Azimuthal correlations between directed and elliptic flow in heavy ion collisions*

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Abstract A method for investigating the azimuthal correlations between directed and elliptic flow in heavy ion collisions is described. The transverse anisotropy of particle emission at AGS energies is investigated within the RQMD model. It is found that the azimuthal correlations between directed and elliptic flow are sensitive to the incident energy and impact parameter. The fluctuations in the initial stage and dynamical evolution of heavy ion collisions are not negligible.

Key words heavy ion collisions, anisotropic flow, azimuthal correlations

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1 Introduction

Determining the nuclear equation of state (EOS) under various conditions has been one of the largest motivations of heavy ion physics in these decades[1, 2]. In noncentral collisions, the initial spatial deformation due to geometry and the pressure developed early cause azimuthal anisotropy in momentum space. As a result, azimuthal angles of outgoing particles are generally correlated with the direction of the impact parameter. Up to now, several kinds of collective flows, especially the directed and elliptic flow, has been proposed to probe high dense matter[3, 4]. The anisotropic collective flow data obtained at AGS energies (1 AGeV $\lesssim E_{\rm beam} \lesssim$ 11 AGeV) show a good landmark to determine EOS[5—7]. In this energy range, the directed flow firstly grows, saturates at around 2 AGeV and then decreases experimentally with increasing in energy. On the other hand, elliptic flow measured experimentally changes its sign from negative to positive as a function of the incident energy.

In noncentral collisions, the two reasons for anisotropic flow are the original asymmetry in the configuration space and rescatterings[8]. The standard methods to measure flow mainly include those using explicitly the concept of the estimated reaction plane and those based on the two-particle azimuthal correlations[9, 10]. The reaction plane is defined by the beam direction and the impact parameter vector, and it can be determined in each event from the emitted particles. However, the dispersion of the estimated reaction plane must be corrected in order to obtain a measurement of the flow. Two-particle correlations based methods suffer from correlations unrelated to the reaction plane. These additional contributions are usually called nonflow effects[11].

In our earlier work on anisotropic flow[12, 13], we have shown that there was a clear signature of multiparticle azimuthal correlations between the two anisotropic components of transverse collective flow in heavy ion collisions. In this paper, we will present the multiparticle azimuthal correlation functions, and the correlations between the directed flow and the elliptic flow will be analyzed at AGS energies. The calculations are carried out within the microscopic transport model RQMD (v2.4, mean-field mode) for Au + Au collisions. A detailed description of the RQMD model can be found elsewhere[14]

2 Multiparticle azimuthal correlation function

Anisotropic collective flow, which describes the

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azimuthal momentum distribution with respect to the reaction plane in noncentral heavy ion collisions, is sensitive to the nuclear EOS[15]. In a certain coordinate system with z-axis along the beam direction, anisotropic flow can be characterized by the Fourier coefficients of the particle azimuthal distribution[16]:

$$v_n = \langle \cos n(\phi - \psi_r) \rangle \ , \tag{1}$$

where ϕ is the azimuthal angle of an outgoing particle, ψ_r is the orientation of the reaction plane, and the angular bracket denotes an average over all particles in a given phase-space region and over many events. The first and second harmonics, v_1 and v_2, characterize the directed flow and elliptic flow, respectively.

For noncentral collisions, anisotropic collective flow describes the azimuthal anisotropy of the event. This means that the reaction plane can be determined independently by using each harmonic of the anisotropic flow itself. The flow vectors $\boldsymbol{Q}_n$ can be reconstructed in a given coordinate system for each event[17, 18]:

$$\boldsymbol{Q}_n = Q_n \cos(n\psi_n)\boldsymbol{e}_x + Q_n \sin(n\psi_n)\boldsymbol{e}_y \ , \tag{2}$$

with

$$Q_n \cos(n\psi_n) = \sum_{j=1}^{M} \omega_j \cos(n\phi_j) = Q_{nx},$$

$$Q_n \sin(n\psi_n) = \sum_{j=1}^{M} \omega_j \sin(n\phi_j) = Q_{ny}.$$

where ω_j are the weights depending on the transverse momentum, the particle mass and the rapidity. ψ_n characterizes the direction of n-th type flow, $0 \leqslant n\psi_n < 2\pi$.

