Splitting of Pseudospin and Spin Partners in the Relativistic Harmonic Oscillator*

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Abstract Pseudospin symmetry and spin symmetry in the relativistic harmonic oscillator are investigated systematically by solving the Dirac equation with scalar and vector potentials and are found to be a good approximation in realistic nuclei such as ^{208}Pb. The pseudospin breaking and spin breaking are shown in correlation with nuclear mean field. The harmonic oscillator frequency and the distance of well-bottom deviation from the center of the potential play an important role in the splittings of energy and wavefunction. The energy-level crossing are found for all the pseudospin partners. The dependence of splittings with quantum numbers is also analyzed.

Key words pseudospin symmetry, spin symmetry, harmonic oscillator, Dirac equation

1 Introduction

About 30 years ago, a quasidegeneracy was observed in heavy nuclei between single-nucleon doublets with quantum numbers $(n,l,j=l+1/2)$ and $(n-1,\ l+2,\ j=l+3/2)$ where n, l, and j are the radial, the orbital, and the total angular momentum quantum numbers, respectively[1, 2]. The quasidegenerative states are suggested as the pseudospin doublets $j=\tilde{l}\pm\tilde{s}$ with the pseudo orbital angular momentum $\tilde{l}$ and the pseudospin angular momentum $\tilde{s}$. Pseudospin has also been discussed in many phenomena, including deformation[3] and superdeformation[4], magnetic moment[5, 6] and identical bands[7—9]. Because of these successes, there have been comprehensive efforts to understand its origin since the discovery of this symmetry. Blokhin et al.[10] performed a helicity unitary transformation in a nonrelativistic single-particle Hamiltonian. They showed that the transformed radial wave functions have a different asymptotic behavior, implying that the helicity transformed mean field acquires a more diffuse surface. The application of the helicity operator to the nonrelativistic single-particle wave function maps the normal states (l,s) onto the pseudostate $(\tilde{l},\tilde{s})$, while keeping all other global symmetries. The same kind of unitary transformation was also considered earlier by Bahri et al. to discuss the pseudospin symmetry in the non-relativistic harmonic oscillator[11]. They showed that a particular condition between the coefficients of spin-orbit and orbit-orbit terms, required to have a pseudospin symmetry in that non-relativistic single-particle Hamiltonian, was consistent with relativistic mean-field (RMF) estimates. Based on the RMF theories, Ginocchio has identified a possible reason for this; namely that the symmetry arises from the near equality in magnitude of an attractive scalar S, and repulsive vector V, relativistic mean field, $S\approx -V$, in which the nucleons move[12]. They revealed that the pseudospin sym-

Received 1 July 2006

* Supported by National Natural Science Foundation of China (10375032) and Doctoral Fund of Ministry of Education of China (20030003090)

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metry is exact when doublets are degenerate. Meng et al.[13, 14] showed that pseudospin symmetry is exact when $d\Sigma/dr = 0$, where $\Sigma = S + V$ and the quality of the pseudospin approximation in real nuclei is connected with the competition between the pseudo-centrifugal barrier and the pseudospin orbital potential. A test of nuclear wave functions for pseudospin symmetry was done in Refs. [15,16]. The pseudospin symmetry in deformed nuclei has already been extensively studied in the framework of the RMF model. The conditions for an exact pseudospin symmetry and the relationships for the lower components of the Dirac eigenfunctions were investigated in Refs. [17,18]. The relationships between the upper and lower components of the two states in the pseudospin doublets have been studied thoroughly for realistic deformed relativistic eigenfunctions[19]. There are also many other investigations on the pseudo-spin symmetry in the framework of RMF theory[20—23]. A briefly review on pseudospin symmetry in RCHB theory could be found in Ref. [24].

