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Relativistic description of single-particle resonances via phase shift analysis

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Abstract Single-particle resonant states in spherical nuclei are studied by the real stabilization method in coordinate space within the framework of self-consistent relativistic mean field theory. Taking ^{122}Zr as an example, the resonant parameters, including the energies and widths are extracted by fitting energy and phase shift. Good agreement with the previous calculations has been found. The details of single-particle resonant states are analysed.

Key words relativistic mean-field, real stabilization method, phase shift, resonance

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1 Introduction

In the past decade, the development of radioactive nuclear beams has extended our knowledge of nuclear physics from the stable nuclei to the unstable nuclei far from the stability line[1, 2]. Intense research in this area shows that there exist lots of unexpected phenomena: strange nuclear structures such as the neutron halo (skin) and the proton halo (skin)[3—11]. In these nuclei, the Fermi surface is usually close to the particle continuum, thus the contribution of the continuum and/or resonances is essential for exotic nuclear phenomena[7, 12—16]. It has been also revealed that the contribution of the continuum mainly comes from single-particle resonant states[17, 18]. Therefore, a proper treatment of resonant states is important for a deeper understanding of the properties in exotic nuclei.

So far, combined with relativistic mean-field (RMF) theory, there are several bound-state-like approaches being developed to deal with resonant states, including the scattering phase shift method (RMF-SPS)[19], analytic continuation in the coupling constant (RMF-ACCC)[20] and the real stabilization method (RMF-RSM)[21]. Furthermore, different physics quantities are proposed to extract the resonance energy and width, including phase shifts[22], density of states[23], etc.

Since ^{122}Zr is the core of the giant neutron halo predicted in the RCHB calculations[8], and the resonant states in ^{122}Zr have been obtained by the scattering method[19], this nucleus has been chosen for the present study. Recently, two simple methods proposed by Maier et al.[23] and Mandelshtam et al.[24] have been adopted in a RMF+RSM calculation to extract the resonance parameters in Ref. [25]. Good agreement with the RMF-SPS and RMF-ACCC predictions has been found.

The phase shift is a very important quantity in the study of the properties of resonant states. Therefore, it is interesting to extract the resonance parameters by explicitly evaluating the phase shifts of single-particle resonances. In this paper, the neutron resonance states in ^{122}Zr will be studied within the RMF-RSM approach in coordinate space. The wave functions, phase shifts, energies and widths of the single-particle resonant states in ^{122}Zr will be investigated, in detail.

2 The model

The standard equation of motion for the nucleon in the relativistic mean-field description of a nucleus

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is given by the Dirac equation [11],

$$[\alpha\cdot\boldsymbol{p}+V(\boldsymbol{r})+\beta(M+S(\boldsymbol{r}))]\psi_\alpha=\epsilon_\alpha\psi_\alpha \ , \qquad (1)$$

where $V(\boldsymbol{r})$ and $S(\boldsymbol{r})$ are the vector and scalar potentials, respectively. ε_α indicates the single particle energy. The Dirac spinor ψ_α, with spherical symmetry, is characterized by the angular momentum quantum numbers l, j, m, the isospin $t=\pm\frac{1}{2}$ for neutron and proton, respectively, and all the remaining other quantum numbers are summarized by i.

$$\psi_\alpha(\boldsymbol{r})=\frac{1}{r}\begin{pmatrix}\mathrm{i}G_i^{lj}(r)Y_{jm}^l(\theta,\phi)\\ -F_i^{lj}(r)(\sigma\cdot\hat{r})Y_{jm}^l(\theta,\phi)\end{pmatrix}\chi_t(t), \qquad (2)$$

where $Y_{jm}^l(\theta,\phi)$ are the spinor spherical harmonics. $G_i^{lj}(r)$ and $F_i^{lj}(r)$ are the radial wave functions for the large and small components, which are determined by the following coupled differential equations,

$$\begin{cases}\left(-\dfrac{\partial}{\partial r}+\dfrac{\kappa}{r}\right)F_i^{lj}(r)+[M+V_p(r)]\,G_i^{lj}(r)=\varepsilon_\alpha G_i^{lj}(r),\\ \left(\dfrac{\partial}{\partial r}+\dfrac{\kappa}{r}\right)G_i^{lj}(r)-[M-V_m(r)]\,F_i^{lj}(r)=\varepsilon_\alpha F_i^{lj}(r),\end{cases} \qquad (3)$$

where, $V_p(r)=V(r)+S(r)$, $V_m(r)=V(r)-S(r)$, $\kappa=(-1)^{j+l+1/2}(j+1/2)$.