Flow vector $\boldsymbol{Q}_n$ can be calculated from experimental data, event by event. Usually, the number M of particles entering the definition of the flow vector $\boldsymbol{Q}_n$ is much large than unity. Then the central limit theorem shows that the distribution of $\boldsymbol{Q}_n$ around its average value $\langle \boldsymbol{Q}_n \rangle$ is a Gaussian[17, 18]

$$\frac{\mathrm{d}P}{\mathrm{d}\boldsymbol{Q}_n} = \frac{1}{\pi\sigma_n^2} \exp\left(-\frac{(\boldsymbol{Q}_n - \langle \boldsymbol{Q}_n \rangle)^2}{\sigma_n^2}\right), \tag{3}$$

where σ_n is the typical magnitude of the fluctuations of $\boldsymbol{Q}_n$, it scales with M like $\sqrt{M}$. A nonvanishing value for the average value of the flow vector $\langle \boldsymbol{Q}_n \rangle$ signals collective flow.

The measurement of $\langle \boldsymbol{Q}_n \rangle$ is not a trivial task because the orientation of the reaction plane is unknown, so the direction of $\boldsymbol{Q}_n$ is unknown. The only measurable quantity is $|\boldsymbol{Q}_n|$, the modulo of $\boldsymbol{Q}_n$. The correlation between every particle and the reaction plane can induce the correlations among the particles. Anisotropic flow is a genuine multiparticle phenomenon. If one considers many-particle correlations instead of two-particle correlations, the relative contribution of nonflow effects would decrease. The azimuthal correlations between the directed and elliptic flow can be computed from Eq. (3) by the joint probability distribution

$$C(\beta) = \int \frac{1}{\pi^2 \sigma_1^2 \sigma_2^2} \exp\left[-\frac{(\boldsymbol{Q}_1 - \langle \boldsymbol{Q}_1 \rangle)^2}{\sigma_1^2} - \frac{(\boldsymbol{Q}_2 - \langle \boldsymbol{Q}_2 \rangle)^2}{\sigma_2^2}\right] Q_1 Q_2 \mathrm{d}Q_1 \mathrm{d}Q_2 \times \mathrm{d}\psi_1 \mathrm{d}(2\psi_2) \delta(\beta - \psi_2 + \psi_1). \tag{4}$$

The relative angle $\beta = \psi_2 - \psi_1$ should fall into the interval $[-2\pi, \pi]$. However, β and $\beta+\pi$ are equivalent in transverse plane, and β can be expressed as

$$\beta = \arccos\left(\frac{\boldsymbol{Q}'_1 \cdot \boldsymbol{Q}'_2}{Q'_1 Q'_2}\right). \tag{5}$$

where $\boldsymbol{Q}'_n = Q_n \cos\psi_n \boldsymbol{e}_x + Q_n \sin\psi_n \boldsymbol{e}_y$. So we can normalize $C(\beta)$ to unity between 0 and π. It is convenient to introduce dimensionless parameter[17] $\chi_n = Q_n/\sigma_n$ and $\bar{\chi}_n = \bar{Q}_n/\sigma_n$, which characterize the relative magnitude of flow and statistical fluctuations. Eq. (4) can be written as

$$C(\beta) = \frac{2}{\pi^2} \exp(-\bar{\chi}_1^2 - \bar{\chi}_2^2) \int_0^{+\infty} \mathrm{d}\chi_1 \int_0^{+\infty} \mathrm{d}\chi_2 \int_0^{2\pi} \mathrm{d}\psi \times \chi_1 \chi_2 \exp(-\chi_1^2 + 2\chi_1 \bar{\chi}_1 \cos\psi) \exp(-\chi_2^2 + 2\chi_2 \bar{\chi}_2 \cos 2(\psi + \beta - (\bar{\psi}_2 - \bar{\psi}_1))). \tag{6}$$

where $\bar{\psi}_1$ and $\bar{\psi}_2$ are the orientations of directed flow and elliptic flow, respectively.