Recently, harmonic oscillator potential has received considerable attention in many fields. The spherical relativistic harmonic oscillator with spin symmetry has been studied previously[25—28]. Chen et al.[29] using a Dirac Hamiltonian with scalar S and vector V potentials quadratic in space coordinates, found a harmonic-oscillator like second order equation which can be solved analytically for $\Delta = V - S = 0$ as considered before by Kukulin[27], and also for $\Sigma = S + V$. Ginocchio has solved the triaxial, axial, and spherical harmonic oscillators for the case $\Delta = 0$ and applied it to the study of antinucleons embedded in nuclei[30]. The case $\Sigma = 0$ is particularly relevant in nuclear physics, since it is usually pointed out as a necessary condition for occurrence of pseudospin symmetry in nuclei[12, 25]. Lisboa et al. have studied the generalized relativistic harmonic oscillator for spin 1/2 particles, obtained the analytical solutions for bound states of the corresponding Dirac equations by setting either Σ or Δ to zero[31]. Moreover, they have researched the perturbation breaking of pseudospin symmetry induced by a tensor potential[32].

However, as stated in some papers[12], the conditions $\Sigma = 0$ or $d\Sigma/dr = 0$ can not be realized in nuclei. Therefore, it is necessary to study the pseudospin symmetry for relativistic nuclear potential. Alberto et al. have researched the nature of pseudospin breaking in nuclei by solving the Dirac equation with vector and scalar potentials of Woods-Saxon type[33]. The roles played by the surface diffuseness, the radius, and the central depth of the sum of the standard vector and scalar nuclear potentials in the energy splitting of pseudospin partners have been analyzed. A systematics for the behavior of the pseudospin splitting is drawn as functions of a, R and V_0. From the behavior of these splitting with Woods-Saxon parameters, they have shown that pseudospin symmetry has a dynamical character. Very recently, pseudospin symmetry for the resonant states in ^{208}Pb is also investigated by solving the Dirac equation with Woods-Saxon vector and scalar potentials[34]. However, pseudospin symmetry not only means single-nucleon energy levels are quasi degenerate but also means the lower components of pseudospin doublets are equal in magnitude. Therefore, Guo et al.[35] researched the nature of the pseudospin energy splittings and peudospin wavefunction splitting by solving the Dirac equation with vector and scalar potentials of harmonic oscillator type and a systematics for the behavior of the pseudospin splitting is drawn as the harmonic oscillator frequency ω and the distance of well-bottom deviation from the center r_0 vary. But their research on the pseudospin wavefunction splitting is incomplete. As we know, spin symmetry is also important. The spherical, triaxial and axially deformed relativistic harmonic oscillators with spin symmetry have been studied in Ref. [30]. Guo et al.[36] have solved the $s(\tilde{s})$-wave Dirac equation for a single particle with spin and pseudospin symmetry moving in a central Woods-Saxon potential and the corresponding radial wavefunctions for the two-component spiner and the energy spectra of the bound states are obtained. We have investigated pseudospin symmetry and spin symmetry in the relativistc woods-saxon[37], very recently. Our purpose here is to go further and investigate Pseudospin symmetry and spin symmetry in the relativistic harmonic

oscillator shaped by the harmonic oscillator frequency and the distance of well-bottom deviation from the center. In the following, we first present the theoretical formalism, then analyze systemically the pseudospin breaking and spin breaking for the relativistic harmonic oscillator.

2 Formalism

The Dirac equation of a nucleon with mass M moving in an attractive scalar potential $S(\boldsymbol{r})$ and a repulsive vector potential $V(\boldsymbol{r})$ can be written as

$$[\boldsymbol{a}\cdot\boldsymbol{p}+\beta(M+S)+V]\Psi=E\Psi. \tag{1}$$

For spherical nuclei, the nucleon angular momentum $\boldsymbol{J}$, and $\boldsymbol{K}=-\beta(\boldsymbol{\sigma}\cdot\boldsymbol{L}+1)$ commute with the Dirac Hamiltonian, where β, $\boldsymbol{\sigma}$ and $\boldsymbol{L}$ are the Dirac matrix, Pauli matrix, and orbital angular momentum, respectively. The wavefunctions can be classified according to there angular momentum j and κ,

$$\Psi_{n\kappa}(\boldsymbol{r})=\begin{pmatrix}f_{n\kappa}\\ g_{n\kappa}\end{pmatrix}=\frac{1}{r}\begin{pmatrix}F_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi)\\ \mathrm{i}G_{n\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\phi)\end{pmatrix}, \tag{2}$$

where n is the radial quantum number, and m is the projection of angular momentum on the third axis. The eigenvalues of $\hat{k}$ are $\kappa=\pm(j+1/2)$ with '$-$' for aligned spin ($s_{1/2},p_{3/2}$, etc.), and '$+$' for unaligned spin ($p_{1/2},d_{3/2}$, etc.). when the angular part was split off, the radial wavefunction satisfy the following equations

