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Triaxial rotation in atomic nuclei*

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Abstract The Projected Shell Model has been developed to include the spontaneously broken axial symmetry so that the rapidly rotating triaxial nuclei can be described microscopically. The theory provides an useful tool to gain an insight into how a triaxial nucleus rotates, a fundamental question in nuclear structure. We shall address some current interests that are strongly associated with the triaxial rotation. A feasible method to explore the problem has been suggested.

Key words shell model, triaxiality, signature inversion, chiral bands, wobbling mode

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1 Introduction

The triaxiality, as the broken axial symmetry, in atomic nuclei have been quite convincing both experimentally and theoretically. However, the problem how a triaxially deformed nucleus rotates has not been solved. A nucleus with the elongated shape rotates favorably around the shortest intrinsic principle axis, and the yrast states can be well described by the theoretical models in which a fixed rotational axis is assumed and the rotation is introduced independently from the mean field to provide the states with angular momentum, e.g., the Cranking Shell Model(CSM)[1]. A nucleus with triaxial deformation rotates in a much more complex way that the rotational axis is not fixed at all and could be changed dramatically with increasing spin, so the theoretical description of such states becomes a great challenge. By treating the rotation and the mean field in a unique uniformity, the present model avoids the difficulty questions of self-consistency between the rotation and the mean field, and thus provides a powerful tool to explore the insight into the triaxial rotation in fully quantum mechanics way.

Recently, the experimental discoveries in the high spin spectra have revived the investigations of the triaxial rotation problems. Among them, the observed wobbling mode has provided an unique evidence for the triaxial motion, yet the stable nuclear triaxial shape. The wobbling excitation quanta were found in triaxial superdeformed (TSD) nucleus ^{163}Lu in 2001[2], and a more firm evidence follows immediately in an upgraded experiment[3]. In the later experiment, the interpretation of TSD2 band as a one-phonon wobbling excitation built on the TSD1 band was strongly supported by the measured strong E2 transitions feeding from the TSD2 band into the TSD1 band, competing with the strong collective intra-band E2 transitions, characterized with large values of $B(\mathrm{E2})_{\mathrm{out}}/B(\mathrm{E2})_{\mathrm{in}}$. The possible two-phonon wobbling excitation built on the TSD1 was found as the TSD3 band. A similar evidence for the wobbling mode was found in superdeformed triaxial nuclei ^{165}Lu[4] and ^{167}Lu[5]. Another most applicable example is the discoveries of the chiral bands. Almost at the same time of the announcement of discovering the nuclear wobbling motion, the experimental evidences for the chiral bands were reported , for example, in odd-odd $N = 75$ isotones, ^{130}Cs, ^{132}La, ^{134}Pr and ^{136}Pm[6]. In contrast to the wobbling modes, the chiral symmetry in nuclei has not been definitely proven by experiment. Nevertheless, the conception of the chiral bands is based on the specific geometry of three components of the total angular momentum,

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therefore, it is strongly related to the triaxial rotation. The signature inversion is a long-standing question, the phenomenon has been widely observed in nuclear rotational spectra, e.g., see Refs.[7,8], but never been convincingly explained. Few interpretations have appeared in the literature, but no one is directly related to the properties of triaxial rotation. Recently, a new interpretation of the phenomenon was given in Ref. [9], which is strongly related to triaxial rotation. In the present paper, we suggest that the signature inversion is one of best probes for the triaxial rotation, and we provide a new method to investigate the subject based on the frame work of shell model.

A brief description of the model is given in Sec.2. Calculation and discussion are given in Sec.3. General conclusions are summarized in Sec.4.

2 Brief description of theory

Atomic nuclei involve many individual nucleons which interact with complex forces, particularly, the strong interaction between them is not simple and not well known. Nevertheless, nuclear spectra present a number of systematic features that imply both single particle motion and collective motion. To describe these features the modelling of nuclear spectra has been a great challenge for theorists. The generally accepted supposition today is that the strongly interacting nucleons form a mean-field which may be deformed and rotating. The difficult questions of self-consistency arise in some useful theoretical models where the field and rotation are treated separately. In the present model this difficulty is avoided by using the projection method, the states of good angular momentum and parity are provided by projecting from the intrinsic mean field, and no external rotation is imposed. The outline of the model is given as follows.

For the present study we do not need the parity projection since the considered intrinsic mean field has a reflection symmetry. And then, the trial wave function may be written as,

$$|\Psi_{IM}\rangle=\sum_{K\kappa} f^{I}_{K_\kappa}\,\hat{P}^{I}_{MK}\,|\Phi_\kappa\rangle\,, \tag{1}$$

where $\hat{P}^{I}_{MK}$ is the three-dimensional angular-momentum-projection operator. The dimension of the summation in Eq. (1) is $K\times\kappa$, where $|K|\leqslant I$ and κ is usually in the order of 10^2. The $|\Phi_\kappa\rangle$ represents a set of multi-qp states associated with the triaxially deformed qp vacuum $|0\rangle$.