At AGS energies, both the directed flow and elliptic flow have been studied extensively. The elliptic flow measured experimentally is relatively weak[6, 7]. In this case, Eq. (6) becomes

$$C(\beta) = \frac{1}{\pi}\left[1 + \frac{\sqrt{\pi}\bar{\chi}_2}{\bar{\chi}_1^2}(\bar{\chi}_1^2 - 1 + \exp(-\bar{\chi}_1^2)) \times \cos 2(\beta + (\bar{\psi}_2 - \bar{\psi}_1))\right]. \tag{7}$$

From Eq. (6) and Eq. (7), $C(\beta)$ can be expressed approximately as

$$C(\beta) = A_0[1 + A_{12} \cos 2(\beta + (\bar{\psi}_2 - \bar{\psi}_1))]. \tag{8}$$

The coefficient A_{12} quantifies the strength of the azimuthal correlations between the directed and elliptic flows. The relative direction of the directed and elliptic flows is parallel, $\bar{\psi}_2 - \bar{\psi}_1 \sim 0$, which corresponds to the definition of positive elliptic flow, while $\bar{\psi}_2 - \bar{\psi}_1 \sim \frac{\pi}{2}$ is the negative elliptic flow.

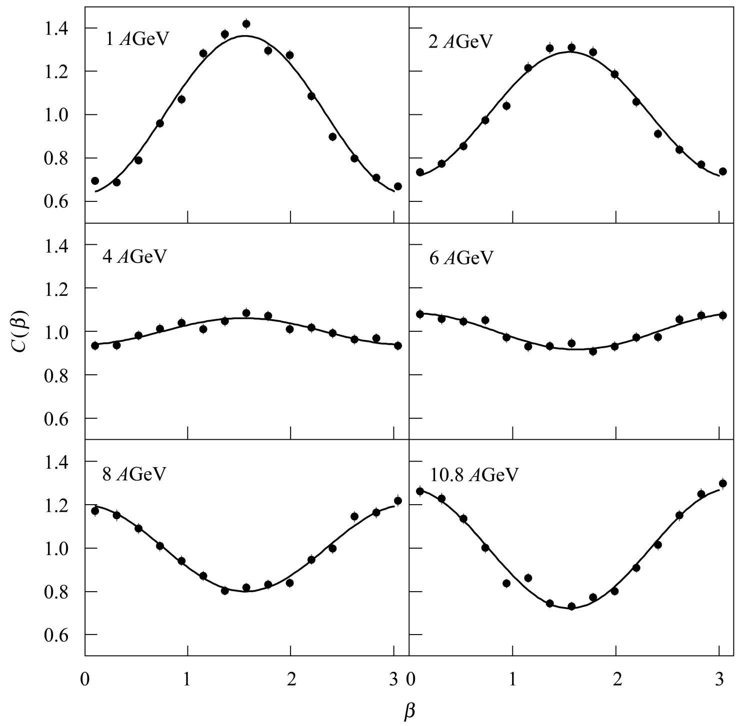

Fig. 1. Azimuthal correlation functions $C(\beta)$ for nucleons from RQMD calculations for Au+Au collisions (b=6 fm) at AGS energies. The curves are fitted by Eq. (8).

3 Incident energy dependence of the azimuthal anisotropy

Both the directed flow and the elliptic flow have been measured and reported for heavy ion collisions at AGS energy[5—7, 15, 19, 20]. The azimuthal correlations between the directed and elliptic flow can be determined without event-by-event estimation of the reaction plane. Fig. 1 shows the azimuthal correlation functions $C(\beta)$ for nucleons from RQMD calculations for Au+Au collisions at incident energy $E_{\rm beam}=1$, 2, 4, 6, 8 and 10.8 AGeV with the impact parameter b=6 fm. The vectors $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are calculated within the normalized rapidity region of $\left|\frac{y}{y_{\rm beam}}\right|_{\rm cm}\geqslant 0.3$ and $\left|\frac{y}{y_{\rm beam}}\right|_{\rm cm}<0.3$, respectively, so the autocorrelation effect in Eq. (5) is removed. The curves in Fig. 1 are obtained using Eq. (8) and the fitting results are shown in Table 1.