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)F_{n\kappa}(r)=(M+E_{n\kappa}-\Delta)G_{n\kappa}(\kappa), \tag{3}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}\right)G_{n\kappa}(r)=(M-E_{n\kappa}+\Sigma)F_{n\kappa}(\kappa), \tag{4}$$

where Δ and Σ are assumed to be radial functions, i.e., $\Delta=V(r)-S(r)$ and $\Sigma=V(r)+S(r)$.We perform a calculation using a relativistic potential of harmonic oscillator type in the Dirac equation. The corresponding scalar and vector components are the mean field central nuclear potential, given as

$$U(r)=U_0+\frac{1}{2}M\omega^2(r-r_0)^2\ , \tag{5}$$

where $U(r)$ stands either for the vector or for the scalar potential. Although this potential is not a full self-consistent relativistic potential derived from meson fields, it is realistic enough to be applied to nuclei. Indeed, most self-consistent potentials have harmonic oscillator-like shape, i.e., one can recognize in them a depth U_0, a harmonic oscillator frequency ω, and a radius r_0 describing the distance of well-bottom deviation from the center. It is known that, in certain isotope chains, as the mass number A increases, the nuclear harmonic oscillator frequency ω decreases according to the $A^{1/3}$ law ($\hbar\omega=41A^{-1/3}$), which means that it is also important to study the role of the parameter ω in pseudospin symmetry and spin symmetry. It is also meaningful to study the parameter r_0, because the nuclear mean field often deviate from the center in many cases, especially for the deformed nuclei. Furthermore, the harmonic oscillator potential can provide fully bound states which are helpful to discuss the symmetry systemically. Therefore the study of pseudospin partners splitting and spin partners splitting as a function of these parameters is meaningful and realistic enough to be applied to most nuclei, at least qualitatively.

3 Results and discussion

Using harmonic oscillator potential for Σ and Δ, we solved numerically the coupled first-order Dirac equations for the radial fields $F(r)$ and $G(r)$. There are altogether six parameters for Σ and Δ, namely, the central depths, Σ_0 and Δ_0, two harmonic oscillator frequencies, and two radius parameters. In order to make the harmonic oscillator potential closer to nuclear mean field, the parameters in harmonic oscillator potential are determined by fitting the scalar and vector potentials derived from the RMF calculations where ^{208}Pb is chosen as a reference. The parameters determined are listed in the Table 1. The quality of our fitting is displayed in Table 2, where the results of the present calculation (harmonic oscillator potential) are shown, together with those obtained by using a relativistic mean field approach with the interactions NL3 for the same set of pseudospin doublets. From Table 2, we can see that the agreement between the present results and the RMF calculations is considerably satisfactory. By using these parameters, the radial wavefunctions of relativistic harmonic

oscillator are obtained with the upper components and lower components are plotted in Fig. 1, in which are found to be quite agreeable for the spin partners and pseudospin partners, as noted previously in Refs. [15,16]. Although such a near agreement of wavefunction has been obtained for the relativistic harmonic oscillator, the splitting between the doublets cannot be neglected, especially on the surface. The extent of splitting is connected with the single particle orbital. For example, when l or $\tilde{l}$ increases, the pseudospin wavefunction splittings increase and the maximum splittings of all the wavefunctions move towards the outside of the nucleus for the spin doublets n=1 in Fig. 1(a) and pseudospin doublets $\tilde{n}$=1 in Fig. 1(b), as was noted for the spherical square well potential in Ref. [12]. The similar case also appears in other pseudospin partners for the doublets $\tilde{l}$=1 and $\tilde{l}$=2, as shown in Fig. 1(c).

