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Bunch transverse emittance increase in electron storage rings*

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Abstract In this paper a theoretical framework to estimate the bunch transverse emittance growing in electron storage rings due to short range transverse wakefield of the machine is established. New equilibrium emittance equations are derived and applied to explain the experimentally obtained results in ATF damping ring. This equation will be useful for linear collider damping ring design.

Key words storage ring, damping ring, linear collider

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1 Introduction

Required by the future e^+e^- linear colliders, damping rings[1] are needed to provide the main linacs with extremely small transverse emittance. In an electron storage ring it is observed that with the increasing bunch current a bunch suffers not only from bunch lengthening, and increase in energy spread, but also from transverse emittance growth. The usual explanation to the transverse emittance growth is based on the intrabeam scattering theory[2—4] according to H. Bruck's idea[5]. Comparison of the emittance growth with the results of the intrabeam scattering theory shows, however, that in the vertical plane the agreement is not satisfactory[6, 7]. In this paper we will draw attention to another important physical cause for the transverse emittance growth in addition to the intrabeam scattering, i.e. the short range transverse wakefield of the machine. It is not difficult to imagine that if the closed orbit is distorted and (or) the vacuum chambers are misaligned from the ideal geometric center, the particles in a bunch will suffer from transverse deflections due to single bunch short range wakefield which results in an emittance growth similar to that in a linac when the axes accelerating structures do not coincide with the trajectory of the passing bunch[8]. In fact, it is also due to this transverse wakefield that there exists a threshold bunch current beyond which the transverse motions of particles inside the bunch will become unstable[9, 10]. In this paper we will estimate this single bunch emittance growth induced by the short range wakefield, restricting ourselves to lepton storage rings. To start with, in section 2, we make a brief recall of the intrabeam scattering theory and point out that the intrabeam scattering phenomenon is not always the dominating physical process for the bunch energy spread and emittance growth in an electron storage ring. In section 3 a Langevin type differential equation for the transverse motion of the particles is established. By solving the Langevin equation one gets the increase of the bunch emittance induced by the short range transverse wakefield and new equilibrium emittance equations. Finally, in section 4 this theory is applied to the analysis of the experimental results obtained from ATF damping ring at KEK[7].

2 Intrabeam scattering

In this section we follow the analysis on intrabeam scattering given in Ref. [4]. The relative r.m.s. energy spread of a bunch due to intrabeam scattering is expressed as

$$\left(\frac{\sigma_{\mathrm{e,s}}(N_{\mathrm{e}})}{E_0}\right)^2 = \frac{N_{\mathrm{e}} r_{\mathrm{e}}^2 c\beta_x \tau_{\mathrm{e}}}{2^5\pi\gamma\sigma_x^2\sigma_y\sigma_z} f(\chi), \qquad (1)$$

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$$f(\chi_m)=\int_{\chi_m}^{\infty}\frac{1}{\chi}\ln\left(\frac{\chi}{\chi_m}\right)\exp(-\chi)\mathrm{d}\chi\,, \tag{2}$$

$$\chi_m=\frac{r_{\mathrm{e}}\beta_x^2}{b_{\max}\gamma^2\sigma_x^2}\,, \tag{3}$$

where $\sigma_{\mathrm{e,s}}$ denotes the energy spread induced by intra beam scattering, E_0 is the particle energy, N_{e} is the particle population inside a bunch, r_{e} is the classical electron radius, β_x is the average beta function, τ_{e} is the synchrotron radiation damping time, γ is the normalized particle energy, $\sigma_{x,y,z}$ are the r.m.s. bunch dimensions, $b_{\max}\approx 1/n^{-1/3}$, and $n=\dfrac{N_{\mathrm{e}}}{2^3\pi^{3/2}\sigma_x\sigma_y\sigma_z}$. Concerning the application of the intrabeam scattering theory to explain the increase in bunch energy spread and bunch transverse emittance growth, we make two observations. Firstly we notice that $\sigma_{x,y,z}$ in Eq. (1) should be functions of N_{e}. To illustrate this point clearly, we take the ATF damping ring at KEK as an example[7]. In Fig. 1, we show the experimentally measured relative bunch energy spread vs bunch population by dots. The dashed line gives the theoretical result from Eq. (1) by fixing the bunch dimensions to their natural values at zero current. The solid line is the theoretical result from Eq. (1) with bunch dimensions varying with bunch population (this information is obtained from the experimentally measured values[7, 11, 12]). It is obvious that the real contribution of the intrabeam scattering to the bunch energy spread is not as large as usually believed. As for the second observation, using again the ATF damping ring as an example[7], we recall the experimental results of the vertical emittances vs the bunch population: the experimental value at the design current is three times larger than that at zero current, which is very difficult to be explained by the intrabeam scattering theory[12, 13]. Based on the two observations we will in the next section examine another physical process which also contributes (even in a dominating way) to the bunch transverse emittance growth. As for the increase of the bunch energy spread, the physical process is discussed in great detail in Ref. [14], where the nonlinear single-bunch short-range wake potential has been regarded as the main physical cause.

