# Variation of the fine-structure constant from the de Sitter invariant special relativity*

CHEN Shao-Xia(陈绍霞) XIAO Neng-Chao(肖能超) YAN Mu-Lin(闫沐霖)[1)]

(Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei 230026, China)

**Abstract** We discuss the variation of the fine-structure constant, $\alpha$. There are obvious discrepancies among the results of $\alpha$-variation from recent Quasi-stellar observation experiments and from the Oklo uranium mine analysis. We use dS Sitter invariant Special Relativity ($\mathcal{SR}_{c,R}$) and Dirac large number hypothesis to discuss this puzzle, and present a possible solution to the disagreement. By means of the observational data and the discussions presented in this paper, we estimate the radius of the Universe in $\mathcal{SR}_{c,R}$ which is about $\sim 2\sqrt{5}\times 10^{11}$l.y.

**Key words** quasi-stellar objects, Oklo nature nuclear reactor, Dirac large number hypothesis, de Sitter invariant special relativity

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## 1 Introduction

The issue of a possible variation of fundamental physical constants has been put on the agenda of contemporary physics. Recently, Webb group reported[1] a varying fine-structure constant $\alpha$ through analyzing the multiple heavy element transitions in the absorption spectra of quasi-stellar objects (QSOs) on the Southern Hemisphere, which agrees with and offers vigorous support to their previous research results[1—7] on the Northern Hemisphere. Compared with the laboratory data on atomic multiplet structures, the observations of the absorption lines of QSOs reveal that the fine-structure constant $\alpha$ has an obvious change in the past several billion years. This confirmation stimulates the further study on the variation of fine-structure constant both from the theoretical interpretation[8, 9] to establish the motivation and to supply more exact model analysis, and the experimental measurements[10] to yield more precise data and more experimental methods.

The possibility of variability of fundamental constants was put forward by Dirac[11—14] in 1937, afterwards, a lot of theoretical illustrations and experimental constraints[15, 16] on the variation of fundamental constants are presented. As we know, the constancy of the fundamental constants plays a significant role in astronomy and cosmology where the redshift measures the look-back time. If ignoring the possibility that the constants are varying we will have a deviated view of our universe. However, if such variations are established, corrections should be applied to the related issues. It is thus necessary to investigate that possibility, especially as the measurements become more precise and/or when the measurements are made on the larger scale.

Besides, a general feature of extra-dimensional theories, such as Kaluza-Klein and string theories, is that the "true" fundamental constants of nature are defined in the full higher dimensional theory, so that the effective 4-dimensional "fundamental constants" depend, among other things, on the structure and sizes of the extra-dimensions. The time and/or space evolution of these sizes would have a significant result that the effective 4-dimensional "fundamental constants" will also depend on the spacetime. What's more, the achievement of experimental constraints on the variation of fundamental constants is dependent on the high energy physics models, to some extent. For two aforementioned reasons, the observation of

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1) E-mail: mlyan@ustc.edu.cn; corresponding author.

the variability of fundamental constants is one of the few ways to test directly the existence of extra-dimensions and to test high energy physics models.

Among all the possible fundamental constants of nature we will focus on the fine-structure constant, $\alpha$, which can be derived from other constants as follows[17]:

$$\alpha = \frac{e^2}{4\pi\hbar c} , \tag{1}$$

where $c$ is the speed of light, $\hbar \equiv h/2\pi$ is the reduced Planck constant, and $e$ is the electron charge magnitude. The value of $\alpha$ measured today on earth is $\alpha_0 \approx 1/137.035$[17].

As the experimental constraints on other fundamental constants, there is also inconsistence among different groups measuring the change of $\alpha$, especially between the nonzero observational results[1—7] from the absorption lines of QSOs and the null bounds[18—23] from the capture cross section of thermal neutron by $^{149}_{62}$Sm in the natural fission reactor that operated about $2\times10^9$yr ago during $(2.3\pm0.7)\times 10^5$yr in the Oklo uranium mine in Gabon. Dirac's large numbers hypothesis[11—14] conjectured that the fundamental constants are functions of the epoch, so it is clear the hypothesis is not able to disentangle the inconsistence in the different observations on its own.