Table 1. The parameters in harmonic oscillator potential determined by fitting the scalar and vector potentials from the RMF calculations with the NL3 for ^{208}Pb. Listed are two central depths (U_0, Σ_0 and Δ_0), two harmonic oscillator frequencies ω and two radius parameters r_0.

	U_0	ω	r_0
$V+S$	−68.887	10.503	2.157
$V-S$	722.787	33.619	1.863

Table 2. Calculated neutron single-particle level in ^{208}Pb, where the parameters in the HO potential are determined by fitting the results from RMF calculation with interactions NL3.

	HO	RMF		HO	RMF
$1s_{1/2}$	−59.065	−59.090	$2s_{1/2}$	−39.660	−40.870
$3s_{1/2}$	−19.892	−18.095	$4s_{1/2}$	−0.332	−0.545
$1d_{3/2}$	−40.756	−43.978	$2d_{3/2}$	−20.504	−18.841
$3d_{3/2}$	−0.707	−0.122	$1d_{5/2}$	−42.685	−45.398
$2d_{5/2}$	−22.108	−20.494	$3d_{5/2}$	−2.076	−0.709
$1g_{7/2}$	−22.171	−24.051	$2g_{7/2}$	−2.053	−0.518
$1g_{9/2}$	−25.548	−28.086	$2g_{9/2}$	−4.939	−2.431
$1i_{11/2}$	−19.745	−2.923	$1i_{13/2}$	−24.368	−9.588
$1p_{1/2}$	−50.080	−52.286	$2p_{1/2}$	−29.994	−29.432
$3p_{1/2}$	−10.140	−7.531	$1p_{3/2}$	−51.142	−52.880
$2p_{3/2}$	−30.867	−30.495	$3p_{3/2}$	−10.880	−8.296
$1f_{5/2}$	−31.395	−34.415	$2f_{5/2}$	−11.172	−9.009
$1f_{7/2}$	−34.102	−37.022	$2f_{7/2}$	−13.454	−11.022
$1h_{9/2}$	−13.143	−13.368	$1h_{11/2}$	−17.092	−18.866

Here, we discuss only the influences of Σ potential on the symmetry to analyze the correlations between the pseudospin symmetry and spin symmetry and the parameters of harmonic oscillator potential. By solving the Dirac equation with the scalar and vector harmonic oscillator potential, the variations of the pseudospin energy splitting ($\Delta E = E_{\tilde{l}j=\tilde{l}-1/2} - E_{\tilde{l}j=\tilde{l}+1/2}$) and the spin energy splitting ($\Delta E = E_{lj=l-1/2} - E_{lj=l+1/2}$) with the parameters are shown in Fig. 2 and Fig. 3. Because the splitting of wavefunction is proportional to the area between the lower components of the doublets, the variations of the wavefunction splitting with the parameters are also drawn in Fig. 2 and Fig. 3. In the figures below, all the pseudospin-orbital partners appear in the order of the energy $1(1\tilde{p}_{3/2},\ 1\tilde{p}_{1/2})$, $2(1\tilde{d}_{5/2},\ 1\tilde{d}_{3/2})$, $3(1\tilde{f}_{7/2},\ 1\tilde{f}_{5/2})$, $4(2\tilde{p}_{3/2},\ 2\tilde{p}_{1/2})$, $5(1\tilde{g}_{9/2},\ 1\tilde{g}_{7/2})$, $6(2\tilde{d}_{5/2},\ 2\tilde{d}_{3/2})$, $7(1\tilde{h}_{11/2},\ 1\tilde{h}_{9/2})$, $8(2\tilde{f}_{7/2},\ 2\tilde{h}_{5/2})$, and $9(3\tilde{p}_{1/2},\ 3\tilde{p}_{3/2})$. All the spin-orbital partners appear in the order of the energy $1(1p_{1/2},\ 1p_{3/2})$, $2(1d_{3/2}, 1d_{5/2})$, $3(1f_{5/2}, 1f_{7/2})$, $4(2p_{1/2}, 2p_{3/2})$, $5(1g_{7/2}, 1g_{9/2})$, $6(2d_{3/2}, 2d_{5/2})$, $7(1h_{9/2}, 1h_{11/2})$, $8(2f_{5/2}, 2f_{7/2})$, $9(3p_{1/2}, 3p_{3/2})$, $10(1i_{11/2}, 1i_{13/2})$, $11(2g_{7/2}, 2g_{9/2})$, and $12(3d_{3/2}, 3d_{5/2})$.