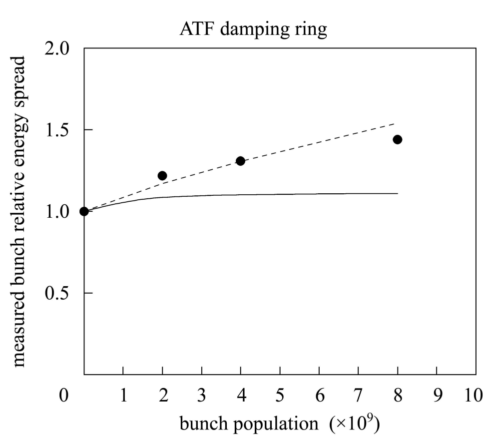

Fig. 1. ATF damping ring single bunch energy spread vs bunch population.

3 Equation for the transverse motion

The differential equation for the transverse motion of a bunch with zero transverse dimension is expressed as

$$\frac{\mathrm{d}^2y(s,z)}{\mathrm{d}s^2}+\frac{2}{\tau_y c}\frac{\mathrm{d}y(s,z)}{\mathrm{d}s}+k(s,z)^2y(s,z)=\frac{1}{m_0c^2\gamma(s,z)}e^2N_{\mathrm{e}}W_{\perp,y}(s,z)Y(s,z), \tag{4}$$

where $y(s,z)$ is the particle's transverse deviation from the closed orbit, s is the longitudinal coordinate of the particle located at the center of a bunch, z denotes a particle's longitudinal position inside the bunch with respect to the bunch center, $k(s,z)$ describes the linear lattice focusing strength, c is the velocity of light, τ_y is the synchrotron radiation damping time in transverse y direction, m_0 is the rest mass of the electron, e is the electron charge, and $Y(s,z)$ is the deviation of the particles from the geometric center of the vacuum chamber.

$$W_{\perp,y}(s,z)=\int_z^{\infty}\rho(z')\mathcal{W}_{\perp,y}(s,z'-z)\mathrm{d}z'\,,$$

where $\mathcal{W}_{\perp,y}(s,z)$ is the point charge wakefield. The bunch line charge density $\rho(z)$ is normalized as $\int_{-\infty}^{\infty}\rho(z')\mathrm{d}z'=1$. Due to synchrotron radiation effect one can treat the particles in a bunch on the same footing by multiplying Eq. (4) with $\rho(z)$ and integrate from $-\infty$ to ∞ over z. As a result one gets

$$\frac{\mathrm{d}^2y(s)}{\mathrm{d}s^2}+\Gamma\frac{\mathrm{d}y(s)}{\mathrm{d}s}+k(s)^2y(s)=\Lambda\,, \tag{5}$$

where

$$\Gamma=\frac{2}{\tau_y c},\quad \Lambda=\frac{e^2N_{\mathrm{e}}k_{\perp,y}(\sigma_z)Y(s)}{m_0c^2\gamma l_s}\,,$$

l_s is the circumference of the storage ring,

$$k_{\perp,y}(\sigma_z)=\int_0^{l_s}\left\{\int_{-\infty}^{\infty}\rho(z)W_{\perp,y}(s,z)\,\mathrm{d}z\right\}\mathrm{d}s\,,$$

and

$$\rho(z)=\frac{1}{\sqrt{2\pi}\sigma_z}\mathrm{e}^{-\frac{z^2}{2\sigma_z^2}}\,.$$

$Y(s)$ is a random variable due to vacuum chamber misalignment error and close orbit distortion with

$\langle Y(s)\rangle = 0$ ($\langle\ \rangle$ denotes the average over s). Eq. (5) can be regarded as Langevin equation similar to that governing the Brownian motion of a molecule.

