

Understanding the Interaction between Dark Energy and Dark Matter^{*}

WANG Bin¹⁾

(Department of Physics, Fudan University, Shanghai 200433, China)

Abstract We have studied the interaction between dark energy and dark matter from the thermodynamical consideration. Assuming the interaction as stable fluctuations around equilibrium and using the logarithmic correction to entropy caused by the fluctuation, we have derived the physical expression of the interaction. We have tested the viability of our scenario on the interaction by confronting with cosmological observations.

Key words dark energy, dark matter, interaction

A variety of cosmological observations suggest a concordant compelling result that our universe is undergoing an accelerated expansion driven by dark energy (DE)^[1]. Despite the robust observational evidence, the theoretical nature and the origin of dark energy are still the source of much debate^[2]. Dark energy is a major puzzle of the modern cosmology which was attracted a lot of efforts to understand it in the past years. The leading interpretation of such a DE is a cosmological constant with equation of state (EOS) $w_D = -1$. Although the cosmological constant is the simplest theoretical solution to DE and is entirely consistent with the current observational constraints, the well-known cosmological constant problem concerning why the vacuum energy is so much smaller than the value from the effective field theory remains unsolved. There are other conjectures relating the DE to a scalar field called Quintessence with $w_D > -1$, or to an exotic field called Phantom with $w_D < -1$. These models are candidates for dynamical DE, which have an edge over the cosmological constant scenario. Recently the analysis of the type Ia supernova data indicates that the time varying DE gives better fit than a cosmological constant^[3]. These

analyses mildly favor the evolution of the DE parameter w_D from $w_D > -1$ to $w_D < -1$ at recent stage. However the quintessence model is plagued by the fine-tuning problem^[2] and the phantom suffers even more theoretical problems^[4]. Besides these models, recently, a new DE model stimulated by the holographic principle has been put forward^[5] and it was found consistent with the observational data^[6].

Most discussions on DE rely on the assumption that its evolution is independent of other matter fields. One might argue that given the unknown nature of both DE and dark matter (DM), an entirely independent behavior of DE and DM is very special. Studies on the interaction between DE and DM have been carried out in^[7–12]. It has been shown that the coupling between the quintessence field and DM can provide a mechanism to generate acceleration and alleviate the coincidence problem^[7, 12]. Investigations on the suitable coupling between holographic DE and DM have presented a theoretical explanation on the observational transition of the DE EOS from $w_D > -1$ to $w_D < -1$ ^[9]. This finding is further supported in Ref. [11] where it is shown that the interaction between DE and DM generically re-

Received 30 March 2007

^{*} Supported by NSFC (10525521) and Ministry of Education of China and Shanghai Education Commission

1) E-mail: wangb@fudan.edu.cn

sults in super-acceleration with an effective DE EOS $w_D < -1$. Thus the devised challenging phantom model describing $w_D < -1$ might be discarded if the interaction is taken into account. The coupling between DE and DM can not only generate acceleration, but also modify the structure formation through the coupling to cold DM density fluctuation^[8, 13], in contrast to minimally coupled DE models. The growth of DM perturbation can be enhanced due to the coupling between DE and DM^[10, 11], which can be used to explain the reason why an old quasar can be observed in the early universe^[10]. Furthermore, it is argued that the appropriate interaction between DE and DM can influence the perturbation dynamics and the lowest multi-poles of the CMB spectrum and account for the observed CMB low l suppression^[10, 14]. Therefore the interaction between DE and DM is a major issue to be confronted in studying the physics of DE.

However, since the nature for both DE and DM is unknown, there is no specified interaction between them from the fundamental principle. The available studies on the interaction either assume a specific coupling from the outset^[11, 12] or determine the structure of the interaction from the phenomenological requirement^[7]. Attempts on providing Lagrangian descriptions of the interaction have been put forward, such as the dependence of the matter field on the scalar field^[15] and writing the cosmological constant as a function of the trace of the energy-momentum tensor^[16], however the exact forms of these dependences are still not specified. In this paper we try to understand such coupling from the thermodynamical consideration. In de Sitter space, the thermodynamics has been well defined. As in the black hole case, there are well-defined concepts corresponding to the thermodynamical quantities exhibited by the de Sitter horizon^[17]. In an eternally accelerating universe with EOS close to -1 , one would expect similar thermodynamical considerations to apply^[18]. We will assume that if there is no interaction between DE and DM, the thermodynamical system of our universe is in the equilibrium. The appearance of the coupling between DE and DM can be considered as small sta-

ble fluctuations around the equilibrium. It is shown that logarithmic corrections to thermodynamic entropy arise in all thermodynamic systems when stable fluctuations around equilibrium are taken into account^[19]. This idea has been applied in black holes in getting the entropy corrections^[19] and in cosmology in obtaining evolution behavior of the cosmological constant^[20]. We will build the relation between the interaction and the logarithmic entropy correction. Our derivation of the interaction is based on the thermodynamical description, which has the physical foundation. We will show that the derived interacting DE model meets the observational requirements.