Fixing Σ_0 , Δ_0 and r_0, we vary ω to see how the pseudospin splitting and spin splitting are sensitive to the diffusivity. These dependencies are shown in Fig. 2. where the horizontal line corresponds to the zero energy splitting one, i.e., $E_{n,l,j=l+1/2} - E_{n-1,l+2,j=l+3/2} = 0$. Below the horizontal line $E_{n,l,j=l+1/2} < E_{n-1,l+2,j=l+3/2}$, and above the line $E_{n,l,j+l+1/2} > E_{n-1,l+2,j+l+3/2}$. From Fig. 2(a) it can be seen that the energy splitting between the doublets is sensitive to the parameter ω, especially for the doublets with smaller $\tilde{n}$ and the doublets with lower $\tilde{l}$. Meanwhile, we observe that the pseudospin partners happen to cross with ω increasing. If we label the crossing point as ω_c, there is a critical point ω_c for all pseudospin partners and the ω_c is different from different doublets. With $\tilde{n}$ or $\tilde{l}$ increasing, ω_c becomes large. When ω passes through ω_c, the energy splitting will invert its sign. This inversion of pseudospin partner splittings is observed experimentally and is also mentioned in Refs. [14,15]. It is

important to note that, once the pseudospin splitting changes its sign, the effect of increasing ω is to drive the pseudospin doublets further apart. This systematics is consistent with the nonrelativistic prediction of Ref. [10]. However, in Fig. 2(b), there is no any critical point ω_c for the spin partners and the energy splitting between the spin doublets is only sensitive to the parameter ω for the doublets with higher l. In Fig. 2(c), the pseudospin wavefunction splittings increase with ω increasing, which is in agreement with the pseudospin energy splitting in Fig. 2(a) for $\omega > \omega_c$. In Fig. 2(d), as ω increasing, the spin wavefunction splittings decrease greatly when ω is relatively small and increase a little when ω is relatively large. which is contrary to the variation of the energy splittings with the parameter ω. This dependence on ω is especially important when comparing different isotopes, since, in many cases, it confines the shapes and range of the mean-field potential that changes most noticeably.

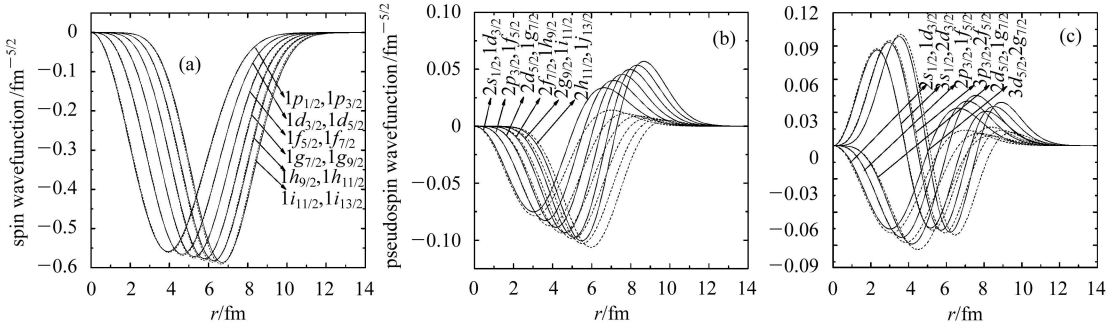

Fig. 1. (a) The radial upper components($F(r)$) of spin partners for the relativistic HO with the parameters determined by fitting the results from the RMF calculations with the interactions NL3 for ^{208}Pb. (b) The same as (a), but for the radial lower components($G(r)$) of pseudospin partners. (c) The same as (b), but for the doublets $\tilde{l}=1$, $\tilde{l}=2$ and $\tilde{l}=3$.

