Theoretical investigation of a backward wave oscillator*

MA Qiao-Sheng(马乔生)1) ZHOU Chuan-Min(周传明) WU Yong(吴勇)

(Institute of Applied Electronics, China Academy of Engineering Physics, Mianyang 621900, China)

Abstract From the linear Vlasov equation, the theoretical investigation on relativistic backward wave oscillator is performed. The relationship between the microwave power and the guiding magnetic field, which accords with the results of the particle simulation and experiments, is deduced.

Key words microwave, guiding magnetic field, Vlasov equation, backward wave oscillator

PACS 41.60.Bq, 47.10.A-

1 Introduction

Relativistic backward wave oscillator (RBWO), whose output microwave power relates closely to the guiding magnetic field[1—3], is one of the most promising microwave devices. There are many theoretical results[4—6] on the relationship between the output microwave power and the parameters of the electron beam, which are attained on the condition of the infinite guiding magnetic field and can't reveal the relationship between the output microwave power and the guiding magnetic field. There are also some theoretical investigations[7, 8] on the relationship between the output power and the guiding magnetic field, but their conclusions are recondite.

Based on the Vlasov equation, the theoretical investigation of RBWO, which gives out a pellucid result on the relationship between the output power and the guiding magnetic field, is performed in this paper.

2 Operation equation of RBWO

In RBWO, the interaction between the electron beam and the microwave electromagnetic field which has the temporal and spatial dependence of a wave in the empty structure causes the electromagnetic wave to vary slowly in time and axial distance. The backward wave of TM_{01} mode can be expressed in the following ways:

$$E_Z=\sum_n E_{Zn}J_h(\Gamma_n r)\mathrm{e}^{\mathrm{i}(k_{Zn}Z+h\theta+\omega t)}\,, \tag{1}$$

$$E_\theta = \sum_n \frac{k_{Zn}h}{\Gamma_n^2 r}E_{Zn}J_h(\Gamma_n r)\mathrm{e}^{\mathrm{i}(k_{Zn}Z+h\theta+\omega t)}\,, \tag{2}$$

$$E_r = \sum_n \frac{-\mathrm{i}k_{Zn}}{\Gamma_n}E_{Zn}J_h'(\Gamma_n r)\mathrm{e}^{\mathrm{i}(k_{Zn}Z+h\theta+\omega t)}\,, \tag{3}$$

$$B_\theta = \sum_n \frac{-\mathrm{i}\omega}{\Gamma_n c^2}E_{Zn}J_h'(\Gamma_n r)\mathrm{e}^{\mathrm{i}(k_{Zn}Z+h\theta+\omega t)}\,, \tag{4}$$

$$B_r = -\sum_n \frac{\omega h}{\Gamma_n^2 c^2 r}E_{Zn}J_h(\Gamma_n r)\mathrm{e}^{\mathrm{i}(k_{Zn}Z+h\theta+\omega t)}\,, \tag{5}$$

$$B_Z = 0\,, \tag{6}$$

where $k_{Zn}=k_z+nk_0$ and $\Gamma_n^2=\left(\frac{\omega}{c}\right)^2-k_{Zn}^2$.

2.1 Expression of the perturbed electron distribution function f_1

When the electrons drift in a slow wave structure (SWS), they would be affected by the guiding magnetic field and the microwave. We will begin our analysis from these two aspects in the article.

Under the affection of the guiding magnetic field, the electrons have an axial drift and a Larmor cyclotron, but its distribution function f_0 is invariable. From Fig. 1, the guiding centre coordinate of the electron can be expressed as[9]

$$\tilde{r}=\left[r^2+r_{\mathrm{L}}^2-2rr_{\mathrm{L}}\sin(\phi-\theta)\right]^{1/2}\,, \tag{7}$$

where $r_{\mathrm{L}}=\dfrac{p_\perp}{m\Omega_{\mathrm{e}}}$ is the Larmor radius, $\Omega_{\mathrm{e}}=\dfrac{eB_0}{m}$ is

Received 3 July 2007

* Supported by State's High-Technology Research and Development Project (863)

1) E-mail: mqshcaep@yahoo.com.cn

the rest-mass cyclotron frequency, and $\Omega_c = \frac{\Omega_e}{\gamma}$ is the cyclotron frequency.

