Microbunch Instability in Wigglers*

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Abstract We studied the microbunch instability in wigglers induced by coherent synchrotron radiation (CSR) theoretically and numerically for the first time. This instability occurs only at very small energy spread, and reaches maximum when the energy of electrons and the peak value of the magnet field adopt specific values. Results show that the instability may slightly exist in the wiggler of Beijing Free Electron Laser (BFEL), but does not happen at all in the wigglers of the proposed China Test Facility (CTF) for the X-ray free electron laser.

Key words microbunch instability, microwave density modulation, coherent synchrotron radiation, wiggler

1 Introduction

The beam instability was assumed to be one challenging issue for the development of electron accelerators and new free electron lasers[1—3]. The microbunch instability induced by coherent synchrotron radiation (CSR) has been studied in storage-rings and bunch compressors[1—4]. Many causes such as the current jitter of the electron gun generate the microbunch density perturbation, which can be amplified through the CSR force. Typically CSR is emitted for wavelengths compared to the length of the electron bunch and leads to a detrimental tailed-head interaction. Thus the instability maximizes at moderate wavelengths of the microwave density perturbation. Some of the studies employed a linear Vlasov equation, where the CSR impedance was introduced which is a function of the curvature radius of the electron trajectory[3]. With these studies, we first argue that whether the microbunch instability exists in a wiggler. This is the motivation of this paper.

In this paper, we first introduce the theoretical methods for studying the microbunch instability in a wiggler. Then we show the numerical results of the practical wiggler of the Beijing Free Electron Laser (BFEL) and wigglers taking other parameters, with brief analysis included. Finally, we make the conclusion and discussion.

2 Theoretical methods

CSR of a bunch in a bunch compressor may lead to the microwave instability producing longitudinal modulation of the bunch with wavelengths which are small compared to the bunch length[5]. The theory of the microbunch instability in the bunch compressors caused by CSR was described in S.Heifets' paper[3], where single particle motion is considered. The magnitude of the density modulation, $G(s) = |g_{\rm k}(s)|$ where $k = \dfrac{2\pi}{\lambda}$ is the perturbation wave number (λ: the perturbation wavelength) and s is the longitudinal position along the reference particle trajectory, is maintained in the integral equation which takes the form of[3]:

$$g_{\rm k}(s) = g_{\rm k}^0(s) + \int_0^s K(s,s')g_{\rm k}(s'){\rm d}s' . \qquad (1)$$

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The value of $G(s)$ measures the amplification of the microwave density perturbation: $G(s)>1$ means the microbunch instability and $G(s)<1$ means no instability. The definition of $g_{\mathrm{k}}^{0}(s)$and $K(s,s')$can be referred to the paper [3], both determined by the beam properties and the environment characteristics[3]. To calculate $g_{\mathrm{k}}^{0}(s)$ and $K(s,s')$, the curvature radius of the beam trajectory $\rho(s)$ plays an important role since it changes the dispersion function $D(s)$, the momentum compressing factor $R_{56}(s)$ and the the CSR wake impedance $Z(k,s)$[3, 6]:

$$Z(k,s)=-\mathrm{i}\frac{k^{1/3}(1.63\mathrm{i}-0.94)}{\rho(s)^{2/3}}\times\frac{Z_0}{4\pi}\ , \tag{2}$$

where $Z_0=377\Omega$. $Z(k,s)$ are employed by $g_{\mathrm{k}}^{0}(s)$ and $K(s,s')$. In the previous cases, whether it is the ring or the bunch compressor, $\rho(s)$ keeps constant in a long range of s; while in a wiggler, $\rho(s)$ changes along the particle trajectory. Though Eq. (2) is correct in the absence of edge effect which requires a long magnet, it can be an estimate of $Z(k,s)$ in our case. This assumption is the basis of our study on the microbunch instability in a wiggler.