By carrying out the variational procedure with respect to the wave function, precisely the coefficients $f^{I}_{K\kappa}$, we obtain the eigenvalue equation,

$$\sum_{K\kappa} f^{I}_{K\kappa}(\langle\Phi_\kappa|HP^{I}_{K'K}|\Phi_\kappa\rangle-E\langle\Phi_\kappa|P^{I}_{K'K}|\Phi_\kappa\rangle)=0. \tag{2}$$

The shell model Hamiltonian considered involves a large number of nucleons moving in a spherical Nilsson potential and an interaction of separable multipole $Q\cdot Q$ plus monopole pairing plus quardrupole pairing,

$$\begin{aligned} H\,=\,&H_0-\frac{1}{2}\sum_{\lambda=2}^{4}\chi_\lambda\sum_{\mu=-\lambda}^{\lambda}Q^{\dagger}_{\lambda\mu}Q_{\lambda\mu}-\\ &G_0P^{\dagger}_{00}P_{00}-G_2\sum_{\mu=-2}^{2}P^{\dagger}_{2\mu}P_{2\mu}. \end{aligned} \tag{3}$$

The standard Nilsson potential used in the calculation contains a proper spin-orbit force[10].

3 Calculation and discussion

The wobbling motion is a well established specific rotational mode and reflects the change of the rotational axis in a fast rotating triaxial nucleus. Therefore, it can be used to explore the triaxial nuclear rotation. We will show through the calculations below that the signature inversion phenomenon is also one of best probes to explore the triaxial rotation in atomic nuclei.

The signature is a quantum number associated with the invariance of a system with intrinsic quadrupole deformation under a rotation of 180° around a principal axis. Due to this symmetry, rotational energies $E(I)$ can be divided into two branches of bands, with $\Delta I=2$, classified by the signature quantum number α. As a rule, the band with a favor signature $\alpha_{\rm f}$ is energetically favored and the partner band with an unfavor signature $\alpha_{\rm u}$ lies higher in energy than the former. The signature inversion (SI) means that the band with $\alpha_{\rm u}$ becomes energetically favored one, lying lower in energy than the band with $\alpha_{\rm f}$. The signature inversion can be shown more clearly by introducing the quantity $S(I)=E(I)–E(I–1)$, which is a staggering function of spin I. The phase of the staggering of $S(I)$ reverses if the signature inversion occurs. The calculated $S(I)$ for typical SI nuclei ^{118}Cs and ^{124}Cs are compared with the experimental data[7, 8] in Fig.1. The deformation parameters used in the calculation are (ε_2,γ)=(0.30,30°) and (0.26,31°) for ^{118}Cs and ^{124}Cs, respectively. The monopole pairing strength G_0 is of a simple form G/A (MeV) with $G=19.6$ for neutrons and 17.2 for protons, respectively. The quadrupole pairing strength

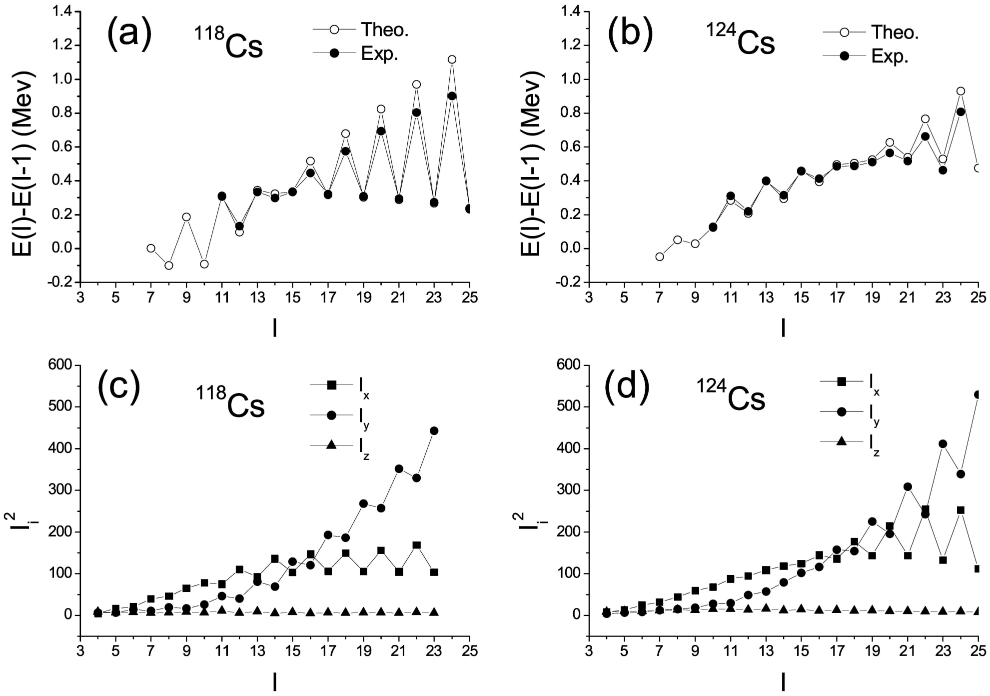

Fig. 1. Calculated quantities S(I) are compared with experimental data for (a) ^{118}Cs and (b) ^{124}Cs. The projections of the total angular momentum on the intrinsic principle axis, calculated with the same wave functions in Figures (a) and (b), are shown in (c) ^{118}Cs and (d) ^{124}Cs.