We will concentrate our attention on the holographic DE model inspired by the holographic idea that the whole energy content of the cosmos cannot exceed the mass of a black hole with the same size of the universe^[21]. Thus it is supposed that the holographic DE energy density satisfies $\rho_D = 3c^2/L^2$, where c is a constant and L is the length scale identified with the future event horizon R_E to accommodate the acceleration of our universe^[5]. The total energy density is $\rho = \rho_m + \rho_D$, where ρ_m is the matter energy density and $\rho_D = 3c^2/R_E^2$ is the DE energy density. If DE and DM do not interact with each other, their energy densities satisfy the conservation laws respectively

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (1)$$

$$\dot{\rho}_D + 3H(1 + w_D^0)\rho_D = 0, \quad (2)$$

where w_D^0 is the EOS of the DE when it evolves independent of DM. The event horizon now is expressed in the form $R_E = a \int_a^\infty da / (Ha^2) = c / (\sqrt{\Omega_D}H)$, where $\Omega_D = \rho_D / (3H^2)$. Taking derivative with respect to the scale factor on both sides of the event horizon expression, we have

$$\frac{H'}{H} = \frac{\sqrt{\Omega_D}}{c} - 1 - \frac{\Omega_D'}{2\Omega_D}. \quad (3)$$

where the prime is the derivative with respect to $x = \ln a$.

Using the Friedmann equation $\Omega_D + \Omega_m = 1$ and (1)–(3), we can obtain the EOS $w_D^0 = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c}$

and the evolution of the DE^[5, 9]

$$\Omega'_D = \Omega_D^2 (1 - \Omega_D) \left[\frac{1}{\Omega_D} + \frac{2}{c\sqrt{\Omega_D}} \right]. \quad (4)$$

With these preparations we can examine how much the event horizon will change over one Hubble time,

$$t_H \frac{\dot{R}_E}{R_E} = \frac{3}{2} - \frac{\Omega'_D}{2\Omega_D(1-\Omega_D)} = 1 - \frac{\sqrt{\Omega_D}}{c}. \quad (5)$$

The event horizon does not change significantly over one Hubble scale and the equilibrium thermodynamical description holds.

The entropy of the dark energy enveloped by the cosmological event horizon is related to its energy and the pressure in the horizon by the Gibb's equation^[22]

$$T dS_D = dE + P dV. \quad (6)$$

Considering $V = 4\pi R_E^3/3$, $E = \rho_D V = c^2 R_E/2$ and using the equilibrium temperature associated with the event horizon $T = 1/(2\pi R_E)$, we get the equilibrium DE entropy described by

$$dS_D^0 = \pi c^2 (1 + 3w_D^0) R_E dR_E. \quad (7)$$

Now we take account of small stable fluctuations around equilibrium and assume that this fluctuation is caused by the interaction between DE and DM. It is shown that due to the fluctuation, there is a leading logarithmic correction to the thermodynamic entropy around equilibrium in all thermodynamical systems, $S_1 = -\frac{1}{2} \ln(CT^2)$, where C is the heat capacity^[19]. In our case, the heat capacity of the DE can be calculated as $C = \frac{1}{T} \frac{\partial S_D}{\partial T} = -(2\pi R_E)^2 \pi c^2 (1 + 3w_D^0) R_E^2$, which is positive since for DE $1 + 3w_D^0 < 0$. Thus the fluctuation is indeed stable and the entropy correction reads

$$S_1 = -\frac{1}{2} \ln [-\pi c^2 (1 + 3w_D^0) R_E^2] = -\frac{1}{2} \ln [2\pi c \sqrt{\Omega_D} R_E^2]. \quad (8)$$

This entropy correction is supposed to arise due to the appearance of the coupling between DE and DM. Now the total entropy enveloped by the event horizon is $S = S_0 + S_1$ and from the Gibb's law we obtain

$$1 + 3w_D = \frac{1}{c^2 \pi R_E} \frac{dS_1}{dR_E} - \frac{2\sqrt{\Omega_D}}{c}, \quad (9)$$

where w_D is the EOS of DE when it has coupling to DM. If there is no interaction, the thermodynamical

system will go back to equilibrium and the system will persist equilibrium entropy and $w_D \rightarrow w_D^0$.