Table 1. The fitted results by using Eq. (8) for Au+Au collisions from RQMD calculations.

beam energy	A_{12}	$\bar{\psi}_2-\bar{\psi}_1$	χ^2/ndf
1 AGeV	0.365±0.008	1.58±0.01	40.85/12
2 AGeV	0.290±0.008	1.59±0.01	18.87/12
4 AGeV	0.061±0.009	1.60±0.07	7.68/12
6 AGeV	0.083±0.009	−0.05±0.05	8.08/12
8 AGeV	0.200±0.009	0.01±0.02	9.25/12
10.8 AGeV	0.277±0.008	0.02±0.02	22.32/12

When the anisotropic flow vanishes, the azimuthal distribution of the nucleons should become isotropic in the whole rapidity range, and the correlation function will be flat. The fact that the curves in Fig. 1 are not flat confirms the multiparticle azimuthal correlation between the two components of the transverse collective motion. It can be seen clearly from Fig. 1 and Table 1 that the azimuthal correlations between the directed and elliptic flow are sensitive to the incident energy. The value of A_{12} firstly decreases and then increases from 1 AGeV to 11 AGeV, and the smallest value is at $E_{\rm beam}\sim 5$ AGeV. The fitted value of $\bar{\psi}_2-\bar{\psi}_1\approx\frac{\pi}{2}$ and $\bar{\psi}_2-\bar{\psi}_1\approx 0$ for the incident energy $E_{\rm beam}\lesssim 5$ AGeV and $E_{\rm beam}\gtrsim 5$ AGeV, respectively. The change of $\bar{\psi}_2-\bar{\psi}_1$ corresponds to the relative orientation of directed and elliptic flow from negative at lower energies to positive elliptic flow at higher energies. The results are consistent with the previous experimental and theoretical studies[6, 7, 15, 19, 20].

In order to quantify the anisotropic emission, we can use a parameter R, which is defined as the ratio of the number of event with elliptic flow parallel to directed flow to the number of event with elliptic flow perpendicular to directed flow

$$R=\frac{C(\beta=0)+C(\beta=\pi)}{C\left(\beta=\frac{\pi}{2}\right)+C\left(\beta=\frac{3\pi}{2}\right)}. \quad (9)$$

From the definition, $R>1$ and $R<1$ are related to positive and negative elliptic flow, respectively. From

Eq. (8) and Eq. (9), one can get:

$$R=\frac{1+A_{12}\cos 2(\bar{\psi}_2-\bar{\psi}_1)}{1-A_{12}\cos 2(\bar{\psi}_2-\bar{\psi}_1)}. \tag{10}$$

The value of R characterizes the correlation strength of anisotropic flow as well as the relative direction of directed and elliptic flow. Fig. 2 shows the dependence of R on the incident energy for the events used in Fig. 1. It can be seen that the ratio R increases and crosses 1 with increasing incident energy, indicating change in the sign of elliptic flow with increasing incident energy. Danielewicz has pointed out that the competition between squeeze-out and in-plane elliptic flow at AGS energies depends on the nature of the nuclear force[20]. At lower energies ($E_{\rm beam}$ <5 AGeV), $R<1$, the elliptic flow is perpendicular to the directed flow since the projectile and the target spectators distort the collective expansion of the 'fireball' in the reaction plane. At higher energies($E_{\rm beam}$ >5 AGeV), $R>1$, the elliptic flow becomes parallel to the directed flow because the projectile and target spectators do not hinder anymore the in-plane expansion of the 'fireball' due to their high velocity.

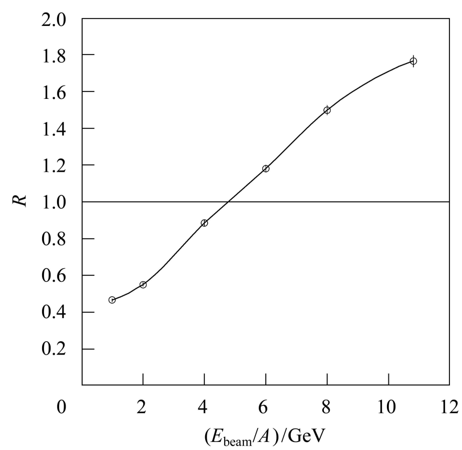

Fig. 2. The values of R as a function of the incident energy for nucleons from RQMD calculations in Au+Au collisions (b=6 fm) at AGS energies.

4 Centrality dependence of the azimuthal anisotropy

An important reason for anisotropic flow in non-central heavy ion collisions is the original asymmetry in the configuration space. The initial asymmetry of the overlap zone is characterized by the impact parameter. The dependence of the ratio R on impact parameter in 2 AGeV Au+Au collisions is shown in Fig. 3. The values of $R<1$ in Fig. 3 indicate that the elliptic flow for nucleons is negative on an average. The collective emission effect is stronger in the semi-central collisions due to the fact that for peripheral collisions, the multiplicity is small, and for central events, the anisotropic flow is small. The impact parameter dependence of collective flow is consistent with those obtained from the experimental data[7].