To get $Z(k,s)$ of the wiggler, we first calculate the function of $\rho(s)$. For any curve in the x-y plane, denote the slope by $y'=\dfrac{\mathrm{d}y}{\mathrm{d}x}$, and $y''=\dfrac{\mathrm{d}y'}{\mathrm{d}x}$, the curvature radius is[7]:

$$\rho(s)=-(1+y'^2)^{3/2}/y''\ . \tag{3}$$

Consider the case of the wiggler. Denote the longitudinal coordinate by x and the transverse by y. Suppose the wiggler period length is λ_{w}, the maximal magnet field is B_{w}, the period number is N, the relativistic electron energy factor is γ, the initial slice energy spread of the beam is σ_{p}, the normalized beam emittance is ε_0. With the first-order approximation, the trajectory curve can be written as $y=Y_{\max}\sin\left(\dfrac{2\pi}{\lambda_{\mathrm{w}}}x\right)$ where $Y_{\max}$ is the amplitude of the transverse motion[8]:

$$Y_{\max}=\frac{e\lambda_{\mathrm{w}}^2B_{\mathrm{w}}}{4\pi^2\gamma m_0c}\ . \tag{4}$$

Here m_0 is the electron mass, e is the electron's electric quantity and c is the light velocity in the vacuum. Then the curvature radius at x is:

$$\rho(s)=-\frac{[1+Y_{\max}^2k_{\mathrm{w}}^2\cos^2(k_{\mathrm{w}}x)]^{3/2}}{Y_{\max}k_{\mathrm{w}}^2\sin(k_{\mathrm{w}}x)} \tag{5}$$

where $k_{\mathrm{w}}=\dfrac{2\pi}{\lambda_{\mathrm{w}}}$.

After that, using Eq. (2) and (1), a computer code can be employed to calculate $g_{\mathrm{k}}(s)$.

3 Numerical results

First we select the wiggler of Beijing Free Electron Laser (BFEL) as the subject investigated. The main parameters of the BFEL wiggler: B_{w}=0.3T, λ_{w}=3cm, N=40; the typical relativistic factor of the electron $\gamma=60$. The dispersion function $D(s)$ and the momentum compressing factor $R_{56}(s)$ are shown in Fig. 1(a) and Fig. 1(b) respectively. Both of their magnitudes are several orders smaller than that of a typical bunch compressor[3], mainly because the BFEL wiggler length (1.2m) is much shorter than the compressor one (∼10m)[3]. Attributed to the wiggler periodicity, $D(s)$ and $R_{56}(s)$ propagate with periodic oscillations. A typical profile of $G(s)$ for the BFEL wiggler is shown in Fig. 1(c). The oscillation amplitude of $G(s)$ slowly increases and the final G exceeds 1.0, indicating the slight amplification of the microwave density perturbation.

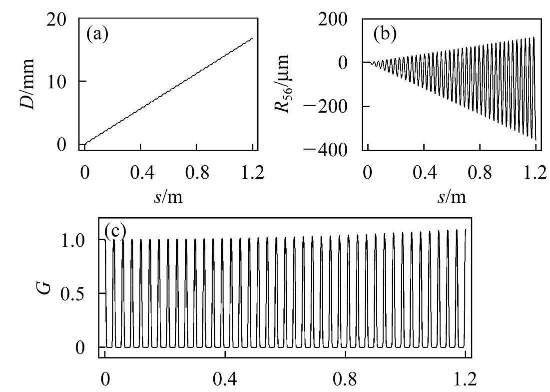

Fig. 1. (a) $D(s)$; (b) $R_{56}(s)$; (c) $G(s)$ for the BFEL wiggler, parameters: $\gamma=60$, B_{w}=0.3T, λ_{w}=3cm, N=40, λ=20μm, ε_0=1μm, $\sigma_{\mathrm{p}}=3\times10^{-5}$.

Second we vary the wiggler parameters, the beam parameters and the perturbation wavelengths to see their respective effect on the microwave density modulation. The final amplification factor of the microwave density perturbation, denoted by G_f, is our major concern here.

Fig. 2 shows the typical dependence of G_f on the perturbation wavelength λ. The $G_\mathrm{f}(\lambda)$ has a single peak at the specific λ, agreeing with the previous findings[3]. At very short wavelengths, the microbunch instability will not occur.

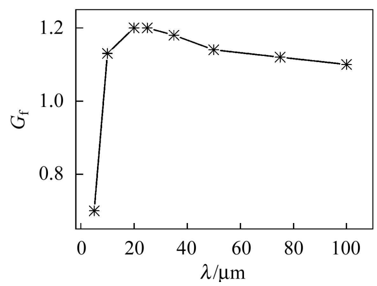

Fig. 2. The dependence of G_f on λ, parameters: γ=60, B_w=0.9T, λ_w=3cm, N=20, ε_0=1μm, $\sigma_\mathrm{p}=3\times10^{-5}$.