With the interaction between DE and DM, ρ_m and ρ_D do not satisfy independent conservation laws, instead they satisfy

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (10)$$

$$\dot{\rho}_D + 3H(1 + w_D^0)\rho_D = -Q, \quad (11)$$

where Q denotes the interaction term which is expected to be derived from the entropy correction. In Ref. [7] a phenomenological coupling form is introduced as $Q = 3b^2 H\rho$, where b is the coupling constant. This phenomenological description of the interaction has been shown consistent with the observations including the Supernova data fitting, small l CMB data analysis, age constraints, etc. by choosing appropriate coupling b ^[9, 10]. We will show that from the entropy correction we can derive the coupling form which is similar to the phenomenological model but with a time-dependent b .

Eqs. (10), (11) can be written as

$$\Omega'_D + \frac{2H'}{H} \Omega_D + 3(1 + w_D) \Omega_D = -\frac{8\pi Q}{3H^3}, \quad (12)$$

$$\Omega'_m + \frac{2H'}{H} \Omega_m + 3\Omega_m = \frac{8\pi Q}{3H^3}. \quad (13)$$

Combining Eqs. (3) and (12), we arrive at

$$1 + 3w_D = -\frac{2\sqrt{\Omega_D}}{c} - \frac{8\pi Q}{3H^3 \Omega_D}. \quad (14)$$

Comparing with Eq. (9), we can obtain the interaction term Q in the form

$$-\frac{8\pi Q}{3H^3} = \frac{\Omega_D}{\pi c^2 R_E} \frac{dS_1}{dR_E}. \quad (15)$$

Thus we have built the relation of the coupling between DE and DM to the entropy correction to the equilibrium state. For the convenience to compare our derived interaction with the phenomenological interaction model, $Q = 3b^2 H\rho$, we can use Eq. (15) to write $b^2 = -\frac{\Omega_D}{3\pi c^2 R_E} \frac{dS_1}{dR_E}$, which is no longer a constant. Employing the holographic DE model, $R_E = c/(H\sqrt{\Omega_D})$ and using Eq. (8), we have

$$\frac{dS_1}{dR_E} = \frac{H(\sqrt{\Omega_D}/c - 1)}{c/\sqrt{\Omega_D} - 1} - \frac{H\Omega'_D}{4\Omega_D(c/\sqrt{\Omega_D} - 1)}. \quad (16)$$

Besides, using the Friedmann equation and (3),

(13) becomes

$$\frac{\Omega'_D}{\Omega_D} + (\Omega_D - 1) + \frac{2\sqrt{\Omega_D}}{c}(\Omega_D - 1) = -\frac{8\pi Q}{3H^3}. \quad (17)$$

Inserting Eqs. (15), (16), it changes to

$$\frac{\Omega'_D}{\Omega_D} \left[1 - \frac{\sqrt{\Omega_D}}{c} + \frac{H^2 \Omega_D^2}{4\pi c^4} \right] = \left(1 - \frac{\sqrt{\Omega_D}}{c} \right) \left[(1 - \Omega_D) \left(1 + \frac{2\sqrt{\Omega_D}}{c} \right) - \frac{H^2 \Omega_D^2}{\pi c^4} \right]. \quad (18)$$