With the vector $V(r)$ and scalar potentials $S(r)$ self-consistently obtained within the mean field and no-sea approximations, the Dirac equation, Eq. (3), is solved in a spherical box of the size $R_{\max}$ using the usual box boundary conditions. Thus the continuum is discretized[21]. When $R_{\max}$ is large enough, the energy of a bound state does not change with $R_{\max}$. In the continuum region, there are some states stable against changes of the size of the box, i.e., the energy of each of such states is almost constant with changing $R_{\max}$. Such stable states correspond to resonances[21].

At large distances, where both the scalar and the vector potentials are zero, the radial equations, Eq. (3), simplify to

$$\begin{cases}\dfrac{\partial^2 G_i^{lj}(r)}{\partial r^2}+\left[\beta^2-\dfrac{\kappa(\kappa+1)}{r^2}\right]G_i^{lj}=0\,,\\ \dfrac{\partial^2 F_i^{lj}(r)}{\partial r^2}+\left[\beta^2-\dfrac{\kappa(\kappa-1)}{r^2}\right]F_i^{lj}=0\,,\end{cases} \qquad (4)$$

where $\beta^2=\varepsilon_\alpha^2-M^2$. The solutions are given by

$$G\ =\ C\beta r\,[\cos(\delta)j_k(\beta r)-\sin(\delta)n_k(\beta r)]\,, \qquad (5)$$

$$F\ =\ D\beta r\,[\cos(\delta)j_{k-1}(\beta r)-\sin(\delta)n_{k-1}(\beta r)]\,, \qquad (6)$$

where C and D are two constants. The phase shift δ is calculated by matching the wave functions obtained by solving Eq. (3) numerically with those obtained by solving Eq. (4) analytically. On the other hand, the phase shift, at energies near a resonance energy, is assumed to have the form[22]

$$\delta_l(E)=\delta_{l,\mathrm{pot}}(E)+\tan^{-1}\left(\frac{\Gamma/2}{E-E_\gamma}\right), \qquad (7)$$

where E_γ and Γ are the energy and the width of the resonance, respectively. The contribution of potential scattering to the phase shift $\delta_{l,\mathrm{pot}}(E)$ is assumed to be a slowly varying function of E near E_γ, and is usually fitted with a polynomial in E. Using Eq. (5), one can determine the phase shift $\delta_l(E)$ for a given E. The resonance parameter: E_γ and Γ, as well as $\delta_{l,\mathrm{pot}}(E)$ can thus be determined by fitting the phase shift to the function given in Eq. (7).

3 Results and discussion

In this section, taking ^{122}Zr as an example, the phase shifts δ_l and the single-particle wave functions of resonance states will be calculated within the RMF+RSM. Based on the phase shift as a function of energy, the corresponding resonance parameters will be determined. The effective interaction NL3[26] is adopted in the RMF calculation for single-particle states. By increasing the box size from 10 fm to 31 fm, the resonance states are determined based on the RSM.