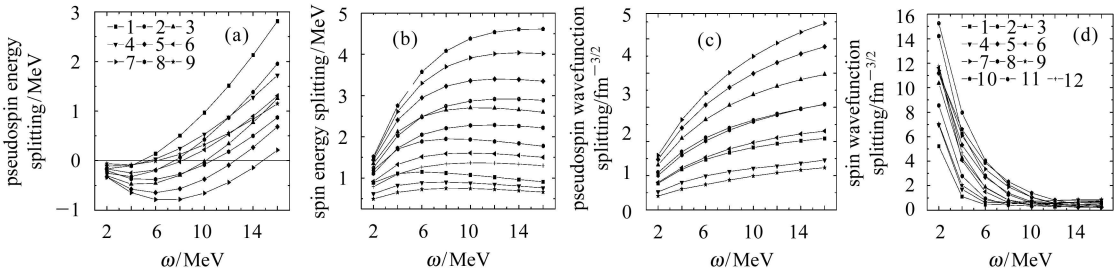

Fig. 2. (a) Pseudospin energy splitting as a function of ω for the different pseudospin doublets, where the horizontal line corresponds to the zero-energy splitting one. (b) Spin energy splitting as a function of ω for the different spin doublets. (c) Pseudospin wavefunction splitting as a function of ω for the radial lower components($G(r)$) of the different pseudospin doublets. (d) Spin wavefunction splitting as a function of ω for the radial upper components($F(r)$) of the different spin doublets.

A similar trend is seen when we vary the parameter r_0 and fix all other parameters. In Fig. 3(a), we observe that the pseudospin energy splitting between the doublets is also sensitive to the parameter r_0. With r_0 increasing, the pseudospin doublets crosses each other and inverts the sign of the energy splitting. If the cross point is labelled as r_0^c, r_0^c is different from different doublets and increases with l or n increasing, which is similar to ω_c. When r_0 passes through r_0^c, the energy splitting will invert its sign. Once the pseudospin splitting changes its sign, the effect of increasing r_0 is to drive the pseudospin doublets further apart. However, in Fig. 3(b), all the spin energy splittings are almost invariant with r_0 increasing. Furthermore, the splittings of wavefunction between doublets are also sensitive to the radius. Fig. 3(c) shows the pseudospin wavefunction splittings decrease significantly as r_0 increases except for the doublets 4 and 9 when $r_0 > 2.4$fm. In Fig. 3(d), the spin wavefunction splittings decrease for the dou-

blets with lower l, but increase for the doublets with higher l.

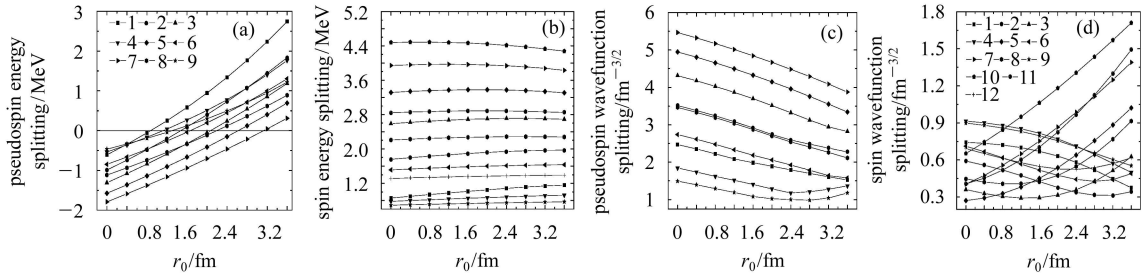

Fig. 3. (a) Pseudospin energy splitting as a function of r_0 for the different pseudospin doublets, where the horizontal line corresponds to the zero-energy splitting one. (b) Spin energy splitting as a function of r_0 for the different spin doublets. (c) Pseudospin wavefunction splitting as a function of r_0 for the radial lower components($G(r)$) of the different pseudospin doublets. (d) Spin wavefunction splitting as a function of r_0 for the radial upper components($F(r)$) of the different spin doublets.