To make an analogy between the movement of the transverse motion of an electron and that of a molecule, we define $P=\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}$, and regard $Y(s)P$ as the particle's "velocity" random increment $\left(\Delta\frac{{\rm d}y}{{\rm d}s}\right)$ over the distance l_s. We assume that the random variable $Y(s)$ follows Gaussian distribution:

$$f(Y(s))=\frac{1}{\sqrt{2\pi}\sigma_Y}\exp\left(-\frac{Y(s)^2}{2\sigma_Y^2}\right) \tag{6}$$

and the velocity (u) distribution of the molecule follows Maxwellian distribution:

$$g(u)=\sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mu^2}{2kT}\right), \tag{7}$$

where m is the molecule's mass, k is the Boltzmann constant, and T is the absolute temperature. The fact that the molecule's velocity follows Maxwellian distribution permits us to get the distribution function for Λl_s[15]:

$$\phi(\Lambda l_s)=\frac{1}{\sqrt{4\pi q l_s}}\exp\left(-\frac{\Lambda^2 l_s^2}{4ql_s}\right), \tag{8}$$

where

$$q=\Gamma\frac{kT}{m}. \tag{9}$$

By comparing Eq. (8) with Eq. (6), one gets:

$$2\sigma_Y^2=\frac{4ql_s}{P^2}, \tag{10}$$

or

$$\frac{kT}{m}=\frac{\sigma_Y^2P^2}{2l_s\Gamma}. \tag{11}$$

Till now one can use all the analytical solutions concerning the random motion of a molecule governed by Eq. (5) by a simple substitution described in Eq. (11). Under the condition, $k^2(s)\gg\frac{\Gamma^2}{4}$ (adiabatic condition), one gets[15]:

$$\begin{aligned}\langle y^2\rangle \;=\; & \frac{kT}{mk^2(s)}+\left(y_0^2-\frac{kT}{mk^2(s)}\right)\times \\ & \left(\cos(k_1s)+\frac{\Gamma}{2k_1}\sin(k_1s)\right)^2\exp(-\Gamma s)= \\ & \frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2+ \\ & \left(y_0^2-\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2\right)\times \\ & \left(\cos(k_1s)+\frac{\Gamma}{2k_1}\sin(k_1s)\right)^2\exp(-\Gamma s),\end{aligned} \tag{12}$$

$$\begin{aligned}\langle y'^2\rangle \;=\; & \frac{kT}{m}+\frac{k(s)}{k_1^2}\left(y_0^2-\frac{kT}{mk^2(s)}\right)\sin^2(k_1s)\times \\ & \exp(-\Gamma s)=\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2+ \\ & \frac{k(s)}{k_1^2}\left(y_0^2-\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_y(\sigma_z)}{m_0c^2\gamma}\right)^2\right)\times \\ & \sin^2(k_1s)\exp(-\Gamma s),\end{aligned} \tag{13}$$

$$\begin{aligned}\langle yy'\rangle \;=\; & \frac{k(s)^2}{k_1}\left(\frac{kT}{mk(s)^2}-y_0^2\right)\Bigg(\cos(k_1s)+ \\ & \frac{\Gamma}{2k_1}\sin(k_1s)\Bigg)\exp(-\Gamma s)= \\ & \frac{k(s)^2}{k_1}\left(\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2-y_0^2\right)\times \\ & \left(\cos(k_1s)+\frac{\Gamma}{2k_1}\sin(k_1s)\right)\exp(-\Gamma s),\end{aligned} \tag{14}$$

where $k_1=\sqrt{k(s)^2-\frac{1}{4}\Gamma^2}$. The asymptotical values for $\langle y^2\rangle$, $\langle y'^2\rangle$, and $\langle yy'\rangle$ for $s\to\infty$ are easily obtained:

$$\langle y^2\rangle=\frac{kT}{mk^2(s)}=\frac{\sigma_Y^2\tau_y}{4T_0k^2(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2, \tag{15}$$

$$\langle y'^2\rangle=k^2(s)\langle y^2\rangle=\frac{\sigma_Y^2\tau_y}{4T_0}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2, \tag{16}$$

$$\langle yy'\rangle=0. \tag{17}$$

Inserting Eqs. (15, 16), and (17) into the definitions of the r.m.s. emittance shown in Eq. (18):

$$\epsilon_{w,y}=\left(\langle y^2\rangle\langle y'^2\rangle-\langle yy'\rangle^2\right)^{1/2}, \tag{18}$$

