Comparison between formulas of rotational band for axially symmetric deformed nuclei *

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Abstract The experimental rotational spectra of the deformed nuclei available in even-even and odd-A nuclei in the rare-earth and actinide regions are systematically analyzed with several rotational spectra formulas, including Bohr-Mottelson's $I(I+1)$-expansion, Harris' ω^2-expansion, ab and abc formulas. It is shown that the simple 2-parameter ab formula is much better than the widely used 2-parameter Bohr-Mottelson's AB formula and Harris' $\alpha\beta$ formula. The available data of the rotational spectra of both ground-state band in even-even nuclei and one-quasiparticle band in odd-A nuclei can be conveniently and rather accurately reproduced by ab formula and abc formula. The moment of inertia and the variation with rotational frequency of angular momentum can be satisfactorily reproduced by ab and abc formulas.

Key words rotational band, moment of inertia, *ab* formula, *abc* formula

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1 Introduction

It is well-established[1] that there exists a large amount of nuclear rotational spectra, particularly in the well-deformed rare-earth and actinide nuclei. Based on the symmetry (axial and reflective) and under adiabatic approximation, Bohr and Mottelson[1] developed the famous angular momentum $I(I+1)$ expansion for nuclear rotational spectra. An angular frequency ω^2 expansion for nuclear rotational spectra was developed by Harris[2]. Later it was found that[3] the convergence of the Harris' ω^2 expansion is better than the $I(I+1)$ expansion. Since then, in addition to Bohr-Mottelson 2-parameter angular momentum AB formula, the Harris 2-parameter ω^2 expansion ($\alpha\beta$) formula has been widely used for analyzing various nuclear rotational spectra. It was shown by A. Klein et al.[4] that the Harris' $\alpha\beta$ formula is equivalent to the variable moment of inertia model presented by Scharff-Goldhaber et al[5]. A simple phenomenological 2-parameter formula for nuclear rotational spectra was presented by Holmberg and Lipas[6]. This type of the above 2-parameter formula was theoretically derived[7, 8] for a well-deformed nucleus with small axial asymmetry ($\sin^2 3\gamma \ll 1$) from Bohr Hamiltonian[9, 10] with a β-separate potential. Then this type of 2-parameter expression is called ab formula. In Ref. [11], a modified 3-parameter expansion (called abc formula) was derived by adding an anharmonic term in Bohr Hamiltonian. In this paper the experimental rotational spectra newly available for the well-deformed rare-earth and actinide nuclei are systematically analyzed by the above formulas. All the experimental data are from the database of Nuclear Data Sheets, NuDat or ENSDF. In Section 2 the ground-state bands (gsb) of even-even nuclei are analyzed. The analyse for the one-quasiparticle rotational bands of odd-A nuclei are given in Section 3. In Section 4 two approaches to extract the moments of inertia (MOI) are presented. The odd-even difference in MOIs between the neighboring nuclei and their variation with rotational angular frequency ω is investigated in Section 4. A brief summary is given in Section 5.

2 The ground state rotational bands of even-even nuclei

The formulas of AB, ABC, $ABCD$ and ab, abc will be used to study the property of the gsb of even-

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even nuclei in this section. Bohr et al.[1] presented the expansion formula of rotational band at the consideration of axial symmetry and adiabatic approximation:

$$E = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \dots\,, \tag{1}$$

In the above equation $\xi^2 = I(I+1) - K^2$, for even-even nuclei, $K = 0$. Usually there are at most four parameters like $ABCD$. The first item of the above equation is purely rotational, and the other items are vibrational corrections. Our calculation shows that the formula is good before the back bending[12].

One of the generally adopted expansion formulas of the rotational bands is Harris ω^2 expansion[2]:

$$E = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 + \dots\,, \tag{2}$$

The definition of ω and its relation with angular momentum I will be discussed below.

The summarization of experimental data indicates that two parameter AB formulas and Harris expansion already have a satisfactory agreement[13], and these two formulas are most popular in the analysis of experimental data.