The recent astronomical observations on supernovae[24, 25] and CMBR[26] manifest that about 73% of the whole energy in the Universe is dark energy and possibly contributed by a tiny positive cosmological constant ($\Lambda$). The observations strongly indicate that the Universe is described by an asymptotically de Sitter (dS) spacetime, which stimulates the interest to reconsider the de Sitter invariant special relativity with two universal constants $c$ and $R$, which is shortly denoted as $\mathcal{SR}_{c,R}$[27—31].

One basis to establish $\mathcal{SR}_{c,R}$ is the principle of special relativity: There exist a set of inertial reference frames, in which the free particles and light signals move with uniform velocity along straight lines, i.e., the inertial motion law holds true in the frames. The other is the postulate of invariant constants: There exist two invariant universal constants, i.e., speed $c$ and length $R$. The key point to set up $\mathcal{SR}_{c,R}$ is that in dS spacetime there is an important kind of coordinate system, called the Beltrami coordinate system, which is analogous to the Minkowski one in a flat spacetime. In the Beltrami coordinate system, test particles and light signals move along the timelike and null geodesics, respectively, with constant coordinate velocity. Therefore, inertial observers and classical observable quantities for these particles and signals in the Beltrami coordinate system can be well defined.

Obviously, the fine-structure constant has not undergone huge variations on Solar system scales and on geological time scales, so one is looking for measurable effects on the larger scale, such as astrophysical and astronomical scale, even cosmological one. In the meanwhile, it is also necessary for the experimental tests to be carried on the large scale for the purpose of distinguishing $\mathcal{SR}_{c,R}$ from Einstein's Special Relativity. These considerations motivate us to investigate the possibility of making use of $\mathcal{SR}_{c,R}$ to illustrate the inconsistence between the observations of variation of $\alpha$ in QSOs and in the Oklo natural reactor.

In this paper, we address the issue of variation of the fine-structure constant in the framework of $\mathcal{SR}_{c,R}$ in order to examine the possibility of solving the inconsistence between the observational constraints from the absorption spectra of QSOs and from the Oklo phenomenon. In addition, through comparing the theoretical derivation and the observational data, we crudely provide the value of the radius of the Universe, $R$.

## 2 Speed of light in de Sitter invariant special relativity

The dS spacetime can be realized as a four dimensional hypersurface embedded in a five dimensional flat space

$$\mathrm{d}s^2 = (\mathrm{d}\xi^0)^2 - (\mathrm{d}\xi^1)^2 - (\mathrm{d}\xi^2)^2 - (\mathrm{d}\xi^3)^2 - (\mathrm{d}\xi^4)^2 , \tag{2}$$

such that

$$(\xi^0)^2 - (\xi^1)^2 - (\xi^2)^2 - (\xi^3)^2 - (\xi^4)^2 = -R^2 . \tag{3}$$

The metric of the 4-dimensional spacetime in Beltrami coordinates has the form

$$\begin{aligned} \mathrm{d}s^2 &= B_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu , \\ B_{\mu\nu} &= \frac{\eta_{\mu\nu}}{\sigma(x,x)} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}x^\alpha x^\beta}{\sigma(x,x)^2 R^2} , \\ \sigma(x,y) &= 1 - \frac{\eta_{\mu\nu}x^\mu y^\nu}{R^2} > 0 , \\ \eta_{\mu\nu} &= \mathrm{diag}(1,\ -1,\ -1,\ -1) , \\ &\alpha,\ \beta,\ \mu,\ \nu = 0,\ 1,\ 2,\ 3. \end{aligned} \tag{4}$$

It is invariant under the fractional linear transformation

$$\begin{aligned} x^\mu \to \bar{x}^\mu &= \frac{\sqrt{\sigma(a)}}{\sigma(a,x)} D^\mu_\nu (x^\nu - a^\nu) , \\ D^\mu_\nu &= L^\mu_\nu + \frac{1}{R^2}\frac{L^\mu_\lambda a^\lambda \eta_{\nu\rho}a^\rho}{\sigma(a)+\sqrt{\sigma(a)}} , \\ L^\mu_\nu &\in SO(1,3) , \\ \sigma(a) &\equiv \sigma(a,a) = 1 - \frac{\eta_{\mu\nu}a^\mu a^\nu}{R^2} . \end{aligned} \tag{5}$$

Here, $(a^0,\ a^1,\ a^2,\ a^3)$ are the spacetime coordinates of the origin of the resulted inertial Beltrami frame in the original inertial Beltrami frame, and we note we use the units in which the speed of light in vacuum in Einstein's Special Relativity is $c=1$.