one gets

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y}{4T_0k(s)}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2, \tag{19}$$

or

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y\langle\beta_y(s)\rangle}{4T_0}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2, \tag{20}$$

where $\langle\beta_y(s)\rangle$ is the average beta function of the machine in y plane. Before proceeding further we remind the reader that we had assumed a zero transverse dimension of the bunch (the bunch is represented as a soft line). In reality, however, a bunch has finite transverse dimension. A particle inside the bunch can move like a molecule in a gas due to quantum effect of synchrotron radiation. In electron storage rings the "banana" shape of the bunch cannot be sustained due to "mixing"; quite different from what happens in a linac and a hadron storage ring where there is no or little synchrotron radiation. To take this fact mathematically into account one rewrites Eq. (20) as follows

$$\epsilon_{w,y}=\frac{\sigma_Y^2\tau_y\langle\beta_y(s)\rangle}{4T_0\mathcal{R}_{\epsilon,y}^3}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2, \qquad (21)$$

where $\mathcal{R}_{\epsilon,y}=\epsilon_{{\rm total},y}/\epsilon_{0,y}$, $\epsilon_{{\rm total},y}$ is the final emittance for a given bunch population $N_{\rm e}$, $\epsilon_{0,y}$ is the emittance zero current and the cubic functional dependence on $\mathcal{R}_{\epsilon,y}$ can be regarded as an Ansatz. Finally we find the expression for the emittance of a bunch corresponding to a given bunch population

$$\epsilon_{{\rm total},y}=\epsilon_{0,y}+\epsilon_{w,y}=$$
$$\epsilon_{0,y}+\frac{\sigma_Y^2\tau_y\langle\beta_y(s)\rangle}{4T_0}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_z)}{m_0c^2\gamma}\right)^2. \qquad (22)$$

If we distinguish now the horizontal plane denoted by the subscript x and the vertical plane denoted by the subscript y, one gets the two emittance equations

$$\mathcal{R}_{\epsilon,x}=\frac{\epsilon_{{\rm total},x}}{\epsilon_{0,x}}=1+\frac{\sigma_X^2\tau_x\langle\beta_x(s)\rangle}{4T_0\epsilon_{0,x}\mathcal{R}_{\epsilon,x}^3}\left(\frac{e^2N_{\rm e}k_{\perp,x}(\sigma_{z0})}{m_0c^2\gamma\mathcal{R}_z^{\Theta}}\right)^2, \qquad (23)$$

$$\mathcal{R}_{\epsilon,y}=\frac{\epsilon_{{\rm total},y}}{\epsilon_{0,y}}=1+\frac{\sigma_Y^2\tau_y\langle\beta_y(s)\rangle}{4T_0\epsilon_{0,y}\mathcal{R}_{\epsilon,y}^3}\left(\frac{e^2N_{\rm e}k_{\perp,y}(\sigma_{z0})}{m_0c^2\gamma\mathcal{R}_z^{\Theta}}\right)^2. \qquad (24)$$

where σ_{z0} is the bunch length for zero current, $\mathcal{R}_z=\sigma_z/\sigma_{z0}$, and $\Theta=0.7$, which corresponds to SPEAR scaling for transverse loss factor[10]. Since $\mathcal{R}_z$ is also a function of $N_{\rm e}$, it is obvious that one can start to solve Eqs. (23) and (24) only if $\mathcal{R}_z(N_{\rm e})$ has been solved from the bunch lengthening equation[14].