Under the affection of microwave, there is a perturbed electron distribution function $f_1(\boldsymbol{x},\boldsymbol{p},t)$ (where the polar coordinates $\boldsymbol{x}$ are (r,θ,Z), and $\boldsymbol{p}$ are $(p_\perp,\phi,p_Z)$) added to the f_0. The f_1 can be gained from the Vlasov equation

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\partial f_1}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f_1}{\partial \boldsymbol{x}} - \frac{e}{m}(\boldsymbol{v}\times\boldsymbol{B_0})\cdot\frac{\partial f_1}{\partial \boldsymbol{v}} = \mathrm{e}(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_p f_0\,, \tag{8}$$

where

$$f_0 = \frac{n_\mathrm{b}}{2\pi p_\perp}\delta(p_\perp - p_{\perp 0})\delta(p_Z - p_{Z0})\,, \tag{9}$$

$$\boldsymbol{v} = \frac{\boldsymbol{p}}{\gamma m} = \frac{1}{\gamma m}(p_\perp \boldsymbol{e}_\perp + p_Z \boldsymbol{e}_Z)\,, \tag{10}$$

$$\nabla_\mathrm{p} f_0 = \frac{\partial f_0}{\partial p_\perp}\boldsymbol{e}_\perp + \frac{\partial f_0}{\partial p_Z}\boldsymbol{e}_Z\,, \tag{11}$$

where n_b is the electron density, and $\boldsymbol{e}_\perp$, $\boldsymbol{e}_Z$ are the unit vectors along $\boldsymbol{p}_\perp$ and axis respectively. It can be seen from Fig. 1 that

$$\boldsymbol{e}_\perp = \cos(\phi-\theta)\boldsymbol{e}_r + \sin(\phi-\theta)\boldsymbol{e}_Z\,. \tag{12}$$

Substituting Eqs. (10), (11) and (12) into Eq. (8), we get

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = e\Big\{E_r\cos(\phi-\theta) + E_\theta\sin(\phi-\theta) + \frac{p_Z}{\gamma m}\big[B_r\sin(\phi-\theta) - B_\theta\cos(\phi-\theta)\big]\Big\}\frac{\partial f_0}{\partial p_\perp} - \frac{ep_\perp}{\gamma m}\big[B_r\sin(\phi-\theta) - B_\theta\cos(\phi-\theta)\big]\frac{\partial f_0}{\partial p_Z} + eE_Z\frac{\partial f_0}{\partial p_Z}\,. \tag{13}$$

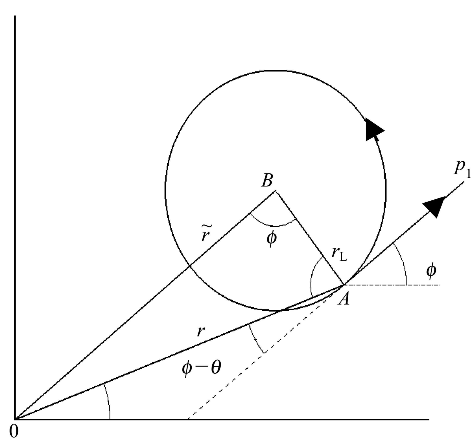

Fig. 1. Projection of the electron orbit (indicated by circle) on the cross-sectional plane. r_L is the Lamor radius. Point 0 is the centre of the waveguide and point B is the centre of the cyclotron orbit. Point A is the instantaneous position of the electron.

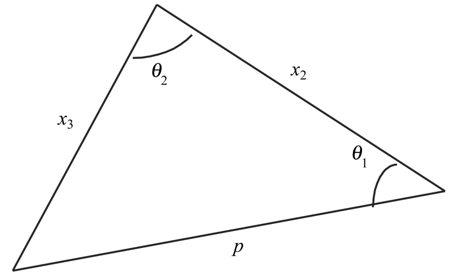

Fig. 2. Geometric representation of the variables used in the Bessel-function summation theorem.