Considering the physical issue we investigate, we can suppose that there aren't relative motion and space rotations between two inertial Beltrami frames, and we can also reduce our discussions into two dimensions, i.e., we only translate the spacetime origin in $x^0-x^1$ plane, $a^2=a^3=0$. Under the assumptions, the coordinate transformation between two inertial frames in Betrami de Sitter spacetime is

$$\begin{aligned} x^0 &\to \bar{x}^0 = \\ &\frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\left[x^0-a^0+\frac{a^0}{R^2}\frac{a^0x^0-a^1x^1+(a^1)^2-(a^0)^2}{\sqrt{\sigma(a)}+\sigma(a)}\right], \\ x^1 &\to \bar{x}^1 = \\ &\frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\left[x^1-a^1+\frac{a^1}{R^2}\frac{a^0x^0-a^1x^1+(a^1)^2-(a^0)^2}{\sqrt{\sigma(a)}+\sigma(a)}\right], \\ x^2 &\to \bar{x}^2 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^2 , \\ x^3 &\to \bar{x}^3 = \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}x^3 . \end{aligned} \tag{6}$$

The energy and momentum of a photon (or light signal) are defined as[29, 32]

$$E \equiv \frac{\epsilon}{\sigma(x)}\frac{\mathrm{d}x^0}{\mathrm{d}\lambda} , \qquad p^i \equiv \frac{\epsilon}{\sigma(x)}\frac{\mathrm{d}x^i}{\mathrm{d}\lambda} , \tag{7}$$

where $\lambda$ is an affine parameter and $\epsilon$ is a constant. It can be proved from the geodesic equation that the energy and momentum of a photon defined in Eq. (7) are conserved along a null geodesic. The velocity of light is the ratio of the energy to the momentum of a photon

$$\tilde{c} \equiv \frac{E}{p} , \tag{8}$$

and the corresponding physical fine-structure constant becomes

$$\alpha = \frac{e^2}{4\pi\hbar\widetilde{c}} . \tag{9}$$

From the coordinate transformation Eq. (6), it is easy to derive the coordinate velocity transformation between two inertial Beltrami frames. Considering photon which moves along the direction of $\boldsymbol{p}=(p^1, 0, 0)$, by Eqs. (8) and (6) we have

$$\widetilde{c} = \frac{E}{p^1} = \frac{\mathrm{d}x^0}{\mathrm{d}x^1} . \tag{10}$$

Setting the earth to be at the origin of the spacetime frame, i.e., $x^0|_{\text{earth}} = x^1|_{\text{earth}} = 0$ (or $a^0|_{\text{earth}} = a^1|_{\text{earth}} = 0$), and QSO and Oklo are at the positions with non-zero $(a^0,\ a^1)$ (see Fig. 1). Eq. (6) serves as space-time transformation from the inertial Beltrami frame of the earth to another inertial frame with $(a^0,\ a^1)$. On the earth, we know $\widetilde{c}(a^0=0, a^1=0) \equiv c = 1$, i.e., $\mathrm{d}x^0/\mathrm{d}x^1 = 1$ (the subscript "earth" is ignored hereafter). Then we can derive the light velocity at $(a^0, a^1)$, $\widetilde{c}(a^0,a^1) \equiv \widetilde{c}$, by the velocity transformation induced from the inertial frame transformation in de Sitter special relativity Eq. (6). By Eq. (6), we have

$$\begin{aligned} \mathrm{d}\widetilde{x}^0 \;=\; & \frac{\sqrt{\sigma(a)}}{\sigma(a,x)^2}\frac{1}{R^2}(a^0\mathrm{d}x^0-a^1\mathrm{d}x^1)\times \\ & \left(x^0-a^0+\frac{a^0}{R^2}\frac{a^0x^0-a^1x^1+(a^1)^2-(a^0)^2}{\sqrt{\sigma(a)}+\sigma(a)}\right)+ \\ & \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\left(\mathrm{d}x^0+\frac{(a^0)^2\mathrm{d}x^0-a^0a^1\mathrm{d}x^1}{R^2(\sqrt{\sigma(a)}+\sigma(a))}\right), \end{aligned} \tag{11}$$

