Systematics on fission fragment mass distribution of neutron induced ^{235}U fission

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Abstract Based on the neutron induced fission fragment mass distribution data up to neutron energy 20 MeV measured with the double kinetic energy method (KEM) and the radio active method (RAM), the systematics of fission fragment mass distribution was investigated by using 5 Gaussian model and the systematics parameters were obtained by fitting the experimental data. With the systematics, the yields of any mass A and at any energy in the region from 0 to 20 MeV of neutron energy can be calculated. The calculated results could well reproduce the experimental data measured with KEM, but show some systematical deviation from the data measured by RAM, which reflects some systematical deviations between the two kinds of measured data.

The error of systematics yield was calculated in an exact error transformation way, including from the error of the experimental yield data to the error of the discrete parameters, then to the systematics parameters, and at last to the yield calculated with systematics.

Key words ^{235}U fission, fission fragment mass distribution, systematics, 5 Gaussian model

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1 Introduction

The research on fission fragment mass distribution is very important for both studying the fission properties and the practical applications. It is going on in three approaches at present: the experimental measurement, the model theory calculation and the systematics[1]. Systematics is most simple and hopeful, if there are enough experimental data as a base. Hambsch's data[2)] are very useful for this purpose.

2 Data base

The following experimental fragment mass distribution or chain yield data were used as the base of the systematics: Hambsch in the energy region from 0.5 to 6.0 MeV by a step 0.5 MeV; R. B. Strittmatter[2], G. Diiorio[3], G. Siegert[4] at thermal energy; W. J. Maeck[5] at 0.4 MeV; J. W. Mandler[6] at 9.0, 15.0 MeV; BAO Jie[7] at 19.1 MeV and FENG Jing[8] at 22.0 MeV.

Among them, Hambsch's data were measured by using the double kinetic energy method(KEM), with which the fragment mass distribution data can be measured simultaneously for more complete mass number A. So the systematical error can be avoided for different mass number A. But the data need to be corrected for the mass (energy) resolution, energy loss, neutron emission and others, which are quite complicated. The other data were measured by using the radio active method (RAM), with which the radio decay γ or β of fission product nuclide is detected and the chain yield of the fission product nuclide is determined one by one. RAM is difficult to measure for more complete mass number A, but there is no problem of mass resolution correction.

Exactly speaking, the fragment mass distribution is different from the chain yield due to the existence of delay neutron emission, but the portion of the delay emission neutron is, in general, quite small, so for our purpose they can be considered approximately the same.

J.W.Mandler's data are the ratio of chain yield to mass chain A=140, the data were calculated into chain yield by using standard chain yield for A=140 evaluated by ourselves[9].

All the data were smoothed[10] for each 9 points

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before they were used for the systematics calculation. Otherwise, it would be difficult to fit, especially for the data measured by RAM.

3 5 Gaussian model on mass distribution

Mass distribution data can be fitted with 5 Gaussian model:

$$Y(A)=\sum_{i=1}^{I} y_i/\sqrt{2\pi}\,\sigma_i \times \exp\left(-(A-\overline{A}+\varDelta_i)\right)^2/2\sigma_i^2$$

where $\boldsymbol{I}=5$ and

$$\overline{A}=(A_{\rm F}-\overline{\nu})/2$$

$\boldsymbol{A}_{\rm F}$ is the mass of fission system and $\overline{\nu}$ is the prompt ν value of the fission nuclide. Due to the symmetry of the distribution, there are altogether 9 adjusted parameters: $\varDelta_1$, $\varDelta_2$, y_1, y_2, y_3, σ_1, σ_2, σ_3, $\overline{A}$. $\varDelta_1$, $\varDelta_2$ are the position shift of the first and second Gaussian distributions ($\varDelta_1=-\varDelta_5$, $\varDelta_2=-\varDelta_4$), y_1, y_2, y_3 are the height of the first, second and third Gaussian distributions ($y_1=y_5$, $y_2=y_4$), σ_1, σ_2, σ_3 are the width of the first, second and third Gaussian distributions ($\sigma_1=\sigma_5$, $\sigma_2=\sigma_4$). One should notice that the correlation nature between the yields of different mass number A is not symmetry, so it must take 15 parameters to calculate the covariance matrix. In this case, the parameter $\boldsymbol{C}$ is a 15-dimension vector and sensitivity matrix $\boldsymbol{F}$ is a 15-column matrix.