By using the Bessel-function identities

$$J_h'(x) = \frac{1}{2}[J_{h-1}(x) - J_{h+1}(x)],$$

$$J_h(x) = \frac{1}{2}\frac{x}{h}[J_{h-1}(x) + J_{h+1}(x)]$$

and the Bessel-function summation theorem (whose variables are defined in Fig. 2)

$$\mathrm{e}^{\pm\mathrm{i}n\theta_1}J_n(x_1) = \sum_q J_{n+q}(x_2)J_q(x_3)\mathrm{e}^{\pm\mathrm{i}q\theta_2}\,.$$

We can get

$$E_r\cos(\phi-\theta) + E_\theta\sin(\phi-\theta) = \sum_n\sum_q E_{Zn}\frac{k_{Zn}(h+q)}{\Gamma_n^2 r_\mathrm{L}}J_{h+q}(\Gamma_n r_\mathrm{L})J_q(\Gamma_n\tilde{r})\times \mathrm{e}^{\mathrm{i}q\tilde{\phi}}\mathrm{e}^{\mathrm{i}\left(k_{Zn}Z + h\phi + \omega t - \frac{h\pi}{2}\right)}\,, \tag{14}$$

$$B_r\sin(\phi-\theta) - B_\theta\cos(\phi-\theta) = -\sum_n\sum_q E_{Zn}\frac{\omega}{c^2}\frac{(h+q)}{\Gamma_n^2 r_\mathrm{L}}J_{h+q}(\Gamma_n r_\mathrm{L})J_q(\Gamma_n\tilde{r})\times \mathrm{e}^{\mathrm{i}q\tilde{\phi}}\mathrm{e}^{\mathrm{i}\left(k_{Zn}Z + h\phi + \omega t - \frac{h\pi}{2}\right)}\,, \tag{15}$$

$$E_Z = \sum_n\sum_q E_{Zn}J_{h+q}(\Gamma_n r_L)J_q(\Gamma_n\tilde{r})\mathrm{e}^{\mathrm{i}q\tilde{\phi}}\times \mathrm{e}^{\mathrm{i}(k_{Zn}Z + h\phi + \omega t - \frac{h\pi}{2})}\,. \tag{16}$$

where $\tilde{\phi} = \arctan\left[\frac{r\cos(\phi-\theta)}{r_\mathrm{L} - r\sin(\phi-\theta)}\right]$.

Substituting Eqs. (14), (15) and (16) into Eq. (13), and for axisymmetric modes $h=0$, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}f_1 = v_{Z0}\frac{\mathrm{d}}{\mathrm{d}Z}f_1 = e\sum_n\sum_q E_{Zn}\mathrm{e}^{\mathrm{i}(k_{Zn}Z + \omega t + q\tilde{\phi})}F_q(\tilde{r},p_\perp,p_Z)\,, \tag{17}$$

where

$$F_q(\tilde{r},p_\perp,p_Z) = J_q(\Gamma_n\tilde{r})J_q(\Gamma_n r_{\rm L})\times \left[\frac{q}{\Gamma_n^2 r_{\rm L}}\left(k_{Zn}-\frac{p_Z}{\gamma mc}\frac{\omega}{c}\right)\frac{\partial f_0}{\partial p_\perp}+\left(1+\frac{p_\perp}{\gamma mc}\frac{\omega}{c}\frac{q}{\Gamma_n^2 r_{\rm L}}\right)\frac{\partial f_0}{\partial p_Z}\right].$$

Since $\mathrm{d}/\mathrm{d}t$ in Eq. (17) denotes time differentiation along the unperturbed electron orbit,

$$f_1 = \frac{e}{v_{Z0}}\sum_n\sum_q E_{Zn}F_q(\tilde{r},p_\perp,p_Z)\times \int_{-\infty}^{0}\mathrm{d}Z'\mathrm{e}^{\mathrm{i}[k_{Zn}Z'+\omega t(Z')+q\tilde{\phi}(Z')]},$$

where

$$Z=v_{Z0}t,\qquad t(Z')=t-\frac{1}{v_{Z0}}(Z-Z'),$$

$$\tilde{\phi}(Z')=\tilde{\phi}-\Omega_{\rm c}\frac{(Z-Z')}{v_{Z0}}.$$

Therefore

$$f_1=e\sum_n\sum_q\frac{E_{Zn}F_q(\tilde{r},p_\perp,p_Z)}{\mathrm{i}(k_{Zn}v_{z0}+q\Omega_{\rm c}+\omega)}\mathrm{e}^{\mathrm{i}(k_{Zn}v_{z0}t+\omega t+q\tilde{\phi})}. \tag{18}$$