Zeng J Y et al.[7, 8] assumed that the β item and γ item in Bohr Hamiltonian could be approximately separated and the γ deformation of nuclei is small, then the following two parameters formula of rotational band is derived:

$$E = a[\sqrt{1 + bI(I+1)} - 1]\,. \tag{3}$$

Considering a higher order anharmonic item in Bohr Hamiltonian, Zeng J Y et al.[11] found an abc formula of rational band:

$$E = a[\sqrt{1 + bI(I+1)} - 1] + cI(I+1)\,. \tag{4}$$

Mottelson et al.[3, 14] believe that two-parameter Harris formula is better than AB formula. The reason is that if we assume $\alpha\beta$ formula and $ABCD$ expansion both agree well with the experimental data, then there must be some definite relations between the parameters, and we could use them to verify whether the $\alpha\beta$ formula is in agreement with the above assumption. Based on the Harris two-parameter expansion

$$E = \alpha\omega^2 + \beta\omega^4\,, \tag{5}$$

one can conclude that the parameters $ABCD$ depend on each other and there are two equations. They are,

$$R_1 \equiv \frac{AC}{4B^2} = 1\,, \tag{6}$$

$$R_2 \equiv \frac{A^2 D}{24B^3} = 1\,. \tag{7}$$

They used these equations to analyze some data of rotational bands and they claimed these relations were well satisfied. However, our systematic analysis of rare-earth nuclei and actinide nuclei does not favor their claim.

Similar argument exists for ab and $ABCD$ formulas. We can get two equations of the parameters involved and they can be tested with the experimentally extracted parameters a, b, A, B, C, D, formula[13]:

$$R_1 = \frac{1}{2}\,, \tag{8}$$

$$R_2 = \frac{5}{24}\,. \tag{9}$$

Obviously the result from Harris expansion is totally different from the ab formula one. The validity of the above four relations of the parameters will be discussed as follows. The ground state rotational bands of rare-earth and actinide even-even nuclei are discussed, with γ transition energy levels before back bending, i.e., $I \leqslant I_c$. These nuclei are $^{148-150}$Ce, $^{150-156}$Nd, $^{152-160}$Sm, $^{154-162}$Gd, $^{156-164}$Dy, $^{160-170}$Er, $^{162-176}$Yb, $^{162-184}$Hf, $^{166-188}$W, $^{170-194}$Os and $^{224-226}$Ra, $^{224-234}$Th, $^{230-238}$U, $^{236-244}$Pu, $^{244-248}$Cm, ^{256}Fm, ^{252}No etc., and there are 95 ground state rotational bands.

Two typical examples are given in Table 1 and 2.

Table 1. The gsb of ^{174}Yb. The units for A, B, C, D, a and c are all keV, b is dimensionless.

^{174}Yb, I_c=20						
I	E_γ/keV	AB	ABC	$ABCD$	ab	abc
20	774.0	758.9	775.5	773.7	768.7	774.0
18	719.0	719.5	717.6	719.7	718.3	719.3
16	660.0	667.6	658.6	660.0	661.6	660.0
14	596.0	604.7	595.2	595.1	597.9	595.3
12	525.0	532.2	525.6	524.5	527.1	524.6
10	446.1	451.5	448.5	447.4	449.0	447.4
8	363.9	364.0	364.1	363.5	363.9	363.4
6	272.9	271.2	272.9	272.9	272.6	272.8
4	176.6	174.4	176.4	176.7	176.1	176.7
2	76.5	75.2	76.3	76.5	76.1	76.5
A	$B\times10^3$	$C\times10^6$	$D\times10^9$	$\chi\times10^2$		
12.56	−3.711			1.239		
12.75	−5.672	3.471		0.23		
12.78	−6.391	6.786	−4.279	0.122		
$a\times10^{-3}$	$b\times10^3$	c		$\chi\times10^2$		
14.54	1.75			0.393		
6.539	2.85	3.469		0.114		

Table 2. The gsb of ^{238}U. The units for A, B, C, D, a and c are all keV, b is dimensionless.

