Study of semileptonic decays $B_s \to D_{s1}(2536)l\bar{\nu}$ and $B_s \to D_{s2}(2573)l\bar{\nu}$ in the CQM model*

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Abstract Based on the assumption that $D_{s1}(2536)$ and $D_{s2}(2573)$ belong to T doublet $(1^+, 2^+)$, we calculate the semileptonic decays of B_s to $D_{s1}(2536)$ and $D_{s2}(2573)$ in terms of the Constituent Quark Meson (CQM) model. For $B_s \to D_{s1}(2536) + l\bar{\nu}$ and $B_s \to D_{s2}(2573) + l\bar{\nu}$, the order of magnitude of the obtained branching ratios is 10^{-3}. Our numerical results of the semileptonic decays of B_s to $D_{s1}(2536)$ and $D_{s2}(2573)$ are large, which implies that two semileptonic decays should be seen in future experiments.

Key words semileptonic decay, CQM model, branching ratios

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1 Introduction

In recent years, both theorists and experimentalists of high energy physics show great interest in the discoveries of exotic mesons $D_{s1}(2536)$ and $D_{s2}(2573)$ which are made up of c and $\bar{s}$[1, 2]. Using heavy quark spin-flavor symmetry, one classifies heavy $Q\bar{q}$ mesons into doublets which can be labeled by the value of the angular momentum s_l of the light degrees of freedom: $s_l = s_{\bar{q}} + l$, with $s_{\bar{q}}$ being the light antiquark spin and l the orbital angular momentum of the light degrees of freedom relative to the heavy quark[3—5]. s_l^P can be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$, when the value of l is 1. Assuming that $D_{s1}(2536)$ and $D_{s2}(2573)$ can be embedded into the $J^P_{s_l} = (1^+, 2^+)_{3/2}$ doublet, Colangelo P et al. studied the transitions $D^+_{s2} \to D^{(*)+}K^0$, $D^{(*)0}K^+$ and $D^+_{s1} \to D^{*+}K^0$, $D^{*0}K^+$ in the Heavy Quark Effective Theory (HQET)[3]. For the width of $D_{s1}(2536)$ the present bound $\Gamma(D_{s1}(2536)) < 2.3$ MeV is given in the data book[6]. Recently, new measurements of the properties of the spin two $D_{s2}(2573)$ meson were reported by the BaBar Collaboration[7], particularly the decay width $\Gamma(D_{s2}(2573) \to DK) = 27.1 \pm 0.6 \pm 5.6$ MeV. The full width of the meson $D_{s2}(2573)$ announced by the Particle Data Group is 15^{+5}_{-4} MeV. So the structures and the properties of exotic mesons $D_{s1}(2536)$ and $D_{s2}(2573)$ are interesting topics, and one needs to use various theoretical approaches to study the productions and decays of $D_{s1}(2536)$ and $D_{s2}(2573)$.

By the QCD sum rules, Aliev A M et al. studied the semileptonic decays $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ and obtained large branching ratios[8, 9]. However, with other reasonable models, studies on the semileptonic decays B_s to D_s family may be useful. In the CQM model, we compute $B_s \to D^{(*)}_s(1968, 2112)l\bar{\nu}$ and $B_s \to D_{sJ}(2317, 2460)l\bar{\nu}$ and get reasonable results[10]. Thus the studies on the production of the higher exotic states $D_{s1}(2536)$ and $D_{s2}(2573)$ are also important, which can not only deepen our understanding about the properties of these states, but also test the reliability of models which are used to calculate the semileptonic decays. The Large Hadron Collider (LHC) is running, from which large amount of data of B_s can be produced. Thus it would be realistic to obtain the information of $B_s \to D_{s1}(2536)l\bar{\nu}$ and $B_s \to D_{s2}(2573)l\bar{\nu}$.

In this work, we calculate the semileptonic decays $B_s \to D_{s1}(2536)l\bar{\nu}$ and $B_s \to D_{s2}(2573)l\bar{\nu}$ in terms of the CQM model. By the assumption that $(D_{s1}(2536), D_{s2}(2573))$ with spin-parity $(1^+, 2^+)$ can be supposed as $T(1^+, 2^+)$ doublet in HQET, we com-

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plete the studies on the semileptonic decays of $\mathrm{B_s}$ to $\mathrm{D_{s1}}(2536)$ and $\mathrm{D_{s2}}(2573)$.

