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On the Difficulty of Comparing Theoretical and Experimental Estimates of Elliptical Flow

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Abstract Elliptic flow is easy to compute in hydrodynamics. However experimentally it is obtained in an indirect way. The question we address in this paper is how comparable are these two approaches. For both cases, our study is done using the hydrodynamical code NeXSPheRIO and simulating nuclear collisions at RHIC.

Key words relativistic nuclear collisions, hydrodynamics, elliptic flow

1 Brief description of NeXSPheRIO

Hydrodynamics is one of the main tools to study the collective flow in high-energy nuclear collisions. Here we discuss results obtained with the hydrodynamical code NeXSPheRIO. It is a junction of two codes: NeXus and SPheRIO. The SPheRIO code is used to compute the hydrodynamical evolution. It is based on Smoothed Particle Hydrodynamics, a method originally developed in astrophysics and adapted to relativistic heavy ion collisions[1].

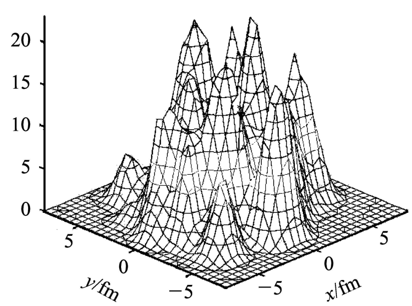

Fig. 1. $\eta=0$ slice for initial energy density of a central RHIC collision with several high density peaks (in GeV/fm^{-3}).

Its main advantage is that any geometry in the initial conditions can be incorporated. The NeXus code is used to compute the initial conditions $T_{\mu\nu}$, j^{μ} and u^{μ} on a proper time hypersurface[2]. An example of initial conditions is shown in Fig. 1.

NeXSPheRIO is run many times, corresponding to many different events or initial conditions. At the end, an average over final results is performed. This mimics experimental conditions. This is different from the canonical approach in hydrodynamics where initial conditions are adjusted to reproduce some selected data and are very smooth. This code has been used to study a range of problems concerning relativistic nuclear collisions: effect of fluctuating initial conditions on particle distributions[3], energy dependence of the kaon effective temperature[4], interferometry at RHIC[5], transverse mass distributions at SPS for strange and non-strange particles[6], effect of the different theoretical and experimental centrality binnings[7], effect of the nature of the quark-hadron transition and of the particle emission mechanism[8]. Here a calculation of elliptic flow is performed[9].

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The version of NeXSPheRIO used has a first order quark-hadron transition, sudden freeze out at $T_{f.\mathrm{out}}$=150MeV (leading to good fits for $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$ and $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta p_\perp \mathrm{d}p_\perp$ for all PHOBOS centrality windows) and no strangeness conservation.

2 Results on $v_2^{\mathrm{b}}(\eta)$ and $v_2^{\mathrm{rec}}(\eta)$

In a hydrodynamical code, the impact parameter $\boldsymbol{b}$ is usually known. The theoretical, or true, elliptic flow parameter at a given pseudo-rapidity η is defined as

$$\langle v_2^{\mathrm{b}}(\eta)\rangle = \left\langle \frac{\int \mathrm{d}^2N/\mathrm{d}\phi\mathrm{d}\eta\cos[2(\phi-\phi_b)]\,\mathrm{d}\phi}{\int \mathrm{d}^2N/\mathrm{d}\phi\mathrm{d}\eta\,\mathrm{d}\phi} \right\rangle, \quad (1)$$

ϕ_b is the angle between $\boldsymbol{b}$ and some fixed reference axis. The average is performed over all events in the centrality bin.

Experimentally, the impact parameter angle ϕ_b is not known. In the so-called standard method, an approximation, ψ_2, is estimated. Elliptic flow parameter with respect to this angle, $v_2^{\mathrm{obs}}(\eta)$, is calculated. Then a correction is applied to $v_2^{\mathrm{obs}}(\eta)$ to account for the reaction plane resolution, leading to the experimentally reconstructed elliptic flow parameter $v_2^{\mathrm{rec}}(\eta)$. For example in a Phobos-like way[10, 11]

$$\langle v_2^{\mathrm{rec}}(\eta)\rangle = \left\langle \frac{v_2^{\mathrm{obs}}(\eta)}{\sqrt{\langle\cos[2(\psi_2^{<0}-\psi_2^{>0})]\rangle}} \right\rangle, \quad (2)$$

where

$$v_2^{\mathrm{obs}}(\eta) = \frac{\sum_i \mathrm{d}^2N/\mathrm{d}\phi_i\mathrm{d}\eta\cos[2(\phi_i-\psi_2)]}{\sum_i \mathrm{d}^2N/\mathrm{d}\phi_i\mathrm{d}\eta}, \quad (3)$$

and

$$\psi_2 = \frac{1}{2}\tan^{-1}\frac{\sum_i \sin 2\phi_i}{\sum_i \cos 2\phi_i}. \quad (4)$$

In the hit-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $\eta<0$ and >0 respectively and if v_2 is computed for a positive (negative) η, the sums in ψ_2, Eq. (4), are over particles with $\eta<0$ $(\eta>0)$. In the track-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $2.05<|\eta|<3.2$, the sums in ψ_2, Eq. (4), are over particles in both sub-events, v_2 is obtained for particles around $0<\eta<1.8$ and reflected (to account for the different multiplicities between a subevent and the sums in Eq. (4), there is also an additional $\sqrt{2\alpha}$ with $\alpha\sim 1$, in the reaction plane correction in Eq. (2)). Since both methods are in agreement but only the hit-based method covers a large pseudo-rapidity interval, we use this latter method.