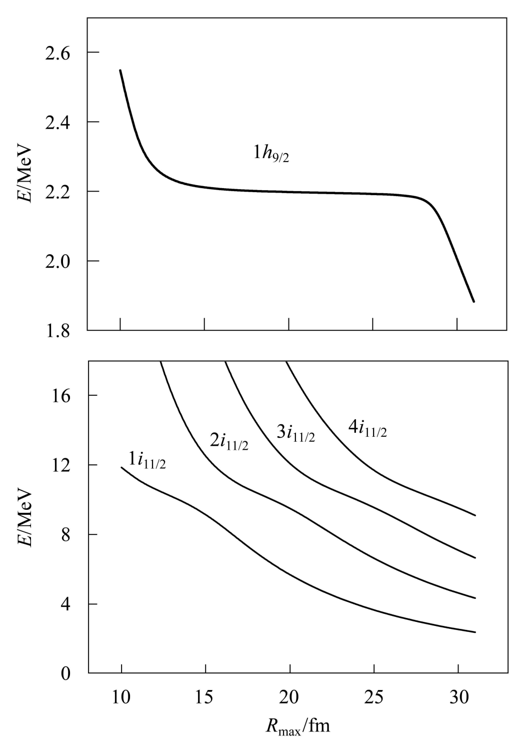

Fig. 1. The energies for $1h_{9/2}$ and $1i_{11/2}$ states in ^{122}Zr as functions of box size $R_{\max}$.

Figure 1 shows the changing energies for the $1h_{9/2}$ and $1i_{11/2}$ states against the box size in ^{122}Zr. It shows that the resonant energy of the $1h_{9/2}$ state is located between 2.190 MeV and 2.202 MeV, while for the $1i_{11/2}$ state, it is between 9.5 MeV and 11.5 MeV.

Given an energy E in this region, it corresponds to a box size $R_{\max}$, in which case, the single-particle wave functions, including the large and small components G and F, are determined by Eq. (3). Thus the obtained wave function G at any point r in this region (where the nuclear potential approximately vanishes) can be substituted into Eq. (5) and the phase shift δ can be determined. Fig. 2 shows the phase shift δ as a function of energy. The energies E_γ and widths Γ of these two resonant states, $1h_{9/2}$ and $1i_{11/2}$, are calculated by fitting their phase shifts to Eq. (7) and obtained as $E_\gamma = 2.196$ MeV, $\Gamma = 0.009$ MeV, and $E_\gamma = 10.352$ MeV, $\Gamma = 2.260$ MeV, respectively.

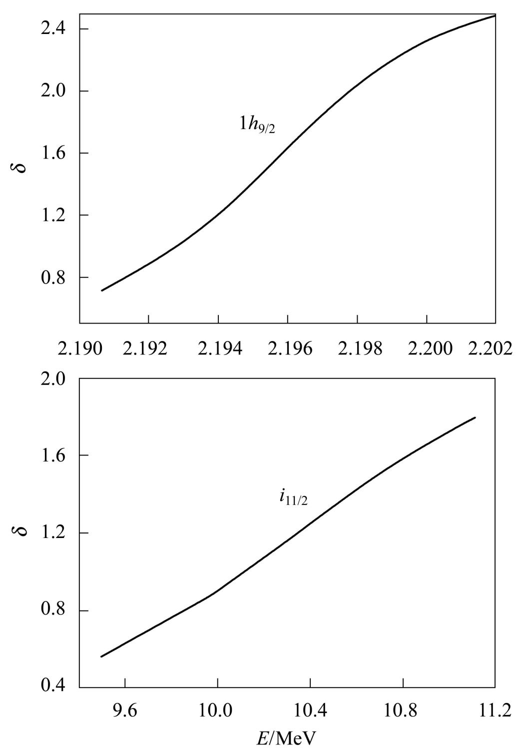

Fig. 2. The phase shifts δ for the $1h_{9/2}$ and $1i_{11/2}$ states in ^{122}Zr as functions of energy.

Figure 3 shows the wave functions for the $1h_{9/2}$ and $1i_{11/2}$ states in ^{122}Zr. The large (solid line) and small (dotted line) components with $r < R_{\max}$ are obtained from Eq. (3). Those with $r > R_{\max}$, as well as the large component (dashed line) of the asymptotic wave function, are obtained from Eq. (5). $R_{\max}$ corresponds to the box size of resonant energy in Fig. 1. It has to be pointed out that a proper choice of the matching point r is essential. The potential at the matching point should vanish. Fig. 3 shows that for the $1h_{9/2}$ and $1i_{11/2}$ states, the bound-state-like wave functions and the asymptotic wave functions in the region with $r > 10$ fm are coincidental.