With Eqs. (18) and (3) at hand, we are in a position to discuss the dependence of the evolution of DE with respect to the coupling to DM. In the numerical calculation, we set $c = 1$. From Fig. 1 we learn that with the interaction between DE and DM, DE starts to be effective earlier. With the interaction, DE and DM follow each other, as displayed in Fig. 2. We see from Fig. 2(a) and 2(b) that with the interaction, $\rho_m = \rho_D$ earlier than the case without interaction. To observe more clearly, we can define the ratio between energy densities $r = \rho_m / \rho_D$ ^[7] and investigate its change with the scale factor. The behavior is shown in Fig. 2(c). r is a monotonous decreasing function of the scale factor, and around the present time it varies very slowly. Compared with the noninteracting case, we find in the recent era the monotonous decrease of r is slower when there is an interaction between DE and DM. This means that in the recent history of the universe DE is being transformed into DM due to their coupling. This result got by employing the derived interaction from entropy correction is consistent with the earlier result in the phenomenological interaction model^[10] and in the study of the interacting quintessence model^[7]. The different evolution of the DM due to its interaction with DE gives rise to the different expansion history of the universe and different evolution of the matter density perturbation which modifies the structure formation. In Ref. [8, 10] the matter density perturbations in the interacting models have been investigated and in Ref. [10] the influence to the DM density perturbation due to the interaction between DE and DM has been used to explain why it is possible, as recently observed, for an old quasar to be observed in the early stages of the universe. As a comparison, in Fig. 1 and 2 we have also included the result by considering the phe-

nomenological interaction between DE and DM with constant coupling b . It is easy to see that the result on the evolution of DE and DM by using the phenomenological model is consistent with the result got by using the interaction derived from the thermodynamical consideration.

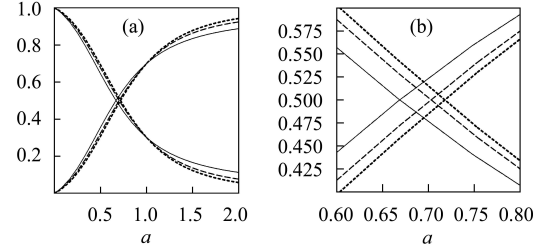


Fig. 1. Evolutions of Ω_D and Ω_m with and without interaction. Lines showing values increasing with a are Ω_D , and the decreasing lines are for Ω_m . The solid line is the result of our model, the dotted line is the result without interaction and the dashed line is for the simple phenomenological model with $b^2 = 0.06$.

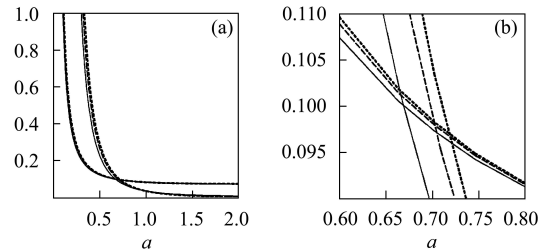


Fig. 2. Evolutions of ρ_D and ρ_m with and without interaction. Before the crossing point, lines on the left are for ρ_D , other bunches of lines are for ρ_m . The solid line is the result of our model, the dotted line is the result without interaction and the dashed line is for the simple phenomenological model with $b^2 = 0.06$.

Including the interaction between DE and DM with the form derived from entropy correction, our model naturally shows that our universe has an accelerated expansion in the late stage and on the other hand it also displays a deceleration in the early time. In Fig. 3, we show the dependence of the deceleration parameter on the coupling between DE and DM. This result is also similar to that got by using the phenomenological interaction^[9].

We now discuss the EOS of DE with the interaction between DE and DM. We found that with our derived interaction form, the DE naturally has a transition as indicated by recent observations with w_D crossing the border -1 . This result is presented in

Fig. 4. Compared with the simple phenomenological model, where appropriate coupling b is needed to be chosen to accommodate the transition of DE EOS^[9], our transition comes natural.

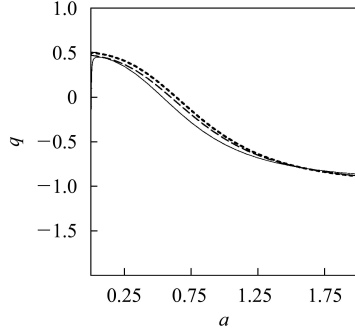


Fig. 3. Dependence of the deceleration parameter on the interaction. The solid line is the result of our model, the dotted line is the result without interaction and the dashed line is for the simple phenomenological model with $b^2 = 0.06$.

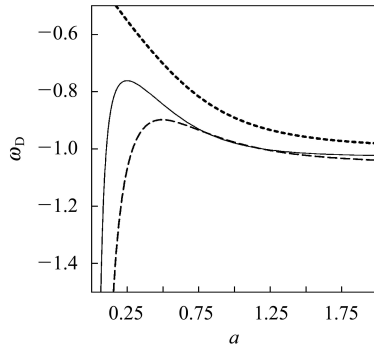


Fig. 4. Behavior of the EOS with and without interaction. The solid line is the result of our model, the dotted line is the result without interaction and the dashed line is for the simple phenomenological model with $b^2 = 0.06$.

