Energy dependence of non-linear dynamical features in e^+e^- collisions*

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Abstract A study of the dynamical fluctuation properties at various c.m. energies in e^+e^- collisions is performed using the Monte Carlo method. The results suggest that, after the normalized factorial moments of 3-dimensional phase space are analyzed using an isotropical phase space partition, the NFM describing non-linear dynamical properties show a power-law scaling, i.e., the dynamical fluctuations in higher dimensional phase space are isotropic. For c.m. energies $\sqrt{s} \leqslant 80$ GeV, the scaling exponents ϕ_q increase rapidly with the c.m. energy and for c.m. energies $\sqrt{s} > 80$ GeV, the ϕ_q gradually saturate.

Key words e^+e^- collisions, non-linear dynamical property, factorial moments, energy dependence

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1 Introduction

It is known that there are large fluctuation events indicating non-linearities in the space-time evolution of high energy collisions. In the observation of nuclear interactions of high energy cosmic rays by the Japanese-American Collaborative Emulsion Experimental group(JACEE) exceptionally large loacal fluctuations in the pseudo-rapidity density distribution were found[1]. Later the experimental groups UA5[2] and NA22[3] successfully observed the so-called "nail" events with large rapidity fluctuations. People began to believe that this phenomenon may point to the existence of certain non-linear dynamical fluctuations during the space-time evolution in high energy collisions; in other words, its intrinsic probability density function has a fractal property. In 1986, Bialas and Pechanski described this kind of non-linear dynamical phenomenon during the space-time evolution in high energy collisions in terms of the self-similar cascade model of the turbulence[4, 5]. In order to investigate this phenomenon, using the experimental data, they introduced the normalized factorial moments (NFM) to eliminate the statistical fluctuations, described the dynamical fluctuations and consequently studied the fractal property of the probability density function by observing the power-law scaling behaviour of the NFM, which are defined as

$$F_q(\delta y) \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q} = \frac{1}{M}\sum_{m=1}^{M}\frac{\int_{\delta y}\cdots\int_{\delta y}\rho_q(y_1,\cdots,y_q)\prod_{i=1}^{q}\mathrm{d}y_i}{\left(\int_{\delta y}\rho_1(y)\mathrm{d}y\right)^q}, \quad (1)$$

where $\delta y = \Omega/M$ is the size of the small phase space interval (y represents the phase space variable) obtained through dividing equally the total phase space region Ω by M (the total number of intervals) and n_m is the multiplicity in the m-th phase space interval.

For the experimental data it is found that for a single particle the single event distribution is in general a non-linear distribution. This non-flat single event distribution has as a consequence that, even though there are no dynamical fluctuations in the system, the NFM show a slight increase with decreasing phase space, leading to "false intermittency phenomena". Hence, this kind of distribution will arouse large systematic errors when there are dynamical fluctuations in the system. In order to eliminate this kind of "false intermittency", a cumulating transform is usually performed to obtain new cumulation

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variables[6]

$$y_{\rm c}=\frac{\displaystyle\int_{y_{\min}}^{y}\rho(y){\rm d}y}{\displaystyle\int_{y_{\min}}^{y_{\max}}\rho(y){\rm d}y} \ . \tag{2}$$

Obviously, the single event distribution of the single particle with new cumulation variables is always flat, i.e., $\rho(y_{\rm c})=1$. Our interest is the power-law scaling behaviour of the probability moments of the system with the change in size of phase space. Although the NFM and the probability moments C_q differ by a constant facor, they have an equivalent power-law scaling behaviour. As a result the power-law scaling behaviour of the NFM can be directly observed. For this reason, working with the cumulation variables allows one to obtain the same scaling behaviour as that in Eq. (1).

In high energy physics the intermittency phenomenon is usually defined in the following way[4, 5, 7]: If the phase space interval δ decreases, the normalized probability moments C_q or NFM satisfy the following power-law scaling behaviour

$$F_q(M)\propto\delta^{-\phi_q}\propto(M)^{\phi_q} \ , \quad (\delta\to0,\ M\to\infty). \tag{3}$$

Here $\phi_q\equiv\phi(q)$ reflects the intensity degree of intermittency and is called the intermittency exponent.