^{238}U, I_c=28.0						
I	E_γ/keV	AB	ABC	$ABCD$	ab	abc
28	499.3	470.8	503.2	497.8	498.2	498.8
26	482.8	477.1	481.5	485.3	484.0	484.2
24	467.0	474.3	463.3	468.2	467.6	467.6
22	448.9	463.0	445.5	448.3	448.6	448.6
20	427.9	443.9	426.0	426.0	426.8	426.6
18	402.6	417.9	402.9	400.9	401.6	401.4
16	372.9	385.5	375.3	372.3	372.6	372.4
14	338.8	347.5	342.3	339.6	339.4	339.2
12	300.6	304.6	303.9	302.0	301.5	301.4
10	257.8	257.7	260.1	259.2	258.7	258.7
8	211.0	207.2	211.4	211.4	211.2	211.2
6	158.8	154.1	158.5	159.1	159.1	159.1
4	103.5	99.0	102.5	103.1	103.2	103.3
2	44.9	42.7	44.3	44.7	44.7	44.8
A	$B\times10^3$	$C\times10^6$	$D\times10^9$	$\chi\times10^2$		
7.122	−1.877			3.274		
7.403	−3.24	1.206		0.776		
7.466	−3.904	2.724	−0.987	0.372		
$a\times10^{-3}$	$b\times10^3$	c		$\chi\times10^2$		
6.546	2.286			0.231		
6.313	2.343	0.092		0.226		

We can learn from Table 1 and 2 that the ab formula is the best in two parameters formulas and $ABCD$ formula also has a satisfactory agreement with the experiment.

Figures 1 and 2 show how relations 6, 7, 8, 9 of A,B,C,D hold for the above rotational bands. It is obvious that ab formula seems better than Harris two parameters formula[15]. Some R_1, R_2 of nuclei are not shown in the figure because they have large deviations and are out of range.

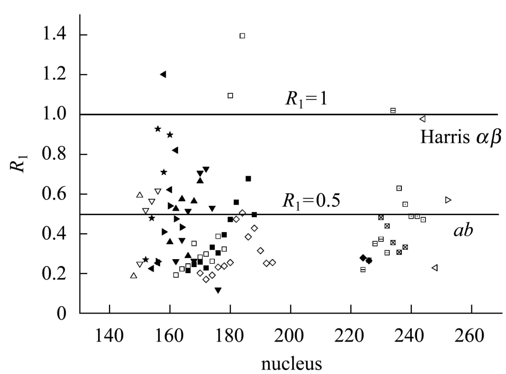

Fig. 1. The distribution of R_1 for the gsbs of even-even nuclei. △: Ce; ▽: Nd; ★: Sm; ◀: Gd; ▶: Dy; ▲: Er; ▼: Yb; □: Hf; ■: W; ◊: Os; ⧫: Ra; ⊟: Th; ⊠: U; ⊡: Pu; ◁: Cm; ▷: No.

Figures 1 and 2 show that R_1 and R_2 derived from ab formula is obviously better than that from Harris expansion, and this means ab formula is a better two parameters formula which agrees well with the experimental data[15].

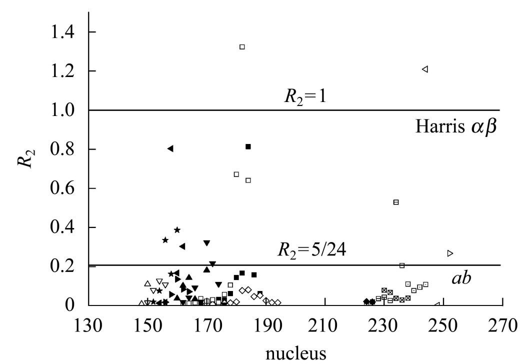

Fig. 2. The distribution of R_2 for the gsbs of even-even nuclei. The illustration is the same as in Fig. 1.

These formulas of rotational band are fine with even-even nuclei[12], but in the following we will see that they are not so good when applied to odd-even nuclei. This is because all these formulas are established with adiabatic approximation[11], which means quasiparticle excitation and rotational excitation can be approximately separated in nuclei. This is also why Xu et al.[13] used only the bands of even-even nuclei. We will discuss odd-A bands as well. The rotational band of even-even nuclei is described better by the above equations for there exists a large energy gap between the vacuum quasiparticle band and the two quasiparticle band, ~1 MeV, and meanwhile the rotational energy is small, ~100 keV. This situation is different for odd-even nuclei, while every odd-even nucleus has some single quasiparticle bands and the energy gaps between these bands are small and comparable to rotational energy, so there may exist a mixing between these single quasiparticle bands. This mixing breaks adiabatic approximation and leads to bad performance of the formulas.