is $G_2 = 0.16G_0$. It is seen from Fig.1 (a) and (b) that a very good agreements between theory and experimental data have been achieved and, in particular, the signature inversion points $I_{\rm rev}$=15 for ^{118}Cs and 19 for ^{124}Cs are reproduced by the calculations. The rotational axis changes with increasing spin in a rotating triaxial nucleus. This situation is demonstrated in Fig.1 (c) and (d), where the calculated I_i^2, $i = x, y, z$, are shown as functions of spin. By noticing that the I_x and I_y crosses each other around the inversion spin $I_{\rm rev}$, we realize that the signature inversion is related to the drift motion of the rotational axis from the x-axis towards the y-axis. The delay of the signature inversion in ^{124}Cs , relative to ^{118}Cs, can be understood as the delay of the crossing of the I_x and I_y in ^{124}Cs, relative to ^{118}Cs. The I_x and I_y are mainly contributed from the 2q.p. alignments and the collective rotation of the core, respectively. In contrast to the case of ^{118}Cs, the increase of the I_y with increasing spin in ^{124}Cs is slower due to its smaller moment of inertia, J_y, caused by a smaller quadrupole deformation. This explains the delay of the crossing of the I_x and I_y as well as the delay of the signature inversion in ^{124}Cs, relative to ^{118}Cs.

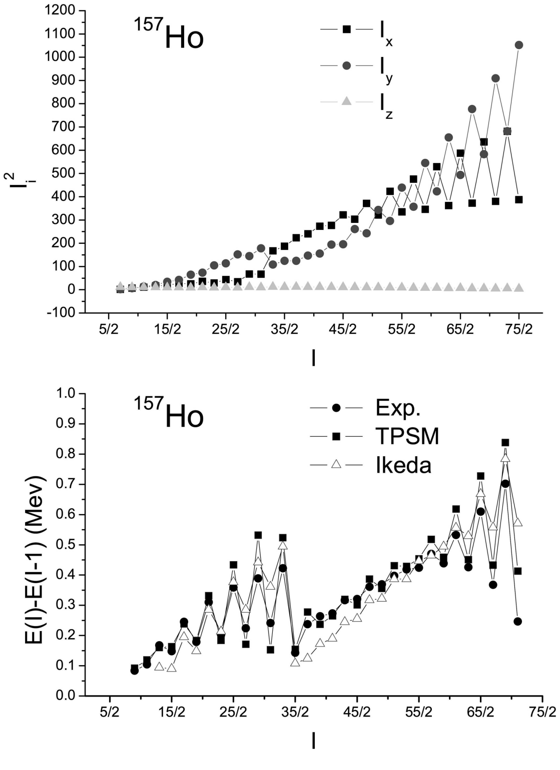

Fig. 2. In the lower part, the experimental quantity $S(I)$ for ^{157}Ho are compared with two model calculations, the present theory (TPSM) and the particle-rotor plus γ-vibration (Ikeda). In the upper part, the projections of the total angular momentum on the intrinsic principle axis, calculated by TPSM are shown as functions of spin.

In the lower part of Fig. 2, the experimental quantity $S(I)$ for ^{157}Ho[11] are compared with two model calculations, the present theory (TPSM) and the particle-rotor plus γ-vibration model (Ikeda)[12]. It can be seen that the TPSM calculation well reproduces the experimental data and Ikeda's calculation does also reproduce the data satisfactorily. However, in the later calculation one has to change the sign of the γ deformation by hand to achieve the signature inversion. In fact, the changing sign of the γ deformation indicates that a change of the rotational axis, from the x-axis to the y-axis, is imposed. It is interesting to note that the present calculation describes automatically the drifts of the rotational axis from the y-axis towards the x-axis and the reverse, see the upper part of Fig.2. The twice crosses of the I_x and I_y are responsible to the twice signature inversions around spins 37/2 and 53/2, respectively. The deformation parameters used in the TPSM calculation are $\varepsilon_2 = 0.26$, $\varepsilon_4 = -0.02$, and $\gamma = 26^\circ$, which reproduce approximately the γ-band energy in the neighbor even-even nuclei. The monopole pairing strength G_0 is of form $G = (g_1 \mp g_2(N-Z)/A)/A$ (MeV) with $g_1 = 20.12$ and $g_2 = 13.12$. The quadrupole pairing strength is $G_2 = 0.22G_0$.

4 Summary

It has been shown that the experimental signature inversion can be well reproduced by the TPSM calculation. The new interpretation of the phenomenon is born out from an analysis of the orientation of the total angular momentum in the intrinsic frame. This analysis leads to the interpretation for the phenomenon, namely, the signature inversion may be explained as a manifestation of the drift of the rotation axis in the intrinsic frame when a triaxial nucleus rotates. We demonstrate that the signature inversion phenomenon can be used to explore a fundamental question in nuclear structure, namely, how atomic nuclei rotate when the axial symmetry is broken in the system.

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