According to the non-linear least square method and using iteration method, the optimal parameters were obtained:

$$\begin{aligned}\overline{C} &= C^{(k+1)}=C^k+(F^{(k)\rm T}V_Y^{-1}F^{(k)})^{-1}\times \\ &\quad F^{(k)\rm T}V_Y^{-1}(Y-Y^{(K)})\ ,\\ \boldsymbol{V}_{\overline{C}}&=(\boldsymbol{F}^{(k)\rm T}\boldsymbol{V}_Y^{-1}\boldsymbol{F}^{(k)})^{-1}\ ,\\ \overline{\boldsymbol{Y}}&=\boldsymbol{Y}^{(k+1)}=\boldsymbol{F}^{(k)}\boldsymbol{C}^{(k)}+\boldsymbol{Y}^{(k)}\ ,\\ \boldsymbol{V}_{\overline{Y}}&=\boldsymbol{F}^{(k)}\boldsymbol{V}_{\overline{C}}\boldsymbol{F}^{(k)\rm T}\ ,\end{aligned}$$

where $\boldsymbol{F}$ is the sensitivity matrix of yield $\boldsymbol{Y}$ to parameters $\boldsymbol{C}$. And the initial values $\boldsymbol{Y}^{(0)}$ were calculated according to the Wahl's formulas[1]. The iteration proceeds until it is convergent. Because it is a problem of searching for optimal values for multi parameters, the solution is not unique. The parameters obtained must be chosen from the multi possible solutions to keep the systematics behavior for each parameter at different energy points. This needs large amount of calculations and some times it is not easy.

The experimental data must be corrected and smoothed before they are fitted to insure that the result is reasonable in physics and the iteration is convergent. In our practical situation, the mass distribution data, measured by KEM, were corrected for mass resolution and smoothed with 7 points, and the chain yield data, measured by RAM, were smoothed with 9 points due to the fact that this kind of data is more fluctuant and the iteration is not convergent with 7 points.

Then the data were fitted by using 5 Gaussian Model with iteration method for the mass distribution data and chain yield data at each energy point. The iteration is processed until it is convergent, and the reduced χ^2 is close to 1 and a set of reasonable parameters, called discrete parameters, are obtained. To research for the systematics rule of the parameters with the energy, the parameters were selected from multi sets of reasonable ones and were adjusted at the neighborhood of the parameter systematics trend sometimes.

4 The systermatics of the parameters with neutron energy

Each parameter obtained at each energy point was fitted as energy function by the least square method with a 2 order function $y=a+bx+cx^2$, whose coefficients a,b,c were determined by a code to make the χ^2 smallest. Each of the 9 parameters was fitted in the energy region from 0 to 20 MeV. The obtained coefficients a, b and c, called systematics parameters, are given in Table 1 and Figs. 1—3, from which the parameters at any energy point can be calculated.

Table 1. The fit coefficients a,b,c and their uncertainty for each parameter.