3 One-quasiparticle rotational bands of odd-A nuclei

The formulas of AB, ABC, $ABCD$ and ab, abc are used to study quasiparticle rotational bands of odd-even nuclei in this section. More than 300 quasiparticle bands are included, in four cases: odd proton rare earth nuclei, $^{153-155}$Eu, $^{153-159}$Tb, $^{159-165}$Ho, $^{159-169}$Tm, $^{163-177}$Lu, $^{165,167,171-181}$Ta, $^{171-185}$Re; odd neutron rare earth nuclei, ^{153}Nd, $^{149-153}$Sm, $^{153-157}$Gd, $^{155-161}$Dy, $^{159-163,167}$Er, $^{163-177}$Yb, $^{165-177}$Hf, $^{171-181}$W, $^{171-185}$Os; odd proton actinoid nuclei, ^{231}Pa, ^{237}Np, ^{241}Am; odd neutron actinoid nuclei, ^{233}Th, $^{233-237}$U, ^{243}Pu. These $K=\frac{1}{2}$ one-quasiparticle bands are not included here and will be discussed later.

Usually we take K as 0 when dealing with odd-even nuclei, the following two tables can show if this assumption is appropriate.

The results in Table 3 and 4 indicate that a difference in K only slightly changes the parameters of these formulas and the error of fitting remains almost the same, so we will take K as 0 while dealing with all rotational bands.

Table 3. $K = 0$.

^{153}Eu, $E_0=0$, $K^\pi=\frac{5}{2}^+$, $\pi\frac{5}{2}[413]$, $\alpha=-1/2$, $I_c=19.5$						
I	E_γ/keV	AB	ABC	$ABCD$	ab	abc
19.5	650.9	639.5	652.5	651.2	655.2	651.6
17.5	621.3	622.6	618.5	620.6	620.2	619.9
15.5	580.6	588.9	579.6	580.5	578.8	580.3
13.5	530.7	540.3	532.3	531.7	529.9	531.9
11.5	473.3	479.0	475.0	473.8	472.5	474.0
9.5	406.9	406.7	406.9	406.2	405.6	406.3
7.5	329.6	325.6	328.7	328.7	329.1	328.7
5.5	241.7	237.5	241.7	242.3	243.2	242.2
A	$B\times10^3$	$C\times10^6$	$D\times10^9$	$\chi\times10^2$		
12.14	−5.149			1.345		
12.47	−7.646	4.184		0.270		
12.55	−8.759	8.783	−5.768	0.169		
$a\times10^{-3}$	$b\times10^3$	c		$\chi\times10^2$		
7.926	3.192			0.375		
12.82	2.345	−2.484		0.192		

Table 4. $K = 2.5$.

^{153}Eu, $E_0=0$, $K^\pi=\frac{5}{2}^+$, $\pi\frac{5}{2}[413]$, $\alpha=-1/2$, $I_c=19.5$						
I	E_γ/keV	AB	ABC	$ABCD$	ab	abc
19.5	650.9	639.5	652.5	651.2	655.2	651.6
17.5	621.3	622.6	618.5	620.6	620.2	619.9
15.5	580.6	588.9	579.6	580.5	578.8	580.3
13.5	530.7	540.3	532.3	531.7	529.9	531.9
11.5	473.3	479.0	475.0	473.8	472.5	474.0
9.5	406.9	406.7	406.9	406.2	405.6	406.3
7.5	329.6	325.6	328.7	328.7	329.1	328.7
5.5	241.7	237.5	241.7	242.3	243.2	242.2
A	$B\times10^3$	$C\times10^6$	$D\times10^9$	$\chi\times10^2$		
12.07	−5.149			1.345		
12.37	−7.567	4.184		0.270		
12.44	−8.596	8.639	−5.768	0.169		
$a\times10^{-3}$	$b\times10^3$	c		$\chi\times10^2$		
8.005	3.129			0.375		
12.91	2.311	−2.484		0.192		

The relations of R_1, R_2 of odd-even nuclei are also studied and the results are listed in Figs. 3 to 6. It shows that even the results of odd-even nuclei is somewhat bad compared with the even-even nuclei, the experimental data can also demonstrate that ab formula is better than Harris two parameters formula. Some R_1, R_2 of nuclei are not shown in Figs. 3 to 6 because they are out of range with large deviations.