Now we test our scenario on the interaction between DE and DM by using some observational results. For the comparison with the phenomenological interaction model, in our model the coupling between DE and DM can be expressed as a counterpart of b as in the phenomenological interaction form. Now the coupling is no longer a constant but a time-dependent parameter described by $b^2 = -\frac{\Omega_D}{3\pi c^2 R_E} \frac{dS_1}{dR_E}$. Its evolution behavior is shown in Fig. 5. During the main process of the universe evolution, the new coupling b in our model is small and it lies within the region of the golden supernova data fitting result $b^2 = 0.00_{-0.00}^{+0.11}$ ^[9] and the observed CMB low l data

constraint^[10]. We have also investigated whether our model can satisfy the current universe age constraints and allow a considerably older universe at high redshift to be compatible with the existence of some old objects such as the old quasar APM0879+5255^[23]. With the Hubble parameter $H_0 = 73.4_{-3.8}^{+2.8}$ km/s/Mpc, at $z = 0$ ^[24], we can get $H_0 t_0 = 1.01418$ which is in consistent with the WMAP three-year results^[24]. At $z = 3.91$, the observed old quasar APM0879+5255 has the age of 2.1 Gyr^[23]. Using the WMAP data on Hubble parameter and the current densities of DE and DM, the dimensionless age of the quasar is in the interval $0.148 \leq T_g \leq 0.162$. In our scenario, it is easy to get that at $z = 3.91$ when our universe was at the age $T = 0.1337$, which is already old enough to accommodate the existence of this old quasar. These results show that our interacting DE scenario is compatible with the observations.

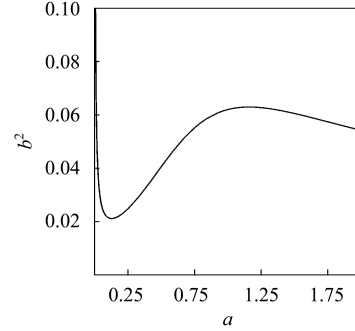


Fig. 5. The corresponding coupling b^2 in our scenario by comparing with the simple phenomenological model.

In summary, we have derived the interaction between DE and DM from the thermodynamical consideration. We assume that the universe is in equilibrium when there is no interaction between DE and DM. The appearance of the coupling between DE and DM is considered as small stable fluctuations to the equilibrium thermodynamical system. Employing the logarithmic entropy correction to the equilibrium state due to the thermodynamical fluctuation caused by the interaction between DE and DM, we have derived the physical expression of the interaction. Compared with the other available descriptions of the DE coupling to DM, we think that our interpretation of the interaction has much solid physical foundation. With the derived interaction form,

we have observed the reasonable behavior of the DE evolution which can explain the transition of the DE EOS from $w_D > -1$ to $w_D < -1$. We have tested our

scenario by using golden SN data, small l CMB data and age constraints at different redshifts and found that our model is consistent with these observations.