It has been found that there are two kinds of fractal structures in high energy collisions, —self-similar and self-affine ones. As an example consider the two-dimensional fractals: If and only if the phase space is contracted in the two directions by the rates $\lambda_{\rm a}$ and $\lambda_{\rm b}$ ($\lambda_{\rm a}\neq\lambda_{\rm b}$) the self-similar property can be observed and thus the power-law scaling behaviour of NFM can be represented as[8]:

$$C(\lambda_{\rm a}x_{\rm a},\lambda_{\rm b}x_{\rm b})=\lambda_{\rm a}^{-\phi_q}\lambda_{\rm b}^{-\phi_q}C(x_{\rm a},x_{\rm b}) \ . \tag{4}$$

This kind of contracting transformation with different contraction rates $\lambda_{\rm a}$, $\lambda_{\rm b}$ ($\lambda_{\rm a}\neq\lambda_{\rm b}$) in two directions is called a self-affine transformation. Under a self-affine transformation the anisotropic fractal system with an abnormal power-law scaling behaviour is called a self-affine system[8]. In case it is contracted in the two directions with the same rate $\lambda_{\rm a}=\lambda_{\rm b}$, a self-similarity property of the fluctuations may be observed. Then the system with its NFM exhibiting an abnormal power-law scaling behaviour is called a self-similar system.

In high energy hadron-hadron collision experiments it has been found that the distribution of the final state particles in phase space is highly anisotropic, which faces oneself with self-affine fractals in higher dimensional phase spaces[9—11]. On the other hand, in e^+e^- collisions at the c.m. energy of $\sqrt{s}=91.2$ GeV , i.e., at the Z^0 decay energy region, the results have shown that the dynamical fluctuations of the final state hadrons in 3-dimensional phase space are isotropic in the two cases, using either the MC simulation[12, 13] or the experimental data from CERN-L3[14] .

It should be noticed that the studies of the non-linear property of the final state particle system in e^+e^- collisions have been made at the Z^0 decay energy region. If the energy in e^+e^- collisions is changed it may lead to certain differences in the evolution process. We may therefore ask the question, wether there are any differences in the non-linear properties of the final state particle system in e^+e^- collisions for different c.m. energies. In particular, do the NFM of the final state hadron system show a power-law scaling behaviour at energy regions, different from the Z^0 decay region,where it has been studied quantitatively, and what are the differences in the fractal properties of the system?

In this thesis we use the Monte Carlo (MC) generator JETSET7.4 to produce the e^+e^- collision events, and then study the non-linear properties of the final state hadron system in e^+e^- collisions at different energies and the fractal property of the final state particle phase space.

2 The scaling behaviour of NFM at different energies

First, we use the MC Simulation generator Jetset7.4 to produce 18 event samples of e^+e^- collisions with c.m. energies of 30, 40, 50, $\cdots$, 200 GeV, each sample having 1 000 000 events. $(y,p_{\rm t},\varphi)$ are chosen as the phase space variables and ($-6\leqslant y\leqslant6$; $0\leqslant p_{\rm t}\leqslant3$ GeV; $-\pi\leqslant\varphi\leqslant\pi$) as the corresponding phase space intervals. With the isotropical phase space partition, the 3-dimensional NFM of order $q=2,\ \cdots,\ 5$ at different c.m. energies are calculated according to Eq. (1) and their abnormal scaling behaviour is shown in a log-log representation in Fig. 1.

It can be seen from Fig. 1 that after omitting the first point to eliminate the influence of momentum conservation[15], the 3-dimensional $\ln F_q(M)$ versus $\ln M$ distributions at different energies show linearity well within the error range. All the 3-dimensional NFM at different energies exhibit a well developed scaling behaviour, which shows that the dynamical fluctuations of the final state particle systems in e^+e^- collisions at different energies are isotropic, i.e., they have self-similarity properties.