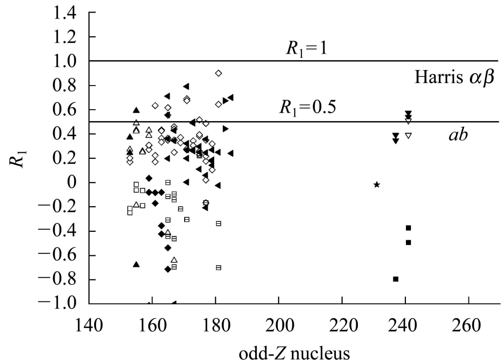

Fig. 3. The distribution of R_1 of rotational bands of odd-Z nuclei.

Here $R_1=\dfrac{AC}{4B^2}$. The legends are as follows: △: $\pi\frac{3}{2}[411]$; ▽: $\pi\frac{3}{2}[521]$; ★: $\pi\frac{3}{2}[651]$; ◀: $\pi\frac{5}{2}[402]$; ▶: $\pi\frac{5}{2}[404]$; ▲: $\pi\frac{5}{2}[413]$; ▼: $\pi\frac{5}{2}[523]$; □: $\pi\frac{5}{2}[532]$; ■: $\pi\frac{5}{2}[642]$; ◇: $\pi\frac{7}{2}[404]$; ◆: $\pi\frac{7}{2}[523]$; ⊟: $\pi\frac{9}{2}[514]$.

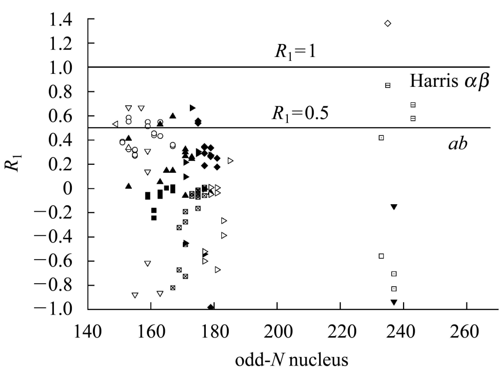

Fig. 4. The distribution of R_1 of rotational bands of odd-N nuclei.

The legends are: △: $\nu\frac{3}{2}[402]$; ▽: $\nu\frac{3}{2}[521]$; ★: $\nu\frac{3}{2}[631]$; ▶: $\nu\frac{5}{2}[512]$; ▲: $\nu\frac{5}{2}[523]$; ▼: $\nu\frac{5}{2}[622]$; □: $\nu\frac{5}{2}[633]$; ■: $\nu\frac{5}{2}[642]$; ◇: $\nu\frac{5}{2}[752]$; ◆: $\nu\frac{7}{2}[514]$; ⊟: $\nu\frac{7}{2}[624]$; ⊠: $\nu\frac{7}{2}[633]$; ⊡: $\nu\frac{7}{2}[743]$; ◁: $\nu\frac{9}{2}[505]$; ▷: $\nu\frac{9}{2}[624]$; ◯: $\nu\frac{11}{2}[505]$.

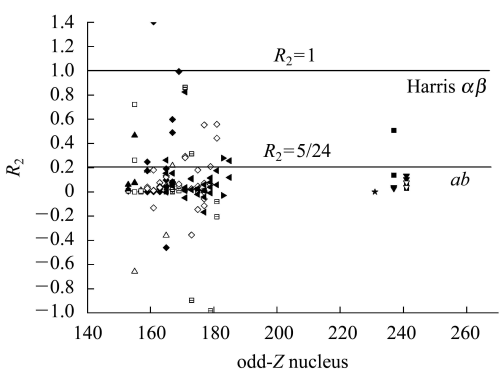

Fig. 5. The distribution of R_2 of rotational bands of odd-Z nuclei. The illustration is the same as in Fig. 3.

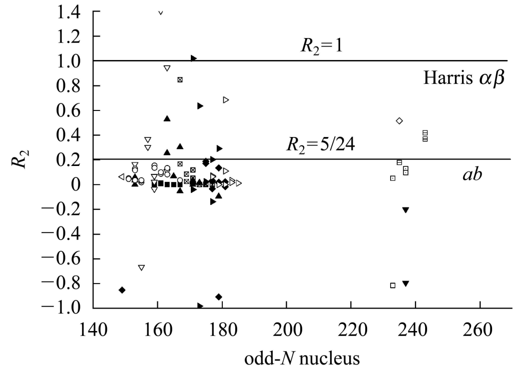

Fig. 6. The distribution of R_2 of rotational bands of odd-N nuclei. The illustration is the same as in Fig. 4.