Finally, if we keep a, R and Δ_0 fixed, but vary Σ_0 to study the sensitiveness of the pseudospin doublets and spin doublets with the depth of the central Σ mean field potential, we can conclude that all the pseudospin doublets and spin doublets are almost invariant with $|\Sigma_0|$ increasing, because Σ_0 is just a constant added to the potential. Therefore, the pseudospin splittings and spin splittings are all insensitive to the depth of the well, which is in accordance with Ginocchio predictions for pseudospin symmetry breaking due to the finiteness of the mean field.

Furthermore, as for a certain nucleus, different partners are in the same potential. So it is necessary to analyze the dependence of splittings with quantum numbers. In Fig. 2(c) and Fig. 3(c), when $\tilde{n}$ is fixed, the pseudospin wavefunction splittings increase with $\tilde{l}$ increasing, while $\tilde{l}$ is fixed, the pseudospin wavefunction splittings decrease with $\tilde{n}$ increasing. In Fig. 2(b) and Fig. 3(b), when $\tilde{n}$ is fixed, the spin energy splittings increase with $\tilde{l}$ increasing, while $\tilde{l}$ is fixed, the spin energy splittings decrease with $\tilde{n}$ increasing, which is due to the particle distribution relative to the spin orbital potential[38]. All these show that both the pseudospin breaking and spin breaking are different for different pseudospin partners and spin partners, respectively. Although some of these results have been derived in specific relativistic field theories[13, 14], these results probably are the general feature of any relativistic model which fits nuclear binding energies, and hence very likely the general feature independent of any model.

4 Conclusion

At the present work, the pseudospin symmetry and the spin symmetry in the relativistic harmonic oscillator are investigated systemically by solving the Dirac equation with scalar and vector radial potentials. The symmetry is found to be a good approximation in realistic nuclei such as ^{208}Pb. The pseudospin breaking and spin breaking are shown in correlation with the nuclear mean field which is shaped by the depth of the well, the harmonic oscillator frequency ω and the distance of well-bottom deviation from the center. The harmonic oscillator frequency ω and the parameter r_0 are found to play an important role in the energy splittings and wavefunction splittings for all the partners. Their influence on the pseudospin splitting is so sensitive that the energy crossing appears in all the pseudospin partners. Furthermore, the dependence of pseudospin splittings and spin splittings with quantum numbers are also analyzed. when l or $\tilde{l}$ increases, the maximum splittings of all the wavefunction move towards the outside of the nucleus for the spin doublets $n=1$ and pseudospin doublets $\tilde{n}=1$. As $\tilde{n}$ is fixed, the pseudospin wavefunction splittings increase with $\tilde{l}$ increasing, while $\tilde{l}$ is fixed, the pseudospin wavefunction splittings decrease with $\tilde{n}$ increasing. When n is fixed, the spin energy splittings increase with l increasing, while l is fixed, the spin energy splittings decrease with n increasing.

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相对论谐振子下的自旋和赝自旋轨道伙伴态劈裂*

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摘要　通过求解具有谐振子势的径向标量势与矢量势的Dirac方程, 分别分析了原子核中赝自旋和自旋双重态的能级劈裂和波函数劈裂随着谐振子的振动频率参数ω和描述谐振子势阱底偏离中心参数r_0的变化关系. 研究发现, 这些参数对于赝自旋和自旋双重态的能级劈裂和波函数劈裂都有着显著的影响. 此外, 也研究了能级劈裂和波函数劈裂随着量子数的变化关系. 由于参数ω与核子数有关, 而参数r_0与形变核有关, 所以以这些参数为变量对于赝自旋劈裂和自旋劈裂的研究是有意义的, 研究的结果至少可以定性地应用到大部分原子核中.

关键词　赝自旋对称性　自旋对称性　谐振子　Dirac方程

2006 – 07 – 01 收稿

∗ 国家自然科学基金(10375032)和国家教育部博士点专项基金(20030003090)资助

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