The CQM model was proposed by Polosa et al.[11] and has been well developed later based on the work of Ebert et al[12]. The effective Lagrangian which incorporates the flavor-spin symmetry for heavy quarks with the chiral symmetry for light quarks is the base of the CQM model. Using the CQM model, the authors[13, 14] have got reasonable results of heavy meson physics, so we can believe that the model is appropriate to our processes and expect to get relatively reasonable results.

The paper is organized as follows. After the introduction, in Section 2, we present our formulation and in Section 3, all the input parameters and the corresponding numerical results of the semileptonic decays $\mathrm{B_s}\to\mathrm{D_{s1}}(2536)\mathrm{l}\bar{\nu}$ and $\mathrm{B_s}\to\mathrm{D_{s2}}(2573)\mathrm{l}\bar{\nu}$ are explicitly given. The last section is devoted to a simple discussion and conclusion. Some detailed expressions are collected in the appendix.

2 Formulation

For the convenience of readers, we give a brief introduction of the CQM model[11]. The model is relativistic and based on an effective Lagrangian which combines the HQET and the chiral symmetry for light quarks

$$\begin{aligned}\mathcal{L}_{\mathrm{CQM}} =\ & \bar{\chi}[\gamma\cdot(\mathrm{i}\partial+V)]\chi+\bar{\chi}\gamma\cdot A\gamma_5\chi-\\ & m_{\mathrm{q}}\bar{\chi}\chi+\frac{f_\pi^2}{8}\mathrm{Tr}[\partial^\mu\Sigma\partial_\mu\Sigma^+]+\bar{h}_{\mathrm{v}}(\mathrm{i}v\cdot\partial)h_{\mathrm{v}}-\\ & [\bar{\chi}(\bar{H}+\bar{S}+\mathrm{i}\bar{T}^\mu\frac{\partial_\mu}{\Lambda_\chi})h_{\mathrm{v}}+\mathrm{h.c.}]+\\ & \frac{1}{2G_3}\mathrm{Tr}[(\bar{H}+\bar{S})(H-S)]+\frac{1}{2G_4}\mathrm{Tr}[\bar{T}^\mu T_\mu],\end{aligned} \tag{1}$$

where the fifth term is the kinetic term of heavy quarks with $\not{v}h_{\mathrm{v}}=h_{\mathrm{v}}$; H and S denote the superfields corresponding to doublets $(0^-,1^-)$ and $(0^+,1^+)$ respectively. The explicit matrix representations of H and S read as

$$H = \frac{1+\not{v}}{2}[P_\mu^*\gamma^\mu-P\gamma_5], \tag{2}$$

$$S = \frac{1+\not{v}}{2}[P_{1\mu}^{*'}\gamma^\mu\gamma_5-P_0], \tag{3}$$

where P, $P^{*\mu}$, P_0 and P_1 are the annihilation operators of pseudoscalar, vector, scalar and axial vector mesons which are normalized as

$$\begin{aligned}\langle 0|P|M(0^-)\rangle &= \sqrt{M_H},\langle 0|P^{*\mu}|M(1^-)\rangle=\sqrt{M_H}\epsilon^\mu,\\ \langle 0|P_0|M(0^+)\rangle &= \sqrt{M_S}\gamma_5,\langle 0|P_1^{*\mu}|M(1^+)\rangle=\sqrt{M_S}\gamma_5\epsilon^\mu.\end{aligned}$$

T is the super-field corresponding to the doublet $(1^+,2^+)$

$$\begin{aligned}T^\alpha =\ & \frac{1+\not{v}}{2}\Bigg[P_2^{*\alpha\beta}\gamma_\beta-\\ & \sqrt{\frac{3}{2}}P_{1\beta}^*\gamma_5\left(g^{\alpha\beta}-\frac{1}{3}\gamma^\beta(\gamma^\alpha-v^\alpha)\right)\Bigg].\end{aligned} \tag{4}$$