4 Extraction of the moments of inertia

The normally deformed rotational bands of even-even and odd-even nuclei are studied in the above sections. Here we will study the odd-even difference of moment of inertia by extracting the bandhead moment of inertia from the experimental data.

4.1 Extraction of MOI directly via the observed rotational spectra

The moment of inertia $J^{(1)}$ can be directly extracted from the experimental data. It can also be extracted from formulas like AB et al[16]. Firstly we will introduce the method of directly extracting the rotational angular frequency and the moment of inertia from experiment. The definition of angular frequency ω is:

$$\hbar\omega=\frac{\mathrm{d}E}{\mathrm{d}I_x}\,, \tag{10}$$

E is the rotational energy, and I_x is the projection of angular momentum I on axis x:

$$I_x(I)=\sqrt{\left(I+\frac{1}{2}\right)^2-K^2}\approx\sqrt{I(I+1)-K^2}\,, \tag{11}$$

Substituting continuous difference with discrete difference, we get

$$\hbar\omega(I)\approx\frac{E(I+1)-E(I-1)}{I_x(I+1)-I_x(I-1)}=\frac{E_\gamma(I+1\to I-1)}{I_x(I+1)-I_x(I-1)}, \tag{12}$$

For $K=0$ band $I_x(I+1)-I_x(I-1)=2$, so

$$\hbar\omega(I)=\frac{1}{2}E_\gamma(I+1\to I-1)\,. \tag{13}$$

The definition of type-I kinematic moment of inertia $J^{(1)}$ is $I_x=\omega J^{(1)}/\hbar$, which means

$$\frac{J^{(1)}}{\hbar^2}=\frac{I_x}{\hbar\omega}=\frac{I_x}{\dfrac{\mathrm{d}E}{\mathrm{d}I_x}}=\frac{1}{2}\frac{\mathrm{d}I_x^2}{\mathrm{d}E}. \tag{14}$$

The definition of I_x is given earlier. Because $\mathrm{d}I_x^2 = 2\left(I+\frac{1}{2}\right)\mathrm{d}I$ and $J^{(1)}$ is the function of I, we have

$$\frac{J^{(1)}(I)}{\hbar^2} = \frac{2I+1}{E_\gamma(I+1\rightarrow I-1)}. \tag{15}$$

For high K rotational band, if the splitting of signature is small, we have

$$\hbar\omega\left(I+\frac{1}{2}\right) = \frac{E_\gamma(I+1\rightarrow I)}{I_x(I+1)-I_x(I)} \approx E_\gamma(I+1\rightarrow I), \quad (I,K\gg 1). \tag{16}$$

$$\frac{J^{(1)}\left(I+\frac{1}{2}\right)}{\hbar^2} = \frac{I+1}{E_\gamma(I+1\rightarrow I)}. \tag{17}$$

The definition of type-II dynamic moment of inertia $J^{(2)}$ is

$$\frac{J^{(2)}}{\hbar^2} = \left(\frac{\mathrm{d}}{\mathrm{d}I_x}\frac{\mathrm{d}E}{\mathrm{d}I_x}\right)^{-1} = \left(\frac{\mathrm{d}^2E}{\mathrm{d}I_x^2}\right)^{-1}, \tag{18}$$

which is applicable to super deformed bands. Considering the following equations,

$$\hbar\omega(I+1) = \frac{E_\gamma(I+2\rightarrow I)}{I_x(I+2)-I_x(I)}, \tag{19}$$

$$\hbar\omega(I-1) = \frac{E_\gamma(I\rightarrow I-2)}{I_x(I)-I_x(I-2)}, \tag{20}$$

thus

$$J^{(2)}(I) \approx \left(\frac{1}{\Delta I_x}\frac{\Delta^2 E}{\Delta I_x}\right)^{-1} = \frac{4}{E_\gamma(I+2\rightarrow I)-E_\gamma(I\rightarrow I-2)}. \tag{21}$$

4.2 Extraction of MOI via formulas of rotational spectra

In this subsection we will discuss the method of extracting moment of inertia from formulas of rotational band like AB, ab, etc.