2.2 The output microwave power

2.2.1 The expression for output power

The output power averaged over time is given by

$$P=\int\mathrm{d}^3x\int\mathrm{d}^3p\frac{\omega}{2\pi}\int\mathrm{d}t[(-ev_ZE_Z^*)f_1], \tag{19}$$

where

$$\int\mathrm{d}^3p=\int_0^\infty p_\perp\mathrm{d}p_\perp\int_0^{2\pi}\mathrm{d}\tilde{\phi}\int_{-\infty}^{\infty}\mathrm{d}p_Z,$$

$$\int\mathrm{d}^3x=\int_0^{r_{\rm b}}\tilde{r}\mathrm{d}\tilde{r}\int_0^{2\pi}\mathrm{d}\theta\int_0^{L_{\rm s}}\mathrm{d}Z.$$

Therefore

$$P=-e^2E^24\pi^2L_{\rm s}\frac{r_{\rm b}^2}{2}\sum_{n,q}A_n\left\{[J_q(\Gamma_nr_{\rm L})]^2\left[[J_q'(\Gamma_nr_{\rm b})]^2+\left[1-\frac{q^2}{(\Gamma_nr_{\rm b})^2}\right][J_q(\Gamma_nr_{\rm b})]^2\right]\int_0^\infty p_\perp\mathrm{d}p_\perp\int_{-\infty}^{\infty}\mathrm{d}p_Z\frac{p_Z}{\gamma m}\times \left[\frac{q}{\Gamma_n^2r_{\rm L}}\left(k_{Zn}-\frac{p_Z}{\gamma mc}\frac{\omega}{c}\right)\frac{\partial f_0}{\partial p_\perp}+\left(1+\frac{p_\perp}{\gamma mc}\frac{\omega}{c}\frac{q}{\Gamma_n^2r_{\rm L}}\right)\frac{\partial f_0}{\partial p_Z}\right]\Big/ (k_{Zn}v_{z0}+q\Omega_{\rm c}+\omega)^2\right\}, \tag{20}$$

where

$$A_n=\frac{1-\cos\left(2\pi\frac{k_{Zn}}{\omega}\delta v_{Z0}\right)}{2\pi k_{Zn}\delta v_{Z0}},\quad \delta=\frac{v_{Z0}-v_{\rm phase}}{v_{Z0}},$$

$r_{\rm b}$ is the averaged electron radius, and $E^2=\sum\limits_n E_{Zn}^2$.

In Eq. (20), we are only interested in two items. The first one is for $q=0$ and $n=-1$, which gives the Cherenkov oscillation denoting the interaction between the electron beam and the -1 harmonic wave of the backward wave. The other one is for $q=-1$ and $n=0$, which denotes the interaction between the electron beam and the fundamental wave of the backward wave. Therefore

$$P\approx\frac{A_{-1}e^2E^2\nu c^2\pi L_{\rm s}r_{\rm b}^2}{\gamma(k_{Z-1}v_{Z0}+\omega)^2}[J_0(\Gamma_0r_{\rm L})]^2\left[[J_1(\Gamma_{-1}r_{\rm b})]^2+[J_0(\Gamma_{-1}r_{\rm b})]^2\right]-A_0e^2E^2\nu c^2\pi L_{\rm s}r_{\rm b}^2[J_1(\Gamma_0r_{\rm L})]^2\times \left\{\left[J_0(\Gamma_0r_{\rm b})-\frac{1}{\Gamma_0r_{\rm b}}J_1(\Gamma_0r_{\rm b})\right]^2+\left[1-\frac{1}{(\Gamma_0r_{\rm b})^2}\right][J_1(\Gamma_0r_{\rm b})]^2\right\}\times \frac{\nu_{Z0}}{\gamma}\frac{k_{Z0}-v_{Z0}\omega/c^2}{(k_{Z0}v_{Z0}-\Omega_{\rm c}+\omega)^2}=P_1-P_2, \tag{21}$$

where $\nu=\dfrac{n_{\rm b}e^2}{mc^2}$ is Budker parameter.