$\chi=\xi q(q=u,d,s)$ is the light quark field and $\xi=\mathrm{e}^{\frac{\mathrm{i}M}{f_\pi}}$, and M is the octet pseudoscalar matrix. We also have

$$V^\mu = \frac{1}{2}(\xi^\dagger\partial^\mu\xi+\xi\partial^\mu\xi^\dagger), \tag{5}$$

$$A^\mu = \frac{-\mathrm{i}}{2}(\xi^\dagger\partial^\mu\xi-\xi\partial^\mu\xi^\dagger). \tag{6}$$

Here we also present the Lagrangian of the decays $\mathrm{H}\to\mathrm{H'M}$, $\mathrm{S}\to\mathrm{H'M}$, $\mathrm{T}\to\mathrm{H'M}$ at leading order of the heavy quark expansion[3].

$$\begin{aligned}\mathcal{L}_H &= g\mathrm{Tr}[\bar{H}_aH_b\gamma_\mu\gamma_5A_{ba}^\mu],\\ \mathcal{L}_S &= h\mathrm{Tr}[\bar{H}_aS_b\gamma_\mu\gamma_5A_{ba}^\mu]+\mathrm{h.c.},\\ \mathcal{L}_T &= \frac{h'}{\Lambda_\chi}\mathrm{Tr}[\bar{H}_aT_b^\mu(\mathrm{i}D_\mu/A+\mathrm{i}/DA_\mu)_{ba}\gamma_5]+\mathrm{h.c.}.\\ D_{\mu ba} &= -\delta_{ba}\partial_\mu+\frac{1}{2}(\xi^\dagger\partial^\mu\xi+\xi\partial^\mu\xi^\dagger)_{ba}.\end{aligned} \tag{7}$$

Categorizing $\mathrm{D_{s1}}(2536)$ and $\mathrm{D_{s2}}(2573)$ as the T-type doublet $(1^+,2^+)$, we compute the semileptonic decays of $\mathrm{B_s}\to\mathrm{D_{s1}}(2536)+\mathrm{l}\bar{\nu}$ and $\mathrm{B_s}\to\mathrm{D_{s2}}(2573)+\mathrm{l}\bar{\nu}$. To calculate the semileptonic decays of $\mathrm{B_s}$ to $\mathrm{D_s}$ mesons, the four-fermion operator of $\mathrm{b}\to\mathrm{c}+\mathrm{l}\bar{\nu}$ is necessary and reads as[15]

$$\mathcal{O}=\frac{G_{\mathrm{F}}V_{cb}}{\sqrt{2}}\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\nu}\gamma_\mu(1-\gamma_5)l. \tag{8}$$

At hadron level, the transition amplitudes of $\mathrm{B_s}\to\mathrm{D_{s1}}(2536)\mathrm{l}\bar{\nu}$ and $\mathrm{B_s}\to\mathrm{D_{s2}}(2573)\mathrm{l}\bar{\nu}$ can be written in the following form

$$\begin{aligned}\mathcal{M} =\ & \langle\mathrm{D_s^{(*)}}\mathrm{l}\bar{\nu}|\mathcal{O}|\mathrm{B_s}\rangle=\\ & \frac{G_{\mathrm{F}}V_{cb}}{\sqrt{2}}\langle\mathrm{D_s^{(*)}}|\bar{c}\gamma^\mu(1-\gamma_5)b|\mathrm{B_s}\rangle\times\\ & \langle l\bar{\nu}|\bar{\nu}\gamma_\mu(1-\gamma_5)l|0\rangle,\end{aligned} \tag{9}$$

where the hadronic matrix element is related to non-perturbative QCD effects. In the HQET symmetries, the hadronic matrix elements can be expressed as

$$\begin{aligned}&\langle\mathrm{D_1^*}(v',\epsilon)|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle=\sqrt{\frac{M_{\mathrm{B}}M_{\mathrm{D_1^*}}}{2}}\times\\ &\tau_{3/2}(\omega)\Big\{(\omega^2-1)\epsilon_\mu^*+(\epsilon^*\cdot v)[3v_\mu-(\omega-2)v'_\mu]-\\ &\mathrm{i}(\omega+1)\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma\Big\},\end{aligned} \tag{10}$$