coefficient	a	Δa	b	Δb	c	Δc
$\varDelta_1$	2.2980×10^{1}	1.1719×10^{-1}	-2.9532×10^{-1}	4.7715×10^{-2}	1.1423×10^{-2}	4.7201×10^{-3}
$\varDelta_2$	1.6029×10^{1}	1.1547×10^{-1}	-1.7469×10^{-1}	4.8911×10^{-2}	9.9681×10^{-3}	4.4094×10^{-3}
y_1	7.9992×10^{1}	1.5588	1.8373	5.8054×10^{-1}	-9.9183×10^{-2}	4.7307×10^{-2}
y_2	1.9843×10^{1}	1.4994	-2.0535	5.4599×10^{-1}	7.2597×10^{-2}	4.2875×10^{-2}
y_3	3.3165×10^{-1}	1.3699×10^{-1}	3.8917×10^{-1}	8.4011×10^{-2}	8.2379×10^{-2}	1.0917×10^{-2}
σ_1	5.0553	4.6429×10^{-2}	1.9431×10^{-1}	2.0065×10^{-2}	-4.9505×10^{-3}	2.2101×10^{-3}
σ_2	2.8878	1.1557×10^{-1}	2.3530×10^{-2}	5.7062×10^{-2}	-1.7867×10^{-3}	6.1554×10^{-3}
σ_3	2.7694×10^{1}	1.5907	-2.6007	4.2610×10^{-1}	8.8630×10^{-2}	1.7585×10^{-2}
$\overline{A}$	1.1785×10^{2}	2.2110×10^{-2}	7.4013×10^{-2}	9.3442×10^{-3}	-8.1667×10^{-3}	8.7519×10^{-4}

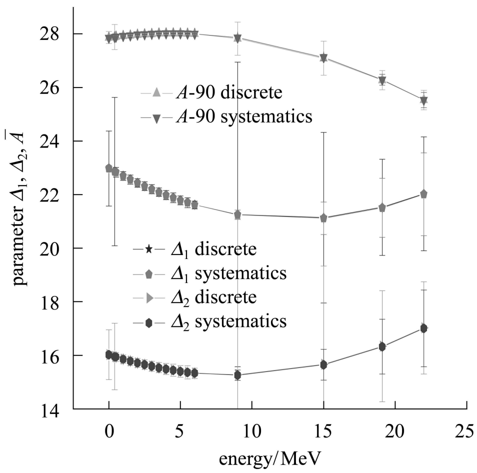

Fig. 1. The dependence of systematics parameters $\overline{A}$, Δ_1, Δ_2 on energy.

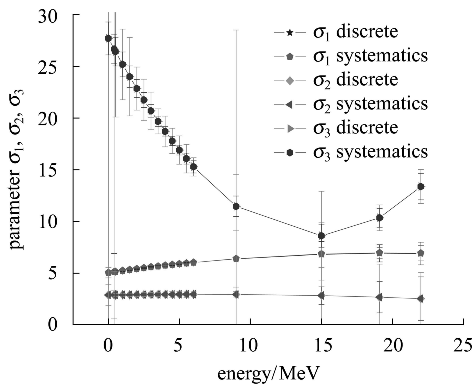

Fig. 2. The dependence of systematics parameters σ_1, σ_2, σ_3 on energy.

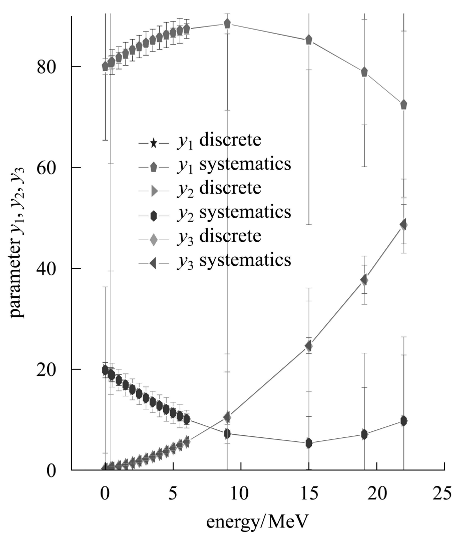

Fig. 3. The dependence of systematics parameters y_1, y_2, y_3 on energy.

The physical meaning can be seen clearly from the Figures for the dependence of some parameters on energy. The parameter $\overline{A}$, the mass number at the symmetric point of the double canal mass distribution, is decreased with increasing energy, which is due to the fact that the fission $\overline{\nu}$ value is increased with increasing energy, so the total mass of the fission product nuclides is decreased. The parameter y_3, the height of the central Gaussian distribution, is increased with increasing energy, which describes a well- known fact that the volley of the double canal mass distribution is increased with the energy increasing. The physical meaning of the other parameters can not be seen directly, because the calculated yield is a comprehensive result of 5 Gaussian, especially 2 pairs of asymmetric Gaussian distribution contributions.