We want to check whether the theoretical and experimental estimates are in agreement, i.e., $\langle v_2^{\mathrm{b}}(\eta)\rangle = \langle v_2^{\mathrm{rec}}(\eta)\rangle$. A necessary condition for this, from Eq. (2), is, $\langle v_2^{\mathrm{b}}(\eta)\rangle \geqslant \langle v_2^{\mathrm{obs}}(\eta)\rangle$. In Fig. 2, we show the results for $\langle v_2^{\mathrm{b}}(\eta)\rangle$ (solid line) and $\langle v_2^{\mathrm{obs}}(\eta)\rangle$ (dashed line).

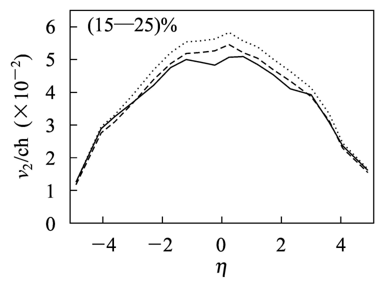

Fig. 2. Comparison between various ways of computing v_2 using NeXSPheRIO for Phobos 15%—25% centrality window[11]: solid line is v_2^{b}, obtained using the known impact parameter angle ϕ_b, dashed (dotted) line is v_2^{obs} (v_2^{rec}), obtained using the reconstructed impact parameter angle ψ_2 without (with) reaction plane correction.

We see that $\langle v_2^{\mathrm{b}}(\eta)\rangle \leqslant \langle v_2^{\mathrm{obs}}(\eta)\rangle$ for most η's. So, as shown also in the figure, dividing by a cosine to get $\langle v_2^{\mathrm{rec}}(\eta)\rangle$ (dotted curve) makes the disagreement worse: $\langle v_2^{\mathrm{b}}(\eta)\rangle$ and $\langle v_2^{\mathrm{rec}}(\eta)\rangle$ are different. This is true for all three Phobos centrality windows: the two methods, $v_2^{\mathrm{b}}(\eta)$ and $v_2^{\mathrm{rec}}(\eta)$, differ by 15%—30% for $\eta=0$ (the highest value is for the most central window). It is also true for elliptic flow as a function of transverse momentum: the two methods differ by 30% for $p_\perp=0.5$GeV. The question that arises then is why is there such a difference. In this context it is interesting to note that there exist other works[12, 13], where it was found that v_2^{b}, calculated using the known quantity $\boldsymbol{b}$ and v_2^{rec}, calculated with the reaction plane method or two-particle cumulant method yield different results. In Ref. [12], Miller and Snellings assume $v_2\propto\epsilon$ (spatial anisotropy) and in Ref. [13], Zhu, Bleicher and Stöcker compute v_2 from UrQMD.

3 Discussion of the reason for $v_2^{\mathrm{b}}(\eta) < v_2^{\mathrm{rec}}(\eta)$

In order to use an equation such as Eq. (2), which we write generically as

$$\langle v_2^{\mathrm{rec}}(\eta)\rangle = \left\langle \frac{v_2(\eta)}{\sqrt{\langle\cos[2(\varPsi^{<}-\varPsi^{>})]\rangle}} \right\rangle,$$

one needs to assume that particles are emitted symmetrically in the plane with inclination $\varPsi$ with respect to the reference axis and containing the beam axis, so that:

$$\frac{\mathrm{d}^2N}{\mathrm{d}\phi\mathrm{d}\eta} = v_0(\eta)\left[1+\sum_n 2v_n(\eta)\cos(n(\phi-\varPsi))\right]. \quad (5)$$

In the past, it was assumed that $\varPsi=\phi_b$, i.e. the particles should be emitted symmetrically with respect to the reaction plane (the plane defined by the impact parameter vector and the beam axis).

In NeXSPheRIO, when we look at the distribution $\mathrm{d}^2N/\mathrm{d}\phi\mathrm{d}\eta$ of a given event (presumably also in a true event), it is not symmetric with respect to the reaction plane, i.e. $\varPsi\neq\phi_b$. Therefore Eq. (5) should not hold for ϕ_b.

This happens because i) the incident nuclei have a granular structure, ii) the number of produced particles is finite. In fact, for NeXSPheRIO as can be seen in Fig. 3, a better approximation would be $\varPsi=\psi_2$, so Eq. (5) should hold for ψ_2. As a consequence, $v_2^{\mathrm{b}}(\eta)\neq v_2^{\mathrm{rec}}(\eta)$.

In a similar way, it was noted by Phobos at QM05[14] that the relevant eccentricity to understand their elliptic flow data seems to be not the standard one (with minor axis in the direction of $\boldsymbol{b}$), but the participant eccentricity (computed considering the participant nucleon ellipse).