In order to quantitatively study the intensity variation of the non-linear dynamical fluctuations in e^+e^- collisions at different energies, the NFM of different orders at different energies are fitted with Eq. (3). The obtained intermittency (fractal) exponents ϕ_q are

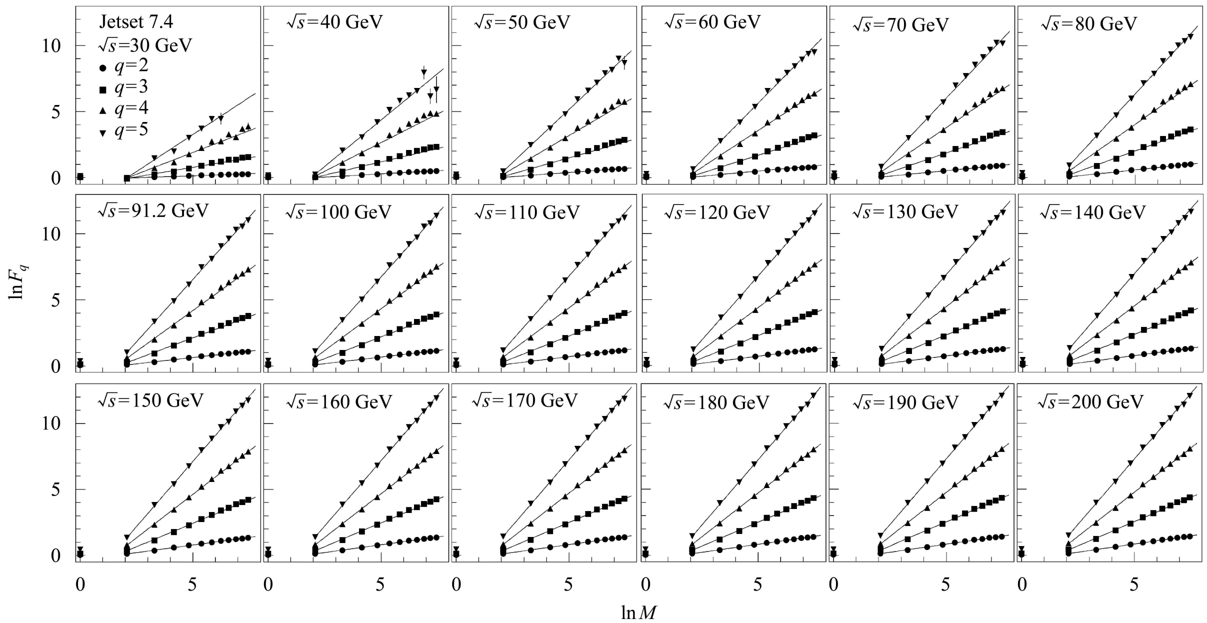

Fig. 1. The 3-dimensional $\ln F_q(M)$ vs. $\ln M$ distribution of order $q=2,\cdots,5$ at different c.m. energies, using Jetset7.4 to produce e^+e^- collision events.

Table 1. The 3-dimensional intermittency (fractal) exponents in e^+e^- collisions at different energies.