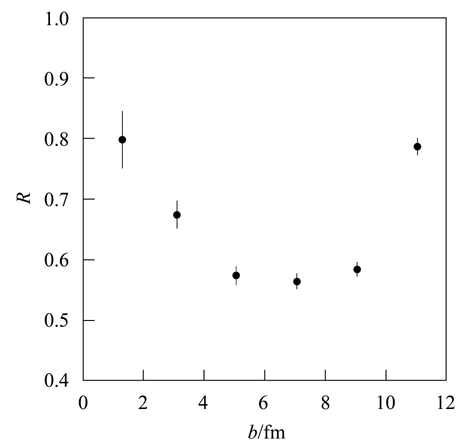

Fig. 3. The value of R as a function of impact parameter for nucleons from RQMD calculations in 2 AGeV Au+Au collisions.

Measurements of the elliptic flow have been predicted to provide the information that is crucial for determining the parameters of the nuclear equation of state. Generally, the study of anisotropic transverse flow requires an averaging in order to obtain a statistically relevant measurement of the flow. The average is done firstly over all particles in one event and then over a large number of events with nearly equal multiplicities. On average, the elliptic flow is negative in 2 AGeV Au+Au collisions[6, 7]. The fact that the shapes of $C(\beta)$ in Fig. 1 are not flat exhibits an event-by-event azimuthal fluctuations also. The dependence of v_2 on azimuthal angle β for nucleons in 2 AGeV Au+Au collisions at $b=6$ fm is shown in Fig. 4.

The elliptic flow is a collective flow pattern which develops in noncentral relativistic heavy ion collisions as a result of the spatial deformation of the initial transverse overlap area. It requires rescatterings among the produced particles as a mechanism to map the initial spatial deformation of the reaction zone onto the finally observed hadron momentum distributions. In Fig. 4, the incident energy and the impact parameter are same for all events. Although the elliptic flow is negative in 2 AGeV Au+Au collisions on the average, the results indicate that the elliptic flow is positive for the events with $\beta\sim 0$ or $\beta\sim\pi$. In a non-central collision, the initial spatial anisotropy gives rise to the final momentum anisotropy, on account of multiple interparticle collisions. So it can indicate that the fluctuations in the initial stage and dynamical evolution of heavy ion collisions are not negligible. The study of multiparticle azimuthal correlations might offer more information about the space-time evolution of the system.

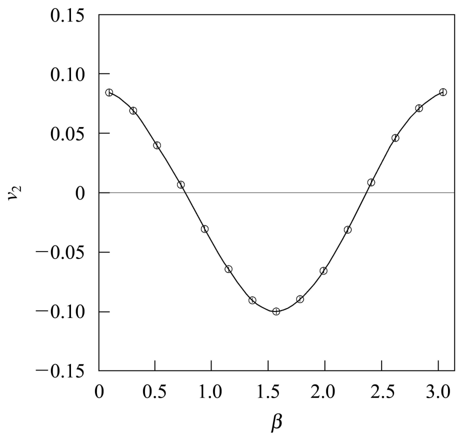

Fig. 4. Elliptic flow as a function of relative azimuthal angle $\beta = \psi_2 - \psi_1$ for nucleons in 2 AGeV Au+Au collisions at $b=6$ fm.

5 Conclusion

The angular correlations generated by collective flow retain some signature of nuclear matter in heavy ion collisions. We studied the azimuthal correlations between the directed and the elliptic flow and a correlation function was presented. We have analyzed the azimuthal correlations for nucleons in Au+Au collisions as a function of the incident energy and impact parameter at AGS energies within the RQMD (v2.4) model. A clear signature of the azimuthal correlations between the directed and the elliptic flow has been observed. The fact that the shapes of the correlation functions are not flat exhibits an event-by-event azimuthal fluctuations. The dependence of v_2 on azimuthal angle between the directed and the elliptic flow for nucleons in 2 AGeV Au+Au collisions at $b = 6$ fm was given. The results showed that v_2 fluctuations existed in events with different β. The fluctuations may be sensitive to the early stage dynamics of heavy ion collisions.

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