$$\langle D_2^*(v',\epsilon')|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle = \sqrt{3M_B M_{D_2^*}}\times \tau_{3/2}(\omega)\Big\{i\varepsilon_{\mu\alpha\beta\gamma}\epsilon'^{*\alpha\eta}v_\eta v'^\beta v^\gamma - [(\omega+1)\epsilon'^*_{\mu\alpha}v^\alpha - \epsilon'^*_{\alpha\beta}v^\alpha v^\beta v'_\mu]\Big\}, \tag{11}$$

where $\omega = v\cdot v'$. In HQET, the dimensionless probability function $\tau_{3/2}(\omega)$ is the famous Isgur-Wise function. Then, the central task in the CQM model is how to extract the Isgur-Wise function from the loop integrals.

In the CQM model, the hadronic matrix element $\langle D_s^{(*)}|\bar{c}\gamma^\mu(1-\gamma_5)b|B_s\rangle$ can be described by the following Feynman diagram (Fig. 1).

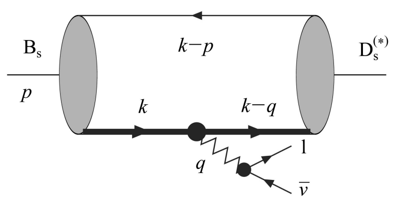

Fig. 1. The Feynman diagram depicts the decays of $B_s \to D_s^{(*)}l\bar{\nu}$. The thick-line denotes the heavy quark propagator.

According to the CQM model[11], the couplings of B_s and $D_{s1}(2536)$ and $D_{s2}(2573)$ with light and heavy quarks are expressed as

$$\frac{1+\not{v}}{2}\sqrt{Z_H M_{B_s}}\gamma_5, \tag{12}$$

$$\frac{1+\not{v}}{2}\sqrt{Z_T M_{D_{s1}}}\sqrt{\frac{3}{2}}\epsilon_\beta\gamma_5\left(g^{\alpha\beta}-\frac{1}{3}\gamma^\beta(\gamma^\alpha-v^\alpha)\right)\frac{i\partial_\alpha}{\Lambda_\chi}, \tag{13}$$

$$\frac{1+\not{v}}{2}\sqrt{Z_T M_{D_{s2}}}\,\epsilon'^{\alpha\beta}\gamma_\beta\frac{i\partial_\alpha}{\Lambda_\chi}, \tag{14}$$

where ϵ denotes the polarization vector of $D_{s1}(2536)$, and ϵ' is the tensor of $D_{s2}(2573)$. Z_H and Z_T are the renormalization constants corresponding to S and T doublets respectively, whose concrete expressions are presented as[11]

$$Z_H^{-1} = (\Delta_H+m)\frac{\partial I_3(\Delta_H)}{\partial\Delta_H}+I_3(\Delta_H), \tag{15}$$

$$Z_T^{-1} = \frac{1}{\Lambda_\chi^2}\left[(\Delta_T^2-m^2)\left((m+\Delta_T)\frac{\partial I_3(\Delta_T)}{\partial\Delta_T}+I_3(\Delta_T)\right)+ (m+\Delta_T)\left(\frac{\partial I_0(\Delta_T)}{\partial\Delta_T}+I_1+2\Delta_T I_3(\Delta_T)\right)+ I_0+\Delta_T I_1\right], \tag{16}$$

where $\Lambda_\chi = 1$ GeV, $N_c = 3$ and m is the mass of the light quark s. To save space the explicit expressions of the functions I_0, I_1 and I_3 are collected in appendix.

Δ_H is a free CQM parameter, which can be fixed by reasonable numerical values, and Δ_T is the function of Δ_H.