5 The yield calculated with systematics

By using the systematics parameters, the yield at any energy point can be calculated.

The comparison among the data calculated with systematics parameters and the experimental mass distribution data, measured with KEM are given in Fig. 4 and the data with RAM are given in Fig. 5.

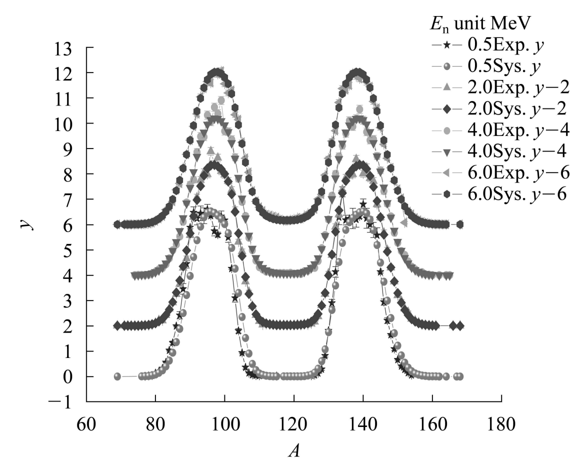

Fig. 4. The comparison of the yield calculated by systematics with the data measured by KEM method.

It can be seen that the systematics can reproduce the data measured with KEM well and the data measured with RAM roughly. The data measured by RAM are not complete for the mass A and also with some fluctuation. It seems that there is some systematical deviation between the systematics and the data measured with RAM, the data calculated with systematics are slightly shift toward to the larger mass number A, especially for the distributions at low energies, which, substantively, reflects the systematical deviation of the two kinds of experimental data. The similar matter was also found for ^{238}U[11]. The data measured by KAM need to be corrected for the mass resolution, the neutron emission from fission compound and fragment, the pulse height effect of the detector, the fragment energy loss in the sample material, etc. These corrections are quite complicated and it is difficult to do accurately. However, this kind of data must be used as the main base of the systematics, because the data are more complete and systematical for the incident neutron energy and fragment mass A.

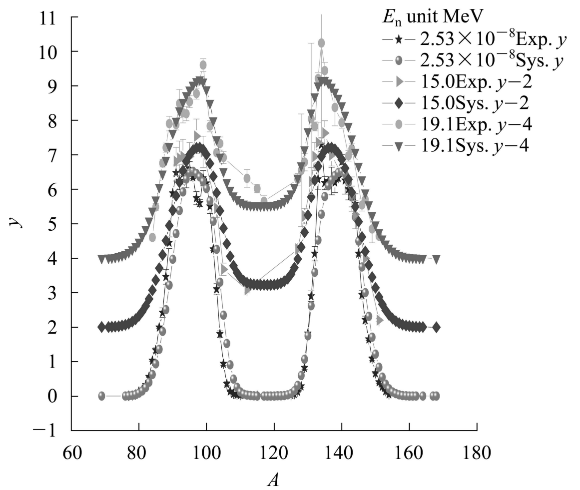

Fig. 5. The comparison of the yield calculated by systematics with the data measured by RAM method.

6 The yield error calculated with systematics

The error of systematics yield was calculated in an exact error transformation way, including from the error of the experimental yield data to the error of the discrete parameters, then from the error of the discrete parameters to the error of the systematics parameters, and at last from the error of the systematics parameters to the error of the yield calculated with systematics.

The typical results are shown in Figs. 6, 7, where the errors calculated with systematics are compared with the experimental ones as inputs.

The following can be seen and concluded:

1) The errors are continuously changed no matter whether the experimental errors are continuous or not (Figs. 6, 7). The errors of the experimental data measured with RAM can be fluctuating largely, because the data are measured one by one for the product nuclide and the measured conditions, for example, the γ rays measured, are different, so the error can be very different. But the errors calculated with systematics are based on not only the error of the yield with mass number A to be calculated, but also the errors of the yields of all mass numbers, which were used for determining the systematics parameters.