We also change the properties of wiggler period, N and λ_w. Fixing other parameters of γ, λ, B_w, ε_0, σ_p at typical values, a typical G_f-N relation is as Fig. 3(a) shows, and a typical G_f-λ_w relation is as shown in Fig. 3(b). Both of the two relations seem monotonous — however, as we will show in the final paragraph of this part, the monotonicity is not always the truth but a typical case.

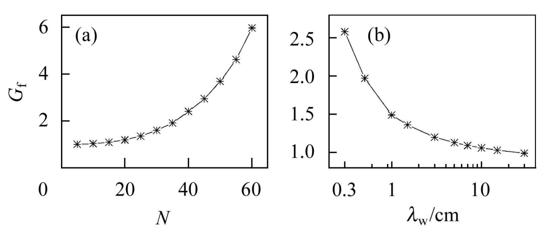

Fig. 3. A typical relation between (a) G_f-N with λ_w=3cm; (b) G_f-λ_w with N=20, the other parameters: γ=60, B_w=0.9T, λ=20μm, ε_0=1μm, $\sigma_\mathrm{p}=3\times10^{-5}$.

By tuning the electron energy γ and the magnet field B_w, $G(s)$ varies attributed to the change of Y_max and $R(s)$. The effect of γ on G_f can be seen in Fig. 4(a), the wiggler parameters are the same as the BFEL wiggler except N=20. We find that G_f maximizes when γ adopt specific values, about 8 here corresponding to 4MeV of electron energy. For very low energy such as 2MeV or below, the microbunch instability vanishes significantly. We also check G_f-γ and G_f-B_w relations with other typical wiggler parameters, and have obtained the similar results as Fig. 4.

The typical effect of B_w on G_f is shown is Fig. 4(b). G_f maximizes at specific B_w values. The BFEL wiggler has B_w=0.3T, locating at the bottom region of G_f-B_w curve, rather immune to the microbunch instability.

The existence of knee point in Fig. 4(a) and Fig. 4(b) can be qualitatively understood as this: According to Eq. (5), if Y_max is extremely large with $Y_\mathrm{max}k_\mathrm{w}\gg1$, $\rho(s)\propto Y^2_\mathrm{max}$ for most values of s; if Y_max is extremely small with $Y_\mathrm{max}k_\mathrm{w}\ll1$, $\rho(s)\propto\dfrac{1}{Y_\mathrm{max}}$. Both cases uniformly have very large ρ. Very large ρ means the modulation factor $K(s,s')\to0$ and $G(s)$ can not be amplified[3]. Only when Y_max is moderate and $K(s,s')$ has observable values, can $G(s)$ be significant. Thus the G_f-Y_max relation peaks at specific Y_max values. Since $Y_\mathrm{max}\propto\dfrac{B_\mathrm{w}}{\gamma}$ according to Eq. (4), the knee points of G_f-γ function in Fig. 4(a) and G_f-B_w function in Fig. 4(b) are easily understood.

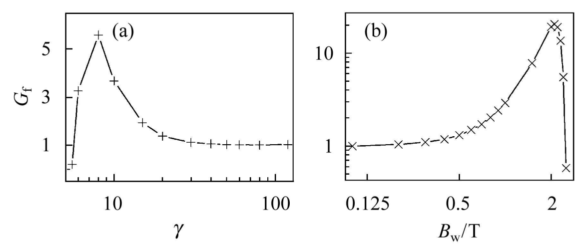

Fig. 4. The typical dependence of G_f on (a) γ with B_w = 0.3T and (b) B_w with γ=60, other parameters: λ_w=3cm, N=20, λ=20μm, ε_0=1μm, $\sigma_\mathrm{p}=3\times10^{-5}$.