$$\begin{aligned} \mathrm{d}\widetilde{x}^1 \;=\; & \frac{\sqrt{\sigma(a)}}{\sigma(a,x)^2}\frac{1}{R^2}(a^0\mathrm{d}x^0-a^1\mathrm{d}x^1)\times \\ & \left(x^1-a^1+\frac{a^1}{R^2}\frac{a^0x^0-a^1x^1+(a^1)^2-(a^0)^2}{\sqrt{\sigma(a)}+\sigma(a)}\right)+ \\ & \frac{\sqrt{\sigma(a)}}{\sigma(a,x)}\left(\mathrm{d}x^1+\frac{-(a^1)^2\mathrm{d}x^1+a^0a^1\mathrm{d}x^0}{R^2(\sqrt{\sigma(a)}+\sigma(a))}\right). \end{aligned} \tag{12}$$

Then, by using $x^0=x^1=0$, $\mathrm{d}x^0=\mathrm{d}x^1$, we obtain the desired result:

$$\tilde{c} = \widetilde{c}(a^0,a^1) = \left.\frac{\mathrm{d}\tilde{x}^0}{\mathrm{d}\tilde{x}^1}\right|_{x^0=x^1=0,\,\mathrm{d}x^0=\mathrm{d}x^1} = \frac{1-\dfrac{(a^0)^2}{R^2}}{\sqrt{\sigma(a)}-\dfrac{a^0a^1}{R^2}} . \tag{13}$$

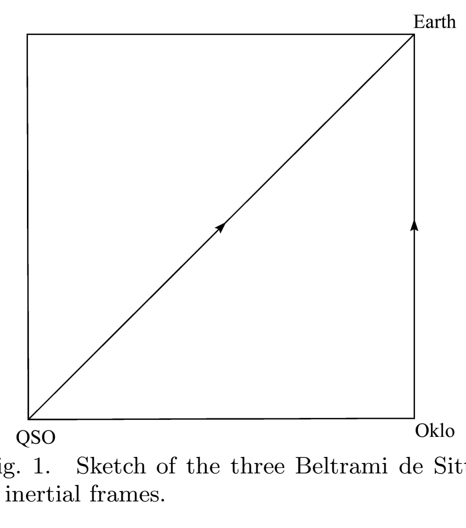


Fig. 1. Sketch of the three Beltrami de Sitter inertial frames.

Note that all derivations of $\tilde{c}$ in the above figure are based on the principle of the de Sitter invariant special relativity. The space-time coordinate transformation between the inertial references Eq. (6), which preserves the Beltrami metric $B_{\mu\nu}(x)$, plays a key role in the derivations. The calculation procedures are similar to those in deriving the velocity transformation between inertial coordinate frames in the Einstein's special relativity, in which the inertial frame transformation is Lorentz transformation which preserves Lorentz metric $\eta_{\mu\nu}$.

## 3 Conclusion

Examining the QSO frame and the earth frame, we have $a^0 = a^1$, therefore,

$$\tilde{c}_{\rm QSO} = 1 \ . \tag{14}$$

It means that the speed of light emitted from QSO to the earth is the same as that observed today on the earth.

Meanwhile, the QSO observations show that the fine-structure constant has a nonzero change $\Delta\alpha/\alpha_0 \equiv (\alpha_{\rm past} - \alpha_0)/\alpha_0 \sim -10^{-5}$, which can only come from the variation of the part $e^2/\hbar$.