Now the hadronic transition matrix element can be written as

$$M^\mu = \frac{(i^5)}{4}\sqrt{M_{B_s}M_{D_s}}\sqrt{Z_H Z_T}N_c\int^{\mathrm{reg}}\frac{d^4k}{(2\pi)^4}\times \frac{\mathrm{Tr}\left[(\not{k}+m)\Gamma'(1+\not{v}')\gamma^\mu(1+\not{v})\gamma^5\right]}{(k^2-m^2)(v'\cdot k+\Delta_H)(v\cdot k+\Delta_H)}, \tag{17}$$

where Γ' should be $\sqrt{\frac{3}{2}}\epsilon_\beta\gamma_5\left(g^{\alpha\beta}-\frac{1}{3}\gamma^\beta(\gamma^\alpha-v^\alpha)\right)\frac{i\partial_\alpha}{\Lambda_\chi}$ and $\epsilon'^{\alpha\beta}\gamma_\beta\frac{i\partial_\alpha}{\Lambda_\chi}$ correspond to $D_{s1}(2536)$ and $D_{s2}(2573)$ respectively.

In order to retrench space in the text, one omits the technical details. After tedious calculation, one finally obtains the Isgur-Wise function $\tau_{3/2}(\omega)$ from Eq. (17) by matching the Eqs. (10) and (11).

$$\tau_{3/2}(\omega) = -\frac{\sqrt{Z_H Z_T}}{\sqrt{3}}\Bigg[m\Bigg(\frac{1}{2(1-\omega)}(I_3(\Delta_H)- I_3(\Delta_T)-(\Delta_H-\Delta_T)I_5(\Delta_H,\Delta_T,\omega))- \frac{1}{2(1+\omega)}(I_3(\Delta_H)+I_3(\Delta_T)+ (\Delta_H+\Delta_T)I_5(\Delta_H,\Delta_T,\omega))\Bigg)- \frac{1}{2(-1-\omega+\omega^2+\omega^3)}(-3S(\Delta_H,\Delta_T,\omega)- (1-2\omega)S(\Delta_T,\Delta_H,\omega)+(1-\omega^2)T(\Delta_H,\Delta_T,\omega)- 2(1-2\omega)U(\Delta_H,\Delta_T,\omega))\Bigg], \tag{18}$$

where S,T,U being linear combinations of I_i, are listed in appendix.

With Eqs. (9)—(11), we deduce the decay widths of the semileptonic decays $B_s \to D_{s1}(2536)l\bar{\nu}$ and $B_s \to D_{s2}(2573)l\bar{\nu}$.

$$d\Gamma\ (B_s\to D_{s1}(2536)l\bar{\nu}) = \frac{G_F^2|V_{cb}|^2}{24\pi^3}M_{D_{s1}}^3\sqrt{(\omega^2-1)}(1+\omega)\tau_{3/2}^2(\omega)\times \big((M_{B_s}^2+M_{D_{s1}}^2)(2\omega^3-\omega^2-2\omega+1)- 2M_{B_s}M_{D_{s1}}(\omega^4-\omega^3+\omega-1)\big)d\omega,$$

$$d\Gamma\ (B_s\to D_{s2}(2573)l\bar{\nu}) = \frac{G_F^2|V_{cb}|^2}{24\pi^3}M_{D_{s2}}^3\sqrt{(\omega^2-1)}(\omega-1)(1+\omega)^2\times \tau_{3/2}^2(\omega)\big((M_{B_s}^2+M_{D_{s2}}^2)(4\omega+1)- 2M_{B_s}M_{D_{s2}}(3\omega^2+\omega+1)\big)d\omega\ .$$

Table 1. The values of Δ_H are taken from[16] and those of Δ_T are calculated from[3, 11]. The values of Z_S and Z_H are obtained from the Eqs. (15) and (16). BR_1 and BR_2 respectively denote the the branching ratios of $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$.

Δ_H/GeV	Δ_T/GeV	Z_H/GeV^{-1}	Z_T/GeV^{-1}	BR_1	BR_2
0.5	0.9	3.99	2.23	2.4×10^{-3}	3.4×10^{-3}
0.6	1.0	2.69	1.29	3.1×10^{-3}	4.4×10^{-3}
0.7	1.1	1.74	0.68	3.7×10^{-3}	5.7×10^{-3}

3 Numerical results

In this section, we present our numerical results. With the formulation we derived in the last section, we numerically evaluate the corresponding decay rates. The input parameters are taken as follows: $G_{\mathrm{F}} = 1.1664\times10^{-5}$ GeV^{-2}, $V_{\mathrm{cb}} = 0.043$, $M_{\mathrm{B_s}} = 5.3696$ GeV, $M_{\mathrm{D_{s1}}} = 2.536$ GeV, $M_{\mathrm{D_{s2}}} = 2.573$ GeV[6], $m = 0.5$ GeV, the UV cutoff $\Lambda = 1.25$ GeV, the IR cutoff $\mu = 0.5$ GeV and $\Delta_T - \Delta_H \sim 0.4$ GeV.