According to Bohr-Mottelson $I(I+1)$ expansion, for $K=0$ band $\xi^2 = I(I+1)$, we have

$$\frac{J^{(1)}(I)}{\hbar^2} = \left(\frac{1}{I_x}\frac{\mathrm{d}E}{\mathrm{d}I_x}\right)^{-1} = \frac{1}{2A}\left(1+\frac{2B}{A}\xi^2+\frac{3C}{A}\xi^4+\ldots\right)^{-1}, \tag{22}$$

$$\frac{J^{(2)}(I)}{\hbar^2} = \left(\frac{\mathrm{d}^2E}{\mathrm{d}I_x^2}\right)^{-1} = \frac{1}{2A}\left(1+\frac{6B}{A}\xi^2+\frac{15C}{A}\xi^4+\ldots\right)^{-1}. \tag{23}$$

For $K\neq 0$ bands, $\xi^2 = I(I+1)-K^2$. The bandhead moment of inertia $J_0 = \frac{\hbar^2}{2A}$.

According to Harris ω^2 expansion,

$$\frac{J^{(1)}(\omega)}{\hbar^2} = 2\alpha+\frac{4}{3}\beta\omega^2+\frac{6}{5}\gamma\omega^4+\ldots, \tag{24}$$

$$\frac{J^{(2)}(\omega)}{\hbar^2} = 2\alpha+4\beta\omega^2+6\gamma\omega^4+\ldots. \tag{25}$$

The bandhead moment of inertia $J_0 = 2\alpha$. Harris two parameters formula is most popular, ie., $J(\omega) = J_0+J_1\omega^2$.

The relation between ω in Harris expansion and I in Bohr-Mottelson expansion can be derived as follows:

$$\omega = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}I_x} = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}\xi}\frac{\mathrm{d}\xi}{\mathrm{d}I_x} \approx \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}\xi}, \tag{26}$$

so

$$\hbar\xi = \int\frac{\mathrm{d}E}{\omega} = \int\frac{1}{\omega}\frac{\mathrm{d}E}{\mathrm{d}\omega}\mathrm{d}\omega = \int(2\alpha+4\beta\omega^2+6\gamma\omega^4+\ldots)\mathrm{d}\omega = 2\alpha\omega+\frac{4}{3}\beta\omega^3+\frac{6}{5}\gamma\omega^5+\ldots. \tag{27}$$

When $K\neq 0$ the above equation can be written as

$$\hbar\sqrt{I(I+1)-K^2} = 2\alpha\omega+\frac{4}{3}\beta\omega^3+\frac{6}{5}\gamma\omega^5+\ldots. \tag{28}$$

Similarly, formulas ab and abc can be used to extract the moment of inertia. The result from ab formula is:

$$\frac{J^{(1)}}{\hbar^2} = J_0(1+b\xi^2)^{\frac{1}{2}} \approx J_0\left(1-\frac{\hbar^2\omega^2}{a^2b}\right)^{-\frac{1}{2}}, \tag{29}$$

$$\frac{J^{(2)}}{\hbar^2} = J_0(1+b\xi^2)^{\frac{3}{2}} \approx J_0\left(1-\frac{\hbar^2\omega^2}{a^2b}\right)^{-\frac{3}{2}}, \tag{30}$$

Here $J_0 = \frac{\hbar^2}{ab}$.

The result from abc formula is:

$$\frac{\hbar^2}{J^{(1)}(I)} = ab(1+b\xi^2)^{-\frac{1}{2}}+2c, \tag{31}$$

$$\frac{\hbar^2}{J^{(2)}(I)} = ab(1+b\xi^2)^{-\frac{3}{2}}+2c. \tag{32}$$

To extract angular frequency ω and type-I moment of inertia $J^{(1)}$ from experiment, we usually use

$$\hbar\omega(I) = \frac{1}{2}E_\gamma(I+1\rightarrow I-1), \tag{33}$$

$$\frac{J^{(1)}(I)}{\hbar^2} = \frac{2I+1}{E_\gamma(I+1\rightarrow I-1)}. \tag{34}$$

in which K is ignored.

Here we will compare the moments of inertia, which are directly extracted from experiment, with the calculated ones from abc formula. Fig. 7 gives an example. It shows that these two results are very close and this demonstrates the validity of ab and abc formulas.