energy/GeV	ϕ_2	ϕ_3	ϕ_4	ϕ_5
30	0.0600 ± 0.0003	0.276 ± 0.002	0.640 ± 0.005	1.124 ± 0.015
40	0.0970 ± 0.0003	0.402 ± 0.001	0.868 ± 0.003	1.405 ± 0.008
50	0.1254 ± 0.0003	0.485 ± 0.001	1.007 ± 0.003	1.607 ± 0.006
60	0.1472 ± 0.0002	0.543 ± 0.001	1.099 ± 0.003	1.733 ± 0.005
70	0.1648 ± 0.0002	0.585 ± 0.001	1.158 ± 0.003	1.806 ± 0.005
80	0.1791 ± 0.0002	0.618 ± 0.001	1.204 ± 0.002	1.854 ± 0.005
91.2	0.1911 ± 0.0002	0.642 ± 0.001	1.237 ± 0.002	1.891 ± 0.004
100	0.1997 ± 0.0002	0.660 ± 0.001	1.262 ± 0.002	1.925 ± 0.004
110	0.2078 ± 0.0002	0.675 ± 0.001	1.277 ± 0.002	1.932 ± 0.004
120	0.2146 ± 0.0002	0.682 ± 0.001	1.289 ± 0.002	1.945 ± 0.004
130	0.2205 ± 0.0002	0.697 ± 0.001	1.304 ± 0.002	1.966 ± 0.003
140	0.2258 ± 0.0002	0.705 ± 0.001	1.310 ± 0.002	1.963 ± 0.003
150	0.2305 ± 0.0002	0.712 ± 0.001	1.317 ± 0.002	1.974 ± 0.003
160	0.2349 ± 0.0002	0.719 ± 0.001	1.323 ± 0.002	1.980 ± 0.003
170	0.2390 ± 0.0002	0.725 ± 0.001	1.332 ± 0.002	1.989 ± 0.003
180	0.2422 ± 0.0002	0.729 ± 0.001	1.334 ± 0.002	1.984 ± 0.003
190	0.2454 ± 0.0002	0.733 ± 0.001	1.337 ± 0.002	1.993 ± 0.003
200	0.2481 ± 0.0002	0.736 ± 0.001	1.338 ± 0.002	1.984 ± 0.003

shown in Table 1. The distributions of the intermittency exponents ϕ_q of the final state particle system in e^+e^- collisions, plotted as a function of energy, are shown in Fig. 2.

It can be seen from Fig. 2 that the intermittency (fractal) exponents of the system obtained from the NFM obviously depend on the c.m. energies in e^+e^- collisions; i.e., they increace with energy. In the low energy region the fractal exponents of NFM increase rapidly with energy, i.e., the dynamical fluctuation increase rapidly. For a c.m. energy $\sqrt{s}>80$ GeV, the fractal exponents saturate.

An extension of the curves to the low energy region shows that all curves nearly intersect at one point near zero, which indicates that the dynamical fluctuations of the final state particle system are close to zero for very low c.m. energies in e^+e^- collisions.

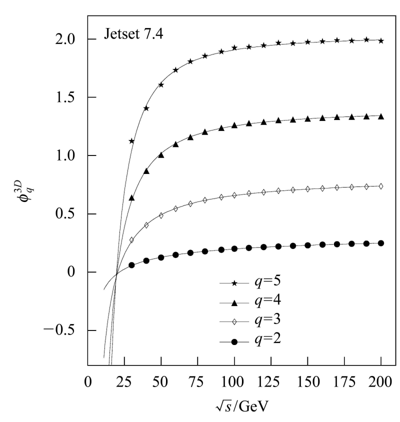

Fig. 2. The distributions of intermittency (fractal) exponents ϕ_q of the final state particle system in e^+e^- collisions at different energies as a function of energy, using Jetset7.4 to produce MC data.

3 Conclusion and discussion

In this thesis, we study the non-linear properties in e^+e^- collisions at different energies using the Monte Carlo simulation method. Through analyzing the 3-dimensional NFM of the final state particles in e^+e^- collisions at different c.m. energies, we found that the NFM describing the non-linear properties of the 3-dimensional final state particle phase space all exhibit a power-law scaling property at different energies. It shows that the dynamical fluctuations of the final state particle system in e^+e^- collisions at different energies are all isotropic and correspondingly the structures of the systems are self-similar fractal. The intermittency (fractal) exponents, reflecting the intensity of the dynamical fluctuations in high energy collisions, show an obvious dependence on the c.m. energy in e^+e^- collisions. For a c.m. energy $\sqrt{s} \leqslant 80$ GeV, the 3-dimensional intermittency exponents ϕ_q increase rapidly with the c.m. energy and for a c.m. energy $\sqrt{s} > 80$ GeV, the 3-dimensional intermittency exponents ϕ_q gradually saturate.

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