4 Application to the analysis of ATF damping ring experimental results

The ATF damping ring is a machine dedicated to feasibility studies of future e^+e^- linear colliders[16]. By applying the theory established above and neglecting intrabeam scattering effects we try to explain the ATF damping ring experimental results[7] with the following machine parameters: $E_0=1.3$ GeV, $\langle\beta_x\rangle=4.2$ m, $\langle\beta_y\rangle=4.6$ m, $\tau_x=18.2$ ms, $\tau_y=29.2$ ms, $\epsilon_{x0}=1.1\times10^{-9}$ mrad, $\epsilon_{y0}=5.8\times10^{-11}$ mrad. The information about the bunch lengthening with respect to $N_{\rm e}$ can be obtained either from experimental results[11, 12] or from analytical results[14] shown in Fig. 2. Assuming $k_{\perp,x}(\sigma_{z0})=k_{\perp,y}(\sigma_{z0})=1020$ V/pC/m (for comparison, in BEPC, $k_{\perp,y}(\sigma_{z0})=215$ V/pC/m at $\sigma_{z0}=1.53$ cm[17]), for $\sigma_{z0}=6.8$ mm, $\sigma_X=0.42$ mm and $\sigma_Y=0.163$ mm the assumed values of σ_X and σ_Y are reasonable as compared with BEPCII quadrupole installation miss-alignment errors1), 0.2 mm required and 0.5 mm measured, here we assume that the vacuum chamber's installation miss-alignment errors are the same as that of quadrupoles, by using Eqs. (23) and (24) one fits the experimentally measured emittance growths vs the bunch population as illustrated in Figs. 3 and 4, where the experimental results correspond to the values denoted in Ref. [7] as "Wire scanner 2001/2/8". It is obvious that both the horizontal and vertical emittances' functional dependence on the bunch population can be well fitted to the experimental results. We stress that $\sigma_{X,Y}^2=\sigma_{x,y,{\rm chamber}}^2+\sigma_{x,y,{\rm co}}^2$, where $\sigma_{x,y,{\rm chamber}}$ are the vacuum chamber misalignment errors and $\sigma_{x,y,{\rm co}}$ are the closed orbit distortion errors. It is obvious that to avoid excessive emittance growth, both the closed orbit distortions and the vacuum chamber misalignment errors should be carefully controlled with the same rigour.

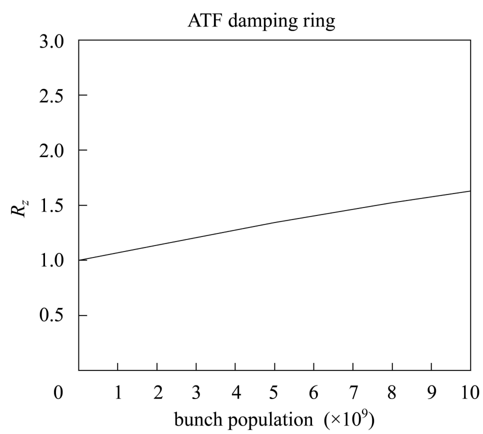

Fig. 2. Bunch lengthening vs bunch population.

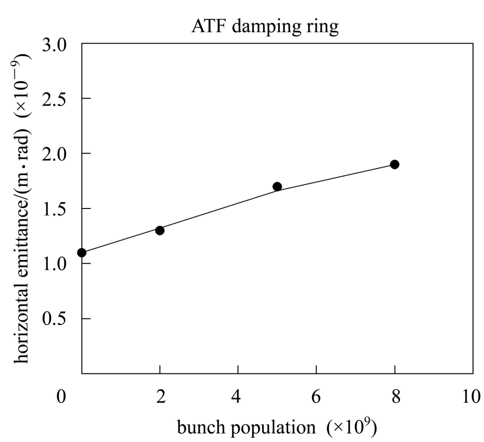

Fig. 3. Horizontal emittance vs bunch population. The dots and solid line correspond to the experimental and theoretical values, respectively.

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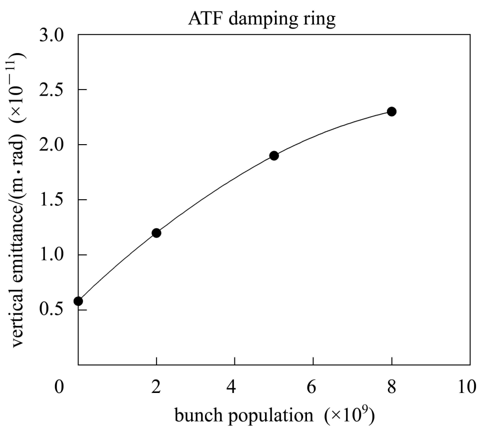

Fig. 4. Vertical emittance vs bunch population. The dots and solid line correspond to the experimental and theoretical values, respectively.

To further check the validity of this theory and to obtain more accurate values for $k_{\perp,x}(\sigma_{z0})$ and $k_{\perp,y}(\sigma_{z0})$ one has to do more experiments by varying $\sigma_{X,Y}$.

5 Conclusion

In this paper we have established a theoretical framework to explain the bunch transverse emittance growth vs the bunch population in an electron storage ring taking into account the transeverse wakefield effect, which is supplementary to intrabeam scattering theory. New equilibrium emittance equations are given and applied to explain the experimental results from the ATF damping ring.

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