The large numbers hypothesis is raised by Dirac[11—14, 16], which argued the fact that some large dimensionless numbers have the same order leads one to believe some fundamental constants vary with the epoch. Based on this hypothesis, we assume $e^2/\hbar$ is only the function of time $t$, so the variations of $\alpha$ in the QSO observations are the results due to the variations of $e^2/\hbar$, that is

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{\rm QSO} \equiv \frac{\alpha_{\rm QSO} - \alpha_0}{\alpha_0} = \frac{\frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_{\rm QSO} \times \left(\frac{1}{\tilde{c}}\right)_{\rm QSO} - \frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_0 \times \left(\frac{1}{\tilde{c}}\right)_0}{\frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_0 \times \left(\frac{1}{\tilde{c}}\right)_0} = \frac{\left(\frac{e^2}{\hbar}\right)_{\rm QSO} - \left(\frac{e^2}{\hbar}\right)_0}{\left(\frac{e^2}{\hbar}\right)_0} \sim -10^{-5} \ , \tag{15}$$

thus

$$\left(\frac{e^2}{\hbar}\right)_{\rm QSO} \sim (1 - 10^{-5}) \times \left(\frac{e^2}{\hbar}\right)_0 \ . \tag{16}$$

In the paper, the quantities with subscript 0 stand for those measured on the earth now.

In the Oklo case, the inertial Beltrami coordinate transformation is between the present earth and the frame whose origin is the spacetime point when and where the Oklo phenomenon took place, so $a^1 = 0$, and Eq. (13) has the form

$$\tilde{c}_{\rm Oklo} = \sqrt{1 - \frac{(a^0)^2}{R^2}} \ . \tag{17}$$

Since the Oklo phenomenon occurred before about $2 \times 10^9$ years, we can cursorily take

$$\left(\frac{e^2}{\hbar}\right)_{\rm QSO} = \left(\frac{e^2}{\hbar}\right)_{\rm Oklo} \ . \tag{18}$$

The Oklo constraints on the variation of fine-structure constant have null results[18—23], therefore

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{\rm Oklo} \equiv \frac{\alpha_{\rm Oklo} - \alpha_0}{\alpha_0} = \frac{\frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_{\rm Oklo} \times \left(\frac{1}{\tilde{c}}\right)_{\rm Oklo} - \frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_0 \times \left(\frac{1}{\tilde{c}}\right)_0}{\frac{1}{4\pi}\left(\frac{e^2}{\hbar}\right)_0 \times \left(\frac{1}{\tilde{c}}\right)_0} = 0 \ . \tag{19}$$

Combining Eq. (16), Eq. (18) and Eq. (19), we obtain

$$\tilde{c}_{\rm Oklo} = \frac{\left(\frac{e^2}{\hbar}\right)_{\rm Oklo}}{\left(\frac{e^2}{\hbar}\right)_0} \sim 1 - 10^{-5} \ . \tag{20}$$

Comparing theoretical calculation Eq. (17) with the result from experimental data Eq. (20), we can have the conclusion preliminarily that $\mathcal{SR}_{c,R}$ can indeed settle the inconsistence between the Oklo and the QSO observational results.

Going a step further, we can estimate the radius of the Universe from the contrast between Eq. (17) and Eq. (20) roughly

$$\sqrt{1 - \frac{(a^0)^2}{R^2}} \sim 1 - 10^{-5} \Rightarrow R \sim 2\sqrt{5} \times 10^{11} {\rm l.y.} \simeq 1.37 \times 10^{11} {\rm pc} = 1.37 \times 10^5 {\rm Mpc} > R_0 \ . \tag{21}$$

Here $R_0 \sim 10^4$ Mpc is the radius (or horizon) of present observable Universe. It is worth noticing that Eq. (21) shows $\mathcal{SR}_{c,R}$ is consistent with the available cosmological observations on the $R_0$.

In conclusion, $\mathcal{SR}_{c,R}$ is really a candidate of the solution to the inconsistence between the observational results of the QSO absorption lines and the Oklo natural reactor on the variation of the fine-structure constant, which is very different from the Einstein's Special Relativity. Furthermore, we obtain a favorable evidence to $\mathcal{SR}_{c,R}$ from the contrast of the theoretical assessment with the observational data, that is, the radius of the Universe, $R$, is greater than the radius (horizon) of the observable Universe, $R_0$. It is anticipated that as more experimental methods are applied and more precise observational data are obtained, $\mathcal{SR}_{c,R}$ will be confronted with more stringent tests, even be proved or disproved. Similarly, as $\mathcal{SR}_{c,R}$ develops, we will have a deeper insight into its application to various experiments, including the experiments on the variation of the fine-structure constant.

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