2.2.2 The discussion on the output power

When $\Omega_{\rm c}$ is big enough, $r_{\rm L}$ inclines to 0. Therefore

$$P\approx\frac{A_{-1}e^2E^2\nu c^2\pi L_{\rm s}r_{\rm b}^2}{\gamma(k_{Z-1}v_{Z0}+\omega)^2}\left[[J_1(\Gamma_{-1}r_{\rm b})]^2+[J_0(\Gamma_{-1}r_{\rm b})]^2\right]. \tag{22}$$

As for Eq. (22), P has the maximum value when $\omega=-k_{Z-1}v_{Z0}=(k_0-k_{Z0})v_{Z0}$ which is considered as the condition for the Cherenkov oscillation.

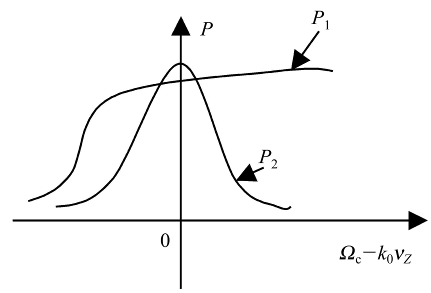

Fig. 3. The relationship between $\Omega_{\rm c}$ and P_1, P_2.

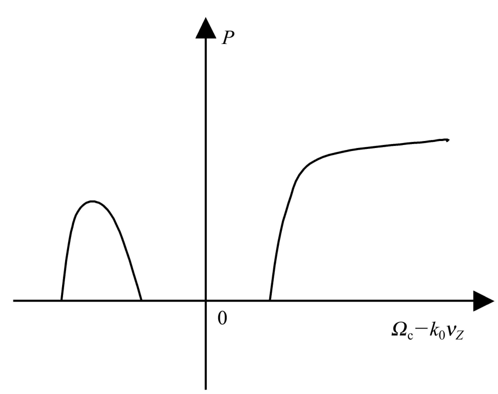

Fig. 4. The relationship between $\Omega_{\rm c}$ and P.

Figure 3 gives the relationships between Ω_c and P_1, P_2 respectively. Fig. 4 gives the relationships between Ω_c and the output power P.

It can be seen from Eq. (21) that we have to elaborate the SWS to let $k_{Z0} - v_{Z0}\omega/c^2 = 0$, so as to maximize the output power.

3 Example

Figure 5 is the sketch of a BWO, which is designed with low magnetic field. Figs. 6(a) and 6(b) are the relationships between the output power and the guiding magnetic field which are respectively attained through simulation and experiment. The results accord with the analyzed result on the whole.

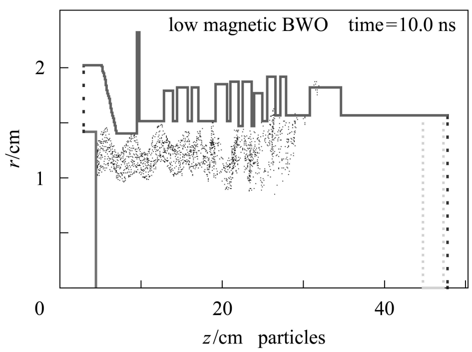

Fig. 5. The sketch of the BWO with low magnetic field.

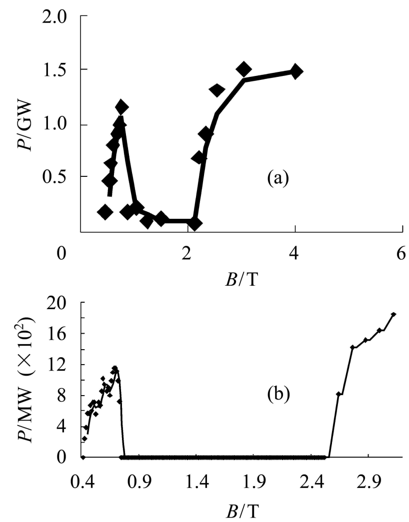

Fig. 6. The relationships between the output power and the guiding magnetic field which are through (a) simulation (b) experiment.

4 Conclusion

Based on the linear Vlasov equation, the relationship between the microwave power and the guiding magnetic field, which accords with the results of the experiments[2, 10, 11], is deduced.

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