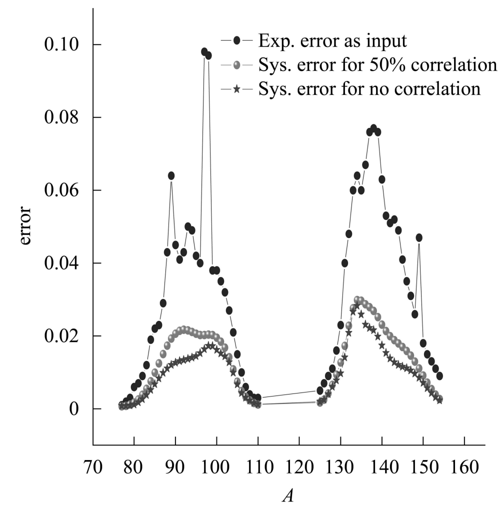

Fig. 6. The error comparison of the systematics and experimental data at thermal energy.

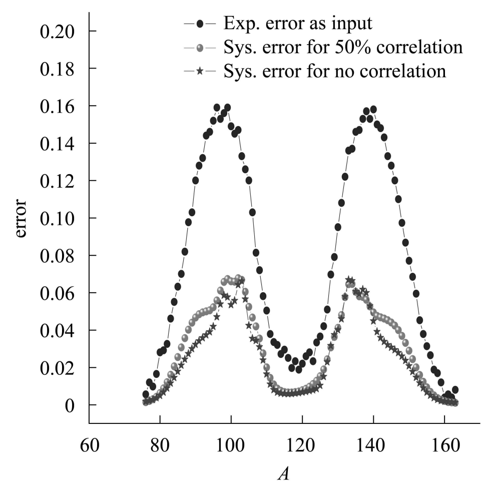

Fig. 7. The error comparison of the systematics and experimental data at 5.0 MeV.

2) The errors calculated by systematics with correlation are usually larger and the curve becomes "flatter" than one without correlation, which means that the changes are larger for the errors corresponding to the smaller ones of the experimental yields. The correlation in the data processing is caused by the systematical error. No correlation in the processing means that the processing is completely statistical. The statistical error is decreased along with the increasing of the data point number N processed, roughly it is proportional to $1/\sqrt{N}$, however the systematical error is not reduced when the data point number processed is increased. Also the "correlation" makes the effecting of the data point with each other stronger. So this result is certain and reasonable in physics.

3) The ratio of the error calculated with systematics over the experimental one is about 0.3—0.5 for

thermal energy, where the data points are more; 0.2—0.5 for 3.0 and 5.0 MeV, where the data points are most; 0.4—0.95 for 15.0 MeV, where the data points are less. It can be concluded that the ratio depends on the data points: the more the data points, the smaller the ratio, which is the result of statistical processing.

Roughly, the average portion is about 0.4. This means that the error of the yields calculated with systematics is smaller than the experimental one, which is only about 40% of it.

7 Conclusion remarks

Based on the mass distribution data measured with KEM and RAM, the systematics on fission fragment mass distribution as a function of incident neutron energy was studied. The mass distribution data were fitted with 5 Gaussian model at each energy point and the discrete parameters were deduced. Each discrete parameter as energy function was fitted with 2 order function and the systematics parameters were determined.

With the systematics, the yields of any mass A and at any energy in the region from 0 to 20 MeV can be calculated. The calculated results could reproduce the experimental data measured with KEM well, but show some systematical deviation from the data measured by RAM, which reflects some systematical deviations between the two kinds of measured data. The possible reason is that the data measured by KEM need more complicated accurate corrections.

The error of systematics yield was calculated in an exact error transformation way, including from the error of the experimental yield data to the error of the discrete parameters, then to the error of the systematics parameters, and at last to the error of the yield calculated with systematics. The error calculated with systematics is continuously changed no matter whether the experimental error is continuous or not. The systematics error is also reduced, depending on the data points, and roughly is 40% of the experimental one. The systematics error with correlation is usually larger and the curve becomes "flatter" than one without correlation.

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