References

- 1 Riess A G et al. *Astron. J.*, 1998, **116**: 1009; Perlmutter S et al. *Astrophys. J.*, 1999, **517**: 565; Perlmutter S et al. *Astrophys. J.*, 2003, **598**: 102; de Bernardis P et al. *Nature*, 2000, **404**: 955
- 2 Padmanabhan T. *Phys. Rept.*, 2003, **380**: 235. hep-th/0212290; Peebles P J E, Ratra B. *Rev. Mod. Phys.*, 2003, **75**: 559. astro-ph/0207347; Sahni V. astro-ph/0403324
- 3 Alam U, Sahni V, Starobinsky A A. *JCAP*, 2004, **0406**: 008; Huterer D, Cooray A. *Phys. Rev.*, 2005, **D71**: 023506; WANG Y, Tegmark M. astro-ph/0501351; Jassal H K, Bagla J S, Padmanabhan T. astro-ph/0506748
- 4 Carroll S M, Hoffman M, Trodden M. *Phys. Rev.*, 2003, **D68**: 023509; Cline J M, Jeon S Y, Moore G D. *Phys. Rev.*, 2004, **D70**: 043543; Hsu S D H, Jenkins A, Wise M B. *Phys. Lett.*, 2004, **B597**: 270
- 5 LI M. *Phys. Lett.*, 2004, **B603**: 1
- 6 HUANG Q G, GONG Y G. *JCAP*, 2004, **0408**: 006; GONG Y G, WANG B, ZHANG Y Z. *Phys. Rev.*, 2005, **D72**: 043510; ZHANG X. astro-ph/0504586; SHEN J Y, WANG B, Abdalla E et al. *Phys. Lett.*, 2005, **B609**: 200; HUANG Z Y, WANG B, Abdalla E et al. *JCAP*, 2006, **0605**: 013; Elizalde E, Nojiri S, Odintsov S D et al. hep-th/0502082; WANG Bin, GONG Yun-Gui, SU Ru-Keng. *Phys. Lett.*, 2005, **B605**: 9—14; WANG Bin, Elcio Abdalla, SU Ru-Keng. *Phys. Lett.*, 2005, **B611**: 21—26
- 7 Sergio del Campo, Ramon Herrera, German Olivares, Diego Pavon. *Phys. Rev.*, 2006, **D74**: 023501; Sergio del campo, Ramon Herrera, Diego Pavon. *Phys. Rev.*, 2005, **D71**: 123529; Diego Pavon, Winfried Zimdahl. *Phys. Lett.*, 2005, **B628**: 206—210; German Olivares, Fernando Atrio-Barandela, Diego Pavon. *Phys. Rev.*, 2005, **D71**: 063523
- 8 Olivares G, Atrio-Barandela F, Pavon D. *Phys. Rev.*, 2006, **D74**: 043521
- 9 WANG Bin, GONG Yun-Gui, Elcio Abdalla. *Phys. Lett.*, 2005, **B624**: 141—146; WANG Bin, LIN Chi-Yong, Elcio Abdalla. *Phys. Lett.*, 2006, **B637**: 357—361
- 10 WANG Bin, ZANG Jia-Dong, LIN Chi-Yong et al. astro-ph/0607126
- 11 Subinoy Das, Pier Stefano Corasaniti, Justin Khoury. *Phys. Rev.*, 2006, **D73**: 083509
- 12 Luca Amendola, Shinji Tsujikawa, Sami M. *Phys. Lett.*, 2006, **B632**: 155—158; Luca Amendola, Claudia Quercellini. *Phys. Rev.*, 2003, **D68**: 023514; Anderson G W, Carroll S M. arXiv:astro-ph/9711288
- 13 Bean R. *Phys. Rev.*, 2001, **D64**: 123516
- 14 Zimdahl W. gr-qc/0505056
- 15 Piazza F, Tsujikawa S. *JCAP*, 2004, **0407**: 004
- 16 Nikodem J. Poplawski. gr-qc/0608031
- 17 Gibbons G W, Hawking S W. *Phys. Rev.*, 1977, **D15**: 2738
- 18 Raphael Bousso. *Phys. Rev.*, 2005, **D71**: 064024
- 19 Saurya Das, Parthasarathi Majumdar, Rajat K. Bhaduri. *Class. Quant. Grav.*, 2002, **19**: 2355—2368
- 20 Cécile Barbachoux, Jérôme Gariel, Gérard Le Denmat. astro-ph/0603299
- 21 Cohen A, Kaplan D, Nelson A. *Phys. Rev. Lett.*, 1999, **82**: 4971
- 22 German Izquierdo, Diego Pavon. gr-qc/0606014; German Izquierdo, Diego Pavon. *Phys. Lett.*, 2006, **B633**: 420—426
- 23 Friaca A, Alcaniz J S, Lima J A S. *Mon. Not. Roy. Astron. Soc.*, 2005, **362**: 1295; Hasinger G, Schartel N, Komossa S. *Astrophys. J.*, 2002, **573**: L77; Komossa S, Hasinger G. in XEUS studying the evolution of the universe
- 24 Spergel D N et al. astro-ph/0603449 ApJ; Spergel D N et al. *Astrophys. J.*, 2003, **148**(Suppl.): 175

理解暗能量和暗物质之间的相互作用*

王斌¹⁾

(复旦大学物理系 上海 200433)

摘要 从热力学角度研究了暗能量和暗物质之间的相互作用. 假设相互作用是平衡态上的涨落并考虑此涨落导致的熵的修正, 导出了相互作用的物理表述, 把我们模型和观测结果作了比较.

关键词 暗能量 暗物质 相互作用

2007 - 03 - 30 收稿

* 国家自然科学基金(10525521)和上海市曙光跟踪计划资助

1) E-mail: wangb@fudan.edu.cn