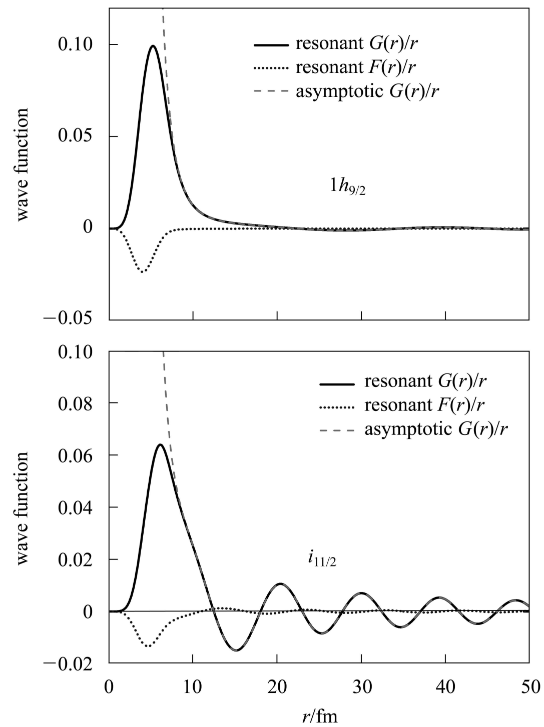

Fig. 3. The wave functions for the $1h_{9/2}$ and $1i_{11/2}$ states in ^{122}Zr. The large (solid line) and small (dotted line) components with $r < R_{\max}$ are obtained from Eq. (3). Those with $r > R_{\max}$, as well as the large component (dashed line) of the asymptotic wave function, are obtained from Eq. (5). $R_{\max}$ corresponds to the box size of resonant energy in Fig. 1.

The calculated energies and widths of the single neutron resonant states, $\nu p_{1/2}$, $\nu f_{5/2}$, $\nu h_{9/2}$, $\nu i_{13/2}$, $\nu i_{11/2}$ in ^{122}Zr are given in Table 1. It shows that $\nu f_{5/2}$, $\nu i_{13/2}$ and $\nu i_{11/2}$ are relatively wide resonant states, while $\nu p_{1/2}$ and $\nu h_{9/2}$ are much narrower. For comparison, the energies and widths of the resonant states predicted by ACCC[20] and SPS[19] are included in the 2nd and 3rd rows as well. Similar RSM calculations[25] but with different effective interaction PK1[27] for the RMF and different methods[23, 24] to extract the resonance parameters are listed in the 4th—5th rows. Although the effective interactions and methods to extract the resonance parameters are quite different, the agreement for the predicted energies and widths of the resonant states is quite good, in particular for $\nu h_{9/2}$, $\nu i_{13/2}$, and $\nu i_{11/2}$ in ^{122}Zr.

Table 1. Predicted energies and widths of single neutron resonant states, $\nu p_{1/2}$, $\nu f_{5/2}$, $\nu h_{9/2}$, $\nu i_{13/2}$, $\nu i_{11/2}$ in ^{122}Zr in comparison with SPS[19], ACCC[20] and RSM[25], respectively.

	$p_{1/2}$		$f_{5/2}$		$h_{9/2}$		$i_{13/2}$		$i_{11/2}$	
	E	Γ	E	Γ	E	Γ	E	Γ	E	Γ
RSM	0.089	0.013	1.210	0.204	2.196	0.009	5.585	0.101	10.352	2.260
SPS[19]	—	—	1.698	0.497	2.485	0.013	5.783	0.097	—	—
ACCC[20]	0.475	0.004	1.6	0.4	2.43	0.014	5.66	0.1	—	—
RSM[25]	—	—	1.419	0.275	2.431	0.011	5.907	0.111	10.648	2.168
RSM[25]	—	—	1.412	0.116	2.430	0.007	5.907	0.092	10.646	1.083

4 Summary

In summary, the RMF-RSM approach has been applied to study the resonance states in ^{122}Zr. The Dirac equation for the neutron is solved in the coordinate space under the box boundary condition. The resonant states are singled out by studying the stable behavior of the positive energy states against changes of the box size. Wave functions and phase shifts of the resonant states are obtained by matching the bound-state-like wave function with the asymptotic solution. The energies and widths are extracted by fitting energy and phase shift. The predicted results are compared with those obtained by the SPS[19], ACCC[20] and RSM[21, 25] approaches and excellent agreement is found.

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