We present the numerical results of the semileptonic decays $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$ in the table.

4 Discussion and conclusion

In this work, the semileptonic decays $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$ are studied in the CQM Model. In order to complete the calculation of the semileptonic decays $\mathrm{B_s}$ to $\mathrm{D_{s1}(2536)}$ and $\mathrm{D_{s2}(2573)}$, we make an assumption that $\mathrm{D_{s1}(2536)}$ and $\mathrm{D_{s2}(2573)}$ belong to T doublet $(1^+, 2^+)$.

For $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$, the order of magnitude of the obtained branching ratios is 10^{-3}. Comparing our results with those obtained by other groups, for the semileptonic decays $\mathrm{B_s}$ to $\mathrm{D_s}$ family, it can be found that the order of our results is reasonable. For $\mathrm{B_s \to D_s(1968)+l\bar{\nu}}$, the experimental data are $(7.9\pm2.4)\%$. Meanwhile one predicts the branching ratio of $\mathrm{B_s \to D_s^*(2112) + l\bar{\nu}}$ which should be at the same order as that of $\mathrm{B_s \to D_s(1968) + l\bar{\nu}}$. In QCD sum rules, the authors[8, 9] calculate $\mathrm{B_s \to D_{sJ}^*(2317)l\bar{\nu}}$ and $\mathrm{B_s \to D_{sJ}(2460)l\bar{\nu}}$, and the order of the corresponding results is about 10^{-3}.

$\mathrm{D_{sJ}^*(2317)}$, $\mathrm{D_{sJ}(2460)}$, $\mathrm{D_{s1}(2536)}$ and $\mathrm{D_{s2}(2573)}$ are all exotic mesons with the orbital angular momentum $l = 1$, and can be categorized as S doublet $(0^+, 1^+)$ and T doublet $(1^+, 2^+)$ respectively. So the branching ratios for the semileptonic decays $\mathrm{B_s}$ to $\mathrm{D_{sJ}^*(2317)}$ and $\mathrm{D_{sJ}(2460)}$ should be close to those of $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$. It is obvious that the order of our numerical results of $\mathrm{B_s \to D_{s1}(2536)l\bar{\nu}}$ and $\mathrm{B_s \to D_{s2}(2573)l\bar{\nu}}$ can match that of $\mathrm{B_s \to D_{sJ}^*(2317)l\bar{\nu}}$ and $\mathrm{B_s \to D_{sJ}(2460)l\bar{\nu}}$ obtained by the QCD sum rules. Both our numerical results and the analyses from the QCD sum rules imply that the semileptonic decays $\mathrm{B_s \to S + l\bar{\nu}}$ and $\mathrm{B_s \to T + l\bar{\nu}}$ have large branching ratios. In the future, these semileptonic decays will be measured in the CDF experiment and LHCb experiment by our experimental colleagues, which can make one understand the nature of those exotic $\mathrm{D_s}$ mesons deeply. Moreover, our understanding of the CQM model can also be improved by further experiments.

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Appendix A

The explicit expressions of I_i, $S(\Delta_1,\Delta_2,\omega)$, $T(\Delta_1,\Delta_2,\omega)$ and $U(\Delta_1,\Delta_2,\omega)$ which are related to our calculations are listed

$$I_0(\Delta) = \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}}\frac{\mathrm{d}^4k}{v\cdot k+\Delta+\mathrm{i}\epsilon} = \frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}s}{s^{3/2}}\mathrm{e}^{-s(m^2-\Delta^2)}\times \left(\frac{3}{2s}+m^2-\Delta^2\right)[1+\mathrm{erf}(\Delta\sqrt{s})] - \Delta\frac{N_c m^2}{16\pi^2}\Gamma\left(-1,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right), \quad \text{(A1)}$$