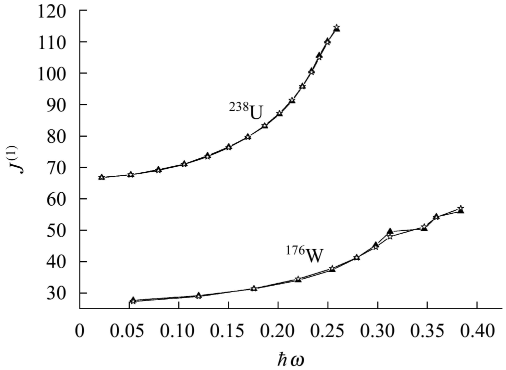

Fig. 7. This figure displays the variation of moment of inertia of the ground state of ^{176}W and ^{238}U with increasing ω. ▲ is extracted from experiment and the other is from *abc* formula.

4.3 Odd-even difference in MOI

The definition of odd-even difference δJ is[17]:

$$\delta J = \frac{J^{(1)}(\text{odd}) - J^{(1)}(\text{even})}{J^{(1)}(\text{even})}, \tag{35}$$

odd means odd-even nucleus and even means even-even ones adjacent.

Figures 8 and 9 give the results.

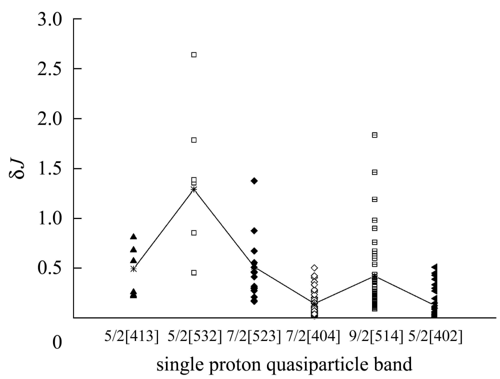

Fig. 8. The odd-even difference of normally deformed band of different single proton quasiparticle bands.
The legends are the same as Fig. 3. $*$ is the average of δJ in the same quasiparticle band. The solid line is the connection of all $*$.

Figure 10 shows, for the $\nu\frac{7}{2}[633]$ band of ^{177}W and adjacent even-even nucleus ^{176}W, how type-I MOI changes while angular frequency ω increases. It is obvious that δJ gradually decreases with the increasing of ω.

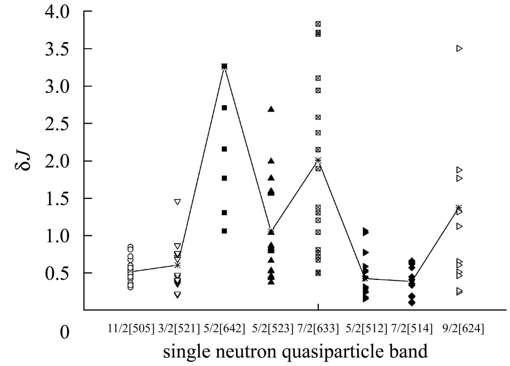

Fig. 9. The odd-even difference of normally deformed band of different single neutron quasiparticle bands.
The legends are the same as Fig. 4. $*$ is the average of δJ in the same quasiparticle band. The solid line is the connection of all $*$.

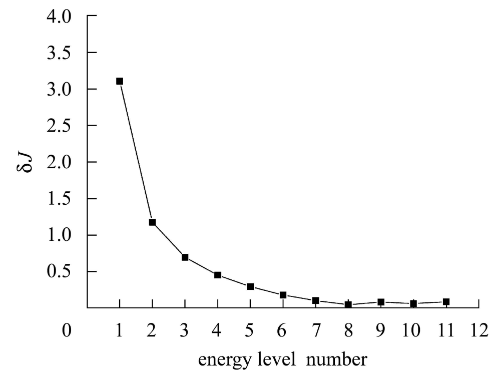

Fig. 10. The change of δJ with the increasing of ω, for the $\nu\frac{7}{2}[633]$ band of ^{177}W and the ground state band of ^{176}W. The abscissa is the δJ between the two bands.

5 Summary

We have discussed the most popular nuclear rotational band formulas, i.e., Bohr-Mottelson's $I(I+1)$-expansion, Harris' ω^2-expansion, as well as *ab* and *abc* formulas. The relations between their parameters are investigated, and used to check the validity of the formulas, in which the newest rotational bands of both even-even and odd-A nuclei are included. Systematic analyse of newest rare earth and actinide nuclear rotational bands are presented, including the odd-even difference of moments of inertia of these bands. The extracted MOIs by *ab* or *abc* formula have explained nuclear odd-even difference satisfactorily.

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