After that, we study the influence of the beam quality parameters: the initial normalized emittance ε_0 and the slice energy spread σ_p, respectively shown in Fig. 5(a) and Fig. 5(b). Smaller emittance ε_0 leads

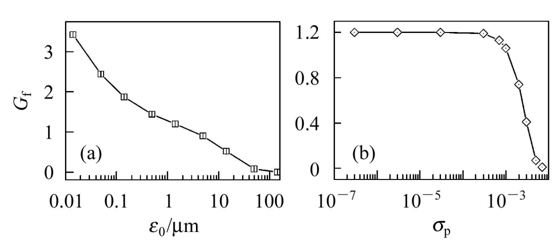

Fig. 5. The dependence of G_f on (a) ε_0 with $\sigma_\mathrm{p}=3\times10^{-5}$; (b) σ_p with ε_0=1μm, other parameters: γ=60, B_w=0.9T, λ_w=3cm, N=20, λ=20μm.

to stronger microbunch instability as shown in Fig. 5(a). By increasing$\sigma_{\rm p}$, $G_{\rm f}$ decreases due to the effect of landau damping. The $G_{\rm f}$-$\sigma_{\rm p}$ dependence in Fig. 5(b) has a saturation when $\sigma_{\rm p}$ is sufficiently small because very small $\sigma_{\rm p}$ cancels the effect of R_{56} uniformly.

Finally we address one remarkable observation. While adjusting all the parameters of γ, $B_{\rm w}$, $\lambda_{\rm w}$, N, λ, ε_0, $\sigma_{\rm p}$ in a large range, the profile of $G(s)$ may be significantly different from Fig. 1(c). With some atypical parameters, $G_{\rm f}$-N is not monotonous as Fig. 3(a) but fluctuate along N, as Fig. 6 shows. This can be observed by tuning the electron relativistic factor γ to very small values, i.e., $\gamma = 5.5$ here. Though practically wigglers do not work with such low-energy electrons even for the infrared FEL, this difference is so remarkable that $G_{\rm f}$ is no longer the extreme point of $G(s)$ and the microbunch instability may occur in the middle of the wiggler but vanishes at the terminal. In this case, the evolution of $G(s)$, especially the maximal value of $G(s)$ is more important than the final $G_{\rm f}$. Its mechanism is not very clear at present.

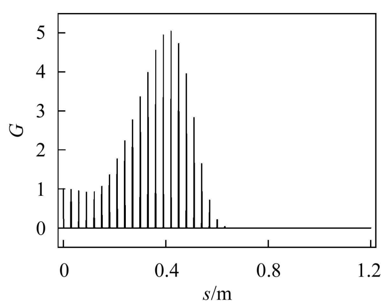

Fig. 6. Typical profile of $G(s)$ in contrast with the typical one in Fig. 1(c). Except $\gamma = 5.5$, the other parameters are the same as Fig. 1(c).

4 Discussion

Succeeded by the previous studies of the microbunch instability driven by CSR in the ring and the bunch compressor[1—5], we study the micronbunch instability in a wiggler for the first time. Wigglers are usually employed to generate radiations of various wavelengths, where the traveling electron beam plays an important role since it is the radiation source. The BFEL wiggler was studied as an example. With the typical parameters, the amplification factor of the microwave density perturbation oscillates periodically, and the oscillating magnitude changes monotonously. For BFEL, this instability is rather slight.

Besides that, we studied the microwave density modulation with different parameters. The typical results are listed. When tuning the incident electron energy as low as 2—4MeV, we observed that the instability may maximize in the middle of the wiggler, instead of the two terminals. Though this is not the usual case, it reflects the complexity of the microwave density modulation behavior driven by CSR. Another significant phenomenon is that the microbunch instability maximizes when the peak value of the wiggler magnet field or the relativistic factor of electrons adopts specific values. We employ this to study the wigglers of the China Test Facility (CTF) for the X-ray free electron laser, one has $\lambda_{\rm w}$=5.8cm, $B_{\rm w}$=1.28T, N=103 and the other has $\lambda_{\rm w}$=3.8cm, $B_{\rm w}$=0.93T, N=303, both with the electron energy of 840MeV, the slice energy spread $\sigma_{\rm p} = 10^{-4}$ and the normalized emittance 1.23μm, We find that the microbunch instability does not occur in the full range of the two wigglers. The microbunch instability observed in our simulation hints that the microbunch instability occurs only at very small energy spread, so it is an effective method to cure this instability by diluting the energy spread.

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扭摆器中的微束团不稳定性*

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摘要 用理论和数值的方法, 首次研究了扭摆器中相干同步辐射效应诱发的微束团不稳定性. 这一不稳定性只存在于小能散的情况, 并当电子能量或峰值磁场取特定值时, 不稳定度达到最大值. 在BFEL的扭摆器中, 可能有轻微的不稳定性存在; 在CTF的扭摆器中, 则不会有不稳定性发生.

关键词 微束团不稳定性 微波密度调制 相干同步辐射 扭摆器

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