$$I_1 = \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}}\frac{\mathrm{d}^4k}{k^2-m^2} = \frac{N_c m^2}{16\pi^2}\Gamma\left(-1,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right), \quad \text{(A2)}$$

$$I_3(\Delta) = -\frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}}\frac{\mathrm{d}^4k}{(k^2-m^2)(v\cdot k+\Delta+\mathrm{i}\epsilon)} = \frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}s}{s^{3/2}}\mathrm{e}^{-s(m^2-\Delta^2)}\times (1+\mathrm{erf}(\Delta\sqrt{s})), \quad \text{(A3)}$$

$$I_5(\Delta_1,\Delta_2,\omega) = \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}}\times \frac{\mathrm{d}^4k}{(k^2-m^2)(v\cdot k+\Delta_1+\mathrm{i}\epsilon)(v'\cdot k+\Delta_2+\mathrm{i}\epsilon)} = \int_0^1\mathrm{d}x\frac{1}{1+2x^2(1-\omega)+2x(\omega-1)}\times \left[\frac{6}{16\pi^2}\int_{1/\Lambda^2}^{1/\mu^2}\mathrm{d}s\,\mathrm{e}^{-s(m^2-2\sigma^2)}s^{-1} + \frac{6}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\mathrm{d}s\,\sigma\,\mathrm{e}^{-s(m^2-\sigma^2)}s^{-1/2}\times (1+\mathrm{erf}(\sigma\sqrt{s}))\right], \quad \text{(A4)}$$

$$I_6(\Delta_1,\Delta_2,\omega) = \frac{\mathrm{i}N_c}{16\pi^4}\int^{\mathrm{reg}}\times \frac{\mathrm{d}^4k}{(v\cdot k+\Delta_1+\mathrm{i}\epsilon)(v'\cdot k+\Delta_2+\mathrm{i}\epsilon)} = I_1\int_0^1\mathrm{d}x\frac{\sigma}{1+2x^2(1-\omega)+2x(\omega-1)} - \frac{N_c}{16\pi^{3/2}}\int_0^1\mathrm{d}x\frac{\sigma}{1+2x^2(1-\omega)+2x(\omega-1)}\times \int_{1/\Lambda^2}^{1/\mu^2}\frac{\mathrm{d}s}{s^{3/2}}\mathrm{e}^{-s(m^2-\sigma^2)}\times \left\{\sigma[1+\mathrm{erf}(\sigma\sqrt{s})]\cdot[1+2s(m^2-\sigma^2)] + 2\sqrt{\frac{s}{\pi}}\mathrm{e}^{-s\sigma^2}\left[\frac{3}{2s}+(m^2-\sigma^2)\right]\right\}. \quad \text{(A5)}$$

$$S(\Delta_1,\Delta_2,\omega) = \Delta_1 I_3(\Delta_2)+\omega(I_1+\Delta_2 I_3(\Delta_2)) + \Delta_1^2 I_5(\Delta_1,\Delta_2,\omega), \quad \text{(A6)}$$

$$T(\Delta_1,\Delta_2,\omega) = m^2 I_5(\Delta_1,\Delta_2,\omega)+I_6(\Delta_1,\Delta_2,\omega), \quad \text{(A7)}$$

$$U(\Delta_1,\Delta_2,\omega) = I_1+\Delta_2 I_3(\Delta_2)+\Delta_1 I_3(\Delta_1) + \Delta_1\Delta_2 I_5(\Delta_1,\Delta_2,\omega). \quad \text{(A8)}$$

In these equations, Γ, $\mathrm{erf}(z)$ and σ are defined by:

$$\Gamma(\alpha,x_0,x_1) = \int_{x_0}^{x_1}\mathrm{e}^{-t}t^{\alpha-1}, \quad \mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z\mathrm{d}x\mathrm{e}^{-x^2},$$

$$\sigma(x,\Delta_1,\Delta_2,\omega) = \frac{\Delta_1(1-x)+\Delta_2 x}{\sqrt{1+2(\omega-1)x+2(1-\omega)x^2}}. \quad \text{(A9)}$$