that for all mass areas, the description of $Z^{1/3}$ law for nucleus radius is significantly better than $A^{1/3}$ law. Isospin dependence $Z^{1/3}$ law gives very similar results to $A^{1/3}$ law. This suggests that for nuclide near β stable line, it is not necessary to add the isospin dependence correction. The reason is that the value of $|\mu-\mu^*|$ is very small for nuclide near β stable line and the isospin dependence correction can be ignored.

We can see from Table 2 that for nuclide with various masses, including the ones far from β stable line, the larger the fraction of weight nucleus, the more significant the correction for the results by $Z^{1/3}$ formula with isospin correction.

4 Summary

This paper calculated the radii of nucleus via some formulae of nucleus charge radius and compared the results derived from different formulae for describing nucleus radius, i.e., the $A^{1/3}$, $Z^{1/3}$, and the isospin dependence $Z^{1/3}$ laws. The results can be concluded as follows:

(1) The result data show that the $Z^{1/3}$ law is more advanced than the $A^{1/3}$ law, but both the two laws can not describe isotope movement effectively.

(2) The $Z^{1/3}$ law can describe the nucleus charge radius when nuclide near β stable line. It is not necessary to take the isospin correction into account. For nuclide far from β stable line, the law with isospin correction can better describe nucleus charge, and the larger the distance from stable line, the better the description.

(3) The isospin dependence nucleus radius formula can describe the nucleus charge radius of nuclide that are both near or far from stable line. It is very important for studies of both nuclear physics and astrophysics, because it can describe effectively the nucleus charge radii of atomics via a simple shape.

References

1 Vries H De, Jager C W De, Vries C De. At. Data & Nucl. Data Tables, 1987, **36**: 495

2 Fricke G et al. At. Data & Nucl. Data Tables, 1995, **60**: 177

3 Aufmuth P, Heilig K, Steudel A. At. Data & Nucl. Data Tables, 1987, **37**: 455

4 Otten E W. Treatise on Heauy-Ion Science vol.8, ed D.A.Bromley. New York: Pleum, 515

5 Billowes J, Campbell P. J. Phys. G, 1995, **21**: 707

6 Schuessler H A, Alousi A, Evans R M et al. Phys. Rev. Lett., 1990, **65**: 1332

7 Levins J M G, Benton D M, Billowes J et al. Phys. Rev. Lett., 1999, **82**: 2476

8 Tanihata 1. Prog.Part. and Nucl. Phys., 1995, **35**: 505

9 Bohr A, Mottelson B R. Nuclear Structure, Benjamn W A Inc., New York, Amesterdam: 1969

10 Nerlo-Pomorska B, Pomorski K. Z. Phys. A, 1993, **344**: 359

11 Warda M, Nerlo-Pomorska B, Pomorski K. Nucl. Phys. A, 1998, **635**: 484

12 ZENG Jin-Yan. Acta Phys. Sin., 1957, **13**: 357 (in Chinese)

13 ZENG Jin-Yan. Acta Phys. Sin., 1975, **24**: 151(in Chinese)

14 ZHANG Shuang-Quan et al. Euro. J. Phys. A, 2002, **13**: 285

15 MENG J, Ring P. Phys. Rev. Lett., 1996, **77**: 3963; 1998, **80**: 460

16 MENG J. Nucl. Phys. A, 1998, **635**: 3; 1999, **654**: 702c