

Harmonic operation of high gain harmonic generation free electron laser^{*}

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Abstract In high gain harmonic generation (HGHG) free electron laser (FEL), with the right choice of parameters of the modulator undulator, the dispersive section and the seed laser, one may make the spatial bunching of the electron beam density distribution correspond to one of the harmonic frequencies of the radiator radiation, instead of the fundamental frequency of the radiator radiation in conventional HGHG, thus the radiator undulator is in harmonic operation (HO) mode. In this paper, we investigate HO of HGHG FEL. Theoretical analyses with universal method are derived and numerical simulations in ultraviolet and deep ultraviolet spectral regions are given. It shows that the power of the 3rd harmonic radiation in the HO of HGHG may be as high as 18.5% of the fundamental power level. Thus HO of HGHG FEL may obtain short wavelength by using lower beam energy.

Key words harmonic operation, HGHG, FEL

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1 Introduction

Free electron lasers (FELs) are devices that use the relativistic electron beams passing through a transverse periodic magnetic field, i.e. the undulator, in order to generate coherent electromagnetic radiation ranging from the infrared to hard X-ray regions^[1]. In recent years, taking self-amplified spontaneous emission (SASE)^[2] and high-gain harmonic generation (HGHG)^[3–5] as two leading candidates for approaching hard X-ray region, people are increasingly interested in the short wavelength FEL. However, the undulator period of the existing FEL is in the order of centimeters^[6–8], and is limited by the practical difficulty of placing very strong and very small magnets together in an alternating array; what is more, the FEL gain falls off rapidly with the decreasing of the undulator period. Thus, it implies that to achieve short-wavelength radiation, the conventional FEL would require a high energy electron beam, which means enormous machines, substantive costs, much time and efforts consuming.

Fortunately, according to the resonant relation-

ship of undulator, the radiation frequency scales as the harmonic number^[9]. Therefore, high harmonics would be an alternative way to obtain short-wavelength instead of high energy electron beam. High harmonics in undulator have attributed to the development of intermediate energy synchrotron radiation (SR) light source^[10]. Compared with the expensive large scale high energy SR light sources like ESRF, APS and Spring-8 where the high brightness hard X-ray radiations are generated by insertion devices operating at the fundamental and low odd harmonics (3 or 5), the intermediate energy SR light sources operating at high harmonics have lower construction and operation costs, and are able to provide comparable performance in the X-ray of 10–20 keV^[11].

Harmonics also exist in the emission spectrum of a FEL undulator operating at the fundamental mode. These kinds of harmonic radiations, nonlinear and weak, have been analytically predicted^[12], numerically simulated^[13, 14] and experimentally measured^[15]. It seems that the output power of the nonlinear harmonic radiation is about

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1% level of the fundamental in conventional FEL. Moreover, a harmonic operation (HO) FEL was proposed by Latham et al.^[16] to obtain large harmonic power, in which a signal at the harmonic frequency is injected into the entrance of FEL undulator and the harmonic radiation is dominant in undulator. Since the feasibility of Latham's scheme absolutely depends on the input signal, it will be useless in the short-wavelength spectral region for the lack of available input laser. However, such a HO scheme may be easily extended in HGHG FEL. Here, we study the HO of HGHG FEL to obtain a promising prospect in the harmonic radiation. With the right choice of the parameters in the modulator, the dispersive section and the seed laser, we make the radiator operate at one of the harmonic frequencies of its resonant wavelength. It indicates that the harmonic radiation is dominant in the radiator for a long distance, and FEL energy-conversion efficiency of the harmonic is enhanced significantly. In comparison with Latham's scheme, for an interested wavelength, a seed laser with much longer wavelength is employed in the HO of HGHG. Furthermore, it has the advantage of HGHG FEL, full longitudinal coherence, the stability of the central wavelength and pure spectrum.

This paper is organized as follows. We first briefly describe the principle of the HO of HGHG in Section 2. Then, in Section 3, using the universal method of FEL radiation, analytical estimates on the HO of HGHG FEL are given, with bunching parameters, coherent radiation, gain and efficiency included. Thirdly, the numerical examples in ultraviolet (UV) and deep ultraviolet (DUV) spectral regions are given in Section 4. Finally, we present our conclusions in Section 5.

2 The principle of HO of HGHG

Conventional HGHG FEL (see Scheme 1 in Fig. 1) consists of three components: a modulator, a dispersive section and a radiator. First, the electron beam enters the modulator, together with the seed laser which modulates the electron beam's energy. Next, the energy-modulated electron beam passes through the dispersive section, where the energy modulation is converted into spatial modulation, producing abundant harmonics bunching in the electron beam density distribution. Finally, when the spatially modulated electron beam enters the radiator which is designed to be resonant to one of the harmonics of the seed laser, rapid coherent emission at this resonant harmonic is produced. This harmonic is further amplified exponentially until saturation.

Contrastively, in the HO of HGHG FEL, as

Scheme 2 in Fig. 1 indicates, the wavelength of the seed laser is not integer times of the resonant wavelength of the radiator but integer times of the wavelength of the 3rd harmonic radiation of the radiator. Thus, when the energy-modulated electron beam enters the radiator with abundant harmonic bunching in the electron beam density distribution, coherent radiation at the 5th harmonic of the seed laser which is the 3rd harmonic component of the radiator radiation is rapidly produced and amplified exponentially until saturation, meanwhile, the fundamental component of radiator radiation, whose wavelength is 3/5 of the seed laser, is expected to start from shot noise.

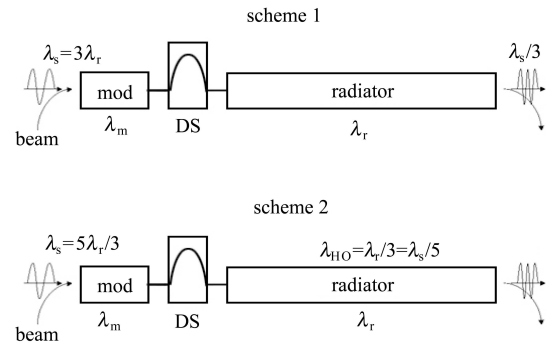


Fig. 1. Schematic of conventional HGHG and HO of HGHG.

3 Analytical estimates on HO of HGHG

In this section, we discuss the analyses of the physical theoretical backgrounds in the HO of HGHG FEL in details. The bunching in the modulator and the dispersive section, the coherent harmonic generation in the first two gain lengths of the radiator, the exponential growth and the FEL power-conversion efficiency are included.

3.1 Bunching parameters

According to the well-known 1D methods describing the electron dynamic and the radiation power evolution in a planar undulator^[17], one may obtain a set of self-consistent equations, harmonic emissions included Ref. [2], expressed as:

$$\frac{d\theta_j}{d\bar{z}} = p_j, \quad (1)$$

$$\frac{dp_j}{d\bar{z}} = -\sum_h (F_h(\xi)A_h e^{i\theta_j} + \text{c.c.}), \quad (2)$$

$$\frac{dA_h}{d\bar{z}} = F_h(\xi)b_h = F_h(\xi)\langle e^{-ih\theta} \rangle, \quad (3)$$

in which j runs over the electron and h over the harmonic components of the field. We have used the

universal scaling introduced in Ref. [17], namely,

$$\begin{aligned}\theta_j &= (k_s + k_w)z_j - ck_s t_j, \\ p_j &= (\gamma_j - \gamma_r)/\rho\gamma_r, \\ A_h &= \omega_s a_h / \omega_p \sqrt{\rho\gamma_r}, \\ \bar{z} &= 2\rho k_w z,\end{aligned}$$

where $\omega_p = (4\pi e^2 n_e / m)^{1/2}$ is the plasma frequency, $k_w = 2\pi/\lambda_w$ is the undulator wave number, $\omega_s = ck_s = 2\pi/\lambda_s$ is the radiation angular frequency, $\gamma_r = (\lambda_w(1+a_w^2)/2\lambda_s)^{1/2}$ is the resonant energy (in units of mc^2), F_h is the difference of Bessel functions defined as $F_h(\xi) = (-1)^{(h-1)/2} [J_{(h-1)/2}(h\xi) - J_{(h+1)/2}(h\xi)]$, $\xi = a_w^2/(1+a_w^2)/2$, $a_w = eB_w/\sqrt{2}k_w mc^2$ is the dimensionless undulator parameter, and $a_h = eE_h \lambda_s / 2\pi mc^2$ is the complex amplitude of dimensionless radiation vector potential. The scaling of A_h has been chosen so that $|A_h|^2 = (|E_h|^2/\rho)/(4\pi n_e \gamma_r mc^2)$ is the power conversion efficiency divided by ρ , and $\rho = (a_w \omega_p / ck_w)^{2/3} / \gamma_r$ is the fundamental FEL parameter.

The term $b_h \equiv \langle \exp(-ih\theta) \rangle$ in Eq. (3) is the bunching parameter of the electron beam in the h -th harmonic field component, and $\langle \rangle$ represents an average over the distribution function of the beam. By an initial electron distribution of the form $f(\theta_0, p_0) = (1/\Psi)(1/\sigma\sqrt{2\pi})\exp[-(p_0 - \delta)^2/2\sigma^2]$ corresponding uniform spread in phase between $[0, \Psi]$ and Gaussian distribution in energy around the value of γ_0 ($\sigma \equiv \sigma_\gamma/\rho\gamma_0$ and $\delta \equiv (\gamma_0 - \gamma_r)/\rho\gamma_r$), following the analytical mode referred in Ref. [18], one can evaluate the bunching out of a dispersive section, of strength $\partial\theta/\partial\gamma$, after a modulator of length \bar{z} . We extend it to the h -th harmonic, making h not necessary to be an integer. Then, the following formulas are given:

$$b_h(\bar{z}) = \int_0^\Psi d\theta_0 \int_{-\infty}^{+\infty} dp_0 f(\theta_0, p_0) \times e^{-ih[\theta(\bar{z}) + (\partial\theta/\partial\gamma)\rho\gamma_r p(\bar{z})]} . \quad (4)$$

Defining

$$A_{\text{III}}(p_0, \bar{z}, \partial\theta/\partial\gamma) = - \int_0^{\bar{z}} dz' A(z') \times [(\partial\theta/\partial\gamma)\rho\gamma_r + \bar{z} - z'] e^{ip_0 z'},$$

Eq. (4) can be rewritten as

$$b_h(\bar{z}) = \int_0^\Psi d\theta_0 \int_{-\infty}^{+\infty} dp_0 \frac{1}{\Psi\sigma\sqrt{2\pi}} \times e^{-(p_0 - \delta)^2/2\sigma^2} e^{-ih[\theta_0 + (\partial\theta/\partial\gamma)\rho\gamma_r p_0 + \delta\bar{z}]} \times e^{i2h|A_{\text{III}}|\sin(\varphi_{\text{III}} + \theta_0 + \pi/2)} . \quad (5)$$

Since the last part of Eq. (5) could be extend as a sum of Bessel functions J_m , and in the limit $\sigma\bar{z} \ll 1$, the

two integrals can be performed independently, yielding the result

$$b_h(\bar{z}) = \frac{1}{\sqrt{2\pi\Psi\sigma}} \int_{-\infty}^{+\infty} dp_0 \times e^{-(p_0 - \delta)^2/2\sigma^2} e^{-ih[(\partial\theta/\partial\gamma)\rho\gamma_r p_0 + \delta\bar{z}]} \times \int_0^\Psi d\theta_0 \sum_{m=-\infty}^{-\infty} e^{i\theta_0(m-h)} e^{im(\varphi_{\text{III}} + \pi/2)} J_m(2h|A_{\text{III}}|) . \quad (6)$$

Defining Ψ not one period 2π but thousand periods corresponding to the length of electron beam and considering the modulator as a small-gain situation, we can obtain the bunching parameters in the h -th harmonic field component

$$|b_h| = \begin{cases} \left| J_h \left(2hA_0 \frac{\partial\theta}{\partial\gamma} \bar{z} \right) \right| e^{-\left(h \frac{\partial\theta}{\partial\gamma} \sigma_\gamma \right)^2 / 2} & (h = \text{int}) \\ 0 & (\text{else}) \end{cases} . \quad (7)$$

After some simple calculations, it is easy to see that the results are the same bunching parameters given by Yu^[3]. As an HO of HGHG FEL showed in Fig. 1, at the entrance of the radiator, $b_{5/3}$ represents the bunching parameter corresponding to the fundamental radiation in the radiator and b_5 represents the bunching parameter corresponding to the 3rd harmonic component of the radiator radiation. It is clear that the former is zero and the latter has a strong initial bunching.

3.2 Coherent harmonic generation

The bunching parameters corresponding to the radiator radiation in the HO of HGHG FEL now can be rewritten in the form of Yu's

$$\begin{cases} b_3 = \exp \left[-\frac{1}{2} \left(5 \frac{\partial\theta}{\partial\gamma} \sigma_\gamma \right)^2 \right] J_5 \left(5 \frac{\partial\theta}{\partial\gamma} \Delta\gamma \right) \\ b_1 = 0 \end{cases} . \quad (8)$$

Here $\Delta\gamma$ is the maximum energy modulation at the end of the modulator, and $\partial\theta/\partial\gamma$ is the total dispersion that contains the contributions from the modulator, the dispersive section and the radiator.

$$\frac{\partial\theta}{\partial\gamma} = \frac{k_{w1}}{\gamma} z_1 + \left(\frac{\partial\theta}{\partial\gamma} \right)_{\text{dispersion}} + \frac{3}{5} \left(\frac{2k_{w2}}{\gamma} z_2 \right) . \quad (9)$$

Therefore, in the HO of HGHG FEL, the fundamental component of radiator radiation would perform as SASE, starting from shot noise because of a zero bunching parameter. The 3rd harmonic component of radiator radiation would perform as HGHG due to the strong initial bunching. To calculate the radiation power of an electron beam passing through the radiator, with an initial bunching $b_h(r, z_2)$, the universal method^[19] can be applied. We make an

assumption that the fundamental and the 3rd harmonic are independent of each other, which would be acceptable as a rough estimate. Thus, the coherently radiated 3rd harmonic power in the first two gain lengths of the radiator P_{coh} would be:

$$P_{\text{coh}} = \frac{I_p^2 Z_0}{8} \left(\frac{K_{\text{rad}} F_3(\xi_{\text{rad}})}{\gamma} \right)^2 \frac{1}{4\pi\sigma_x^2} |I_b|^2, \quad (10)$$

with $Z_0=377 \Omega$ the vacuum impedance, I_p the electron beam peak current, K_{rad} the radiator parameter in terms of maximum field, $F_3(\xi_{\text{rad}})$ the difference of Bessel function defined in Eq. (2), σ_x the rms electron beam size, and I_b the bunching integral over two gain lengths L_{g3} of the 3rd harmonic radiation in the large beam size limit:

$$I_b \cong \int_0^{2L_{g3}} \frac{1}{4\pi\sigma_x^2} dz \int_0^{2\sigma_x} b_3 J_0 \left(\frac{2.4r}{2\sigma_x} \right) d^2r. \quad (11)$$

3.3 Exponential gain and FEL efficiency

In a HO of HGHG FEL, our goal is to choose the optimal parameters so that the overall gain of the 3rd harmonic component is as large as possible while still providing significant amplification of the 3rd harmonic. In fact, a “good” HO of HGHG FEL design is one in which the 3rd harmonic component is dominant almost in the whole process and the amplitude of the fundamental component remains small enough that nonlinear effects are not important. Consequently, one can easily obtain the output power depending on the length of the radiator

$$\begin{cases} P_1 = P_{\text{noise}} \exp(z/L_{g1}) \\ P_3 = CP_{\text{coh}} \exp(z/L_{g3}) \end{cases}, \quad (12)$$

with z being the length of the radiator, L_{g1} and L_{g3} the gain length of the fundamental and the 3rd harmonic component respectively, P_{noise} the estimated shot noise power, and $C \approx 3.72/12 \approx 1/3$ the coupling coefficient.

For the coherent radiation generation calculation, we have ignored the energy spread and angular spread, but we could take all these effects into account for the exponential growth region. We take the energy spread in the radiator as an effort contributed by the initial energy spread and energy modulation at the modulator. As a rough approximation, we use $\Delta\gamma$ at the radial position $r = \sigma_x$, here

$$\sigma_{\gamma^2} = \sqrt{\sigma_\gamma^2 + \frac{\Delta\gamma^2}{2}}. \quad (13)$$

Now we pay our attention to the gain length of the fundamental and the 3rd harmonic component. For the fundamental, a FEL gain universal scaling function to calculate the gain length L_{g1} including 3D effects has been known^[20]. For the 3rd harmonic, a similar scaling function to calculate the linear gain

length of the 3rd harmonic radiation L_{g3} has been achieved and checked by simulation^[21]. One may obtain

$$\frac{\text{Im}(\mu_3)}{D_3} = \frac{1}{2k_w L_{g3} D_3} = G \left(k_s \varepsilon, \frac{\sigma_\gamma}{D_3}, \frac{k_\beta}{k_w D_3}, \frac{\omega - \omega_s}{\omega_s D_3} \right), \quad (14)$$

where μ_3 is the complex growth rate, ε is the electron beam emittance, k_β is the betatron wave number, and D_3 is the defined scaling parameters.

Finally, one must pay some attention to the FEL power-conversion efficiencies in the HO of HGHG. FEL saturation and synchrotron oscillation are due to the electron trapped by the ponder motive potential well, in the relativistic case, the efficiency of h -th harmonic radiation can be estimated from the relationship^[22]

$$\eta_h \cong \frac{\rho}{h} \left(\frac{F_h(\xi)}{F_1(\xi)} \right)^{2/3}. \quad (15)$$

Thus, we present a full analytical approach to calculate the performance of HO of HGHG FEL.

4 Numerical examples

A one dimensional FEL code, based on Eqs. (1), (2) and (3), is developed to simulate the HO process of HGHG FEL, in which the electron phase is uniformly loaded between $[0, 2000\pi]$ by “mirroring” method used in 3D FEL code GENESIS1.3^[23] to remove the effect of finite electron numbers, and the gain reduction due to the transverse emittance is equivalent to an external initial energy spread^[24]. To determine the radiation in the radiator, we integrate Eqs. (1) — (3) for two field components, $h=1$ and 3, using two example parameters listed in Table 1 which is selected carefully.

Firstly, we consider the HO of HGHG FEL with the parameters of UV example. According to the results of simulation, Fig. 2 shows the bunching parameters corresponding to the fundamental and the 3rd harmonic component in the radiator for parameters of UV example. We obtain that the 3rd harmonic radiation starts with a strong spatial bunching and the fundamental starts from a zero bunching. And as the power growth seen in Fig. 3, the 3rd harmonic radiation experiences the quadratic growth, the exponential growth (but grows less slowly than the fundamental) and the saturation region, which are well recognized in HGHG^[3], contrastively, the fundamental radiation performs as a SASE FEL and can be neglected, which agrees with what we have expected above and would be an amazing character of the HO of HGHG FEL. Moreover, from the comparisons in Fig. 3, we can see that the general dependence of the

analytical estimates is correct, and to some extent, we can conclude that it can be a guide for simulation.

Table 1. Numerical examples of HO of HGHG FEL.

	UV example	DUV example
seed laser parameters		
λ_s/nm	476.5	145
P/MW	5.0	11.4
electron beam parameters		
E/MeV	160	300
I_p/A	300	400
$\varepsilon/(\mu\text{m}\cdot\text{rad})$	6	4
σ_γ/γ	1×10^{-4}	1×10^{-4}
modulator undulator		
λ_w/cm	5.0	5.0
L/m	0.8	0.8
K	1.19	1.19
radiator undulator		
λ_w/cm	2.5	2.5
K	1.41	1.41
calculated FEL performance		
$\lambda_{\text{HO}}/\text{nm}$	87.3	29
$\partial\theta/\partial\gamma$	2.50	2.00
P_3/MW	47.4	129
P_1/MW	157	266
$P_{\text{SASE}}/\text{MW}$	256	682
$P_3/P_{\text{SASE}}/(\%)$	18.5(S) 16.1(T)	18.9(S) 17.1(T)

(S) represents the results obtained by the numerical simulation. (T) represents the results obtained by the analytical estimate.

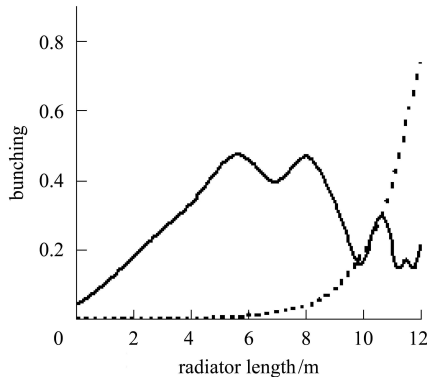


Fig. 2. Comparison of the simulated bunching parameters of the fundamental (the dashed) and the 3rd harmonic (the solid) in HO of HGHG with the parameters of UV example.

In more detail, what we really care are the output powers of the fundamental 262 nm radiation and the 3rd harmonic 87.3 nm radiation. Since the energy of the electron beam yielding the optimum performance at the fundamental and the 3rd harmonic radiation is different, we adjust the energy to obtain the optimum output power of the 3rd harmonic radiation, where 87.3 nm radiation with saturation power of 47.4 MW and 262 nm radiation with saturation power of 157 MW are obtained. Thus the output power ratio of the 3rd harmonic to the fundamental is mainly 30.2%. However, for the sake of deviation

from the energy of the electron beam yielding the optimum performance at the fundamental and the existence of significant power at the 3rd harmonic, the fundamental radiation is obviously degraded. In further study under the optimum condition, we obtain the fundamental 262 nm radiation with saturation power of 256 MW. Then the optimum power ratio of the 3rd harmonic to the fundamental is 18.5%. It is consistent with 16.1% derived from the analytical estimates. Thus the 3rd harmonic radiation efficiency is enhanced from 1%—2% to 18.5% of the fundamental level, which testifies the most attractive aspects of the HO of HGHG.

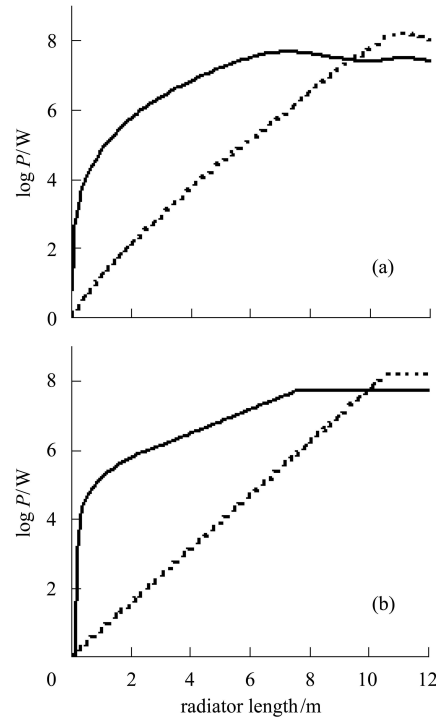


Fig. 3. Radiation power vs. radiator length in HO of HGHG FEL with the parameters of UV example. The dashed curve is the fundamental and the solid is the 3rd harmonic radiation. (a) is the simulation results and (b) is the theoretical results.

A DUV example has also been calculated. The results are listed in Table 1. Good agreement and promising prospects are also found. For the sake of space, we don't present the figures of these results.

5 Concluding remarks

Compared with the conventional FEL operating at the fundamental frequency, HO of HGHG FEL produces significant power and short-wavelength radiation using a lower energy electron beam. In this paper, two numerical examples in UV and DUV spectral regions are given. It indicates that the power of the 3rd harmonic radiation in HO of HGHG is as

high as 18.5% of the fundamental level. Contrastively in the conventional FEL^[12–15], the power of the 3rd nonlinear harmonic generation is 1%–2% of the fundamental power level. This is the most attractive feature of HO of HGHG FEL.

Since we are interested in the high harmonic radiation in the HO of HGHG, it is very sensitive to the strength of dispersive section. And we could advance or delay the saturation of the 3rd harmonic relative to the fundamental by adjusting the dispersive strength. As mentioned above, we could make the harmonic radiation dominant for a long distance where the fundamental could be negligible, which is another attractive character of the HO of HGHG. Thus in the short-wavelength production by cascading stages of HGHG^[25], some stages could be replaced by the HO of HGHG, which may attribute to miniaturization and simplification of FEL scheme for the

sake of using a lower energy electron beam.

More generally, the HO of HGHG can be extended to higher harmonic number 5, 7 and so on with the right choice of the parameter. Since low efficiency in higher harmonic situation is predicted, we present a description of the principle of the HO of HGHG FEL and perform some preliminary calculations of the HO of HGHG FEL corresponding to the 3rd harmonic. There are still many questions to be addressed before any detailed design or experiment can be carried out. Moreover, the HO of HGHG FEL is very sensitive to the electron beam quality, which is a point of view we concluded from the simulation. Since rigorous requirement over electron beam for ultra short-wavelength FEL radiation, such as emittance, can't be satisfied at low energy, the HO of HGHG FEL may work well in the region from ultraviolet down to soft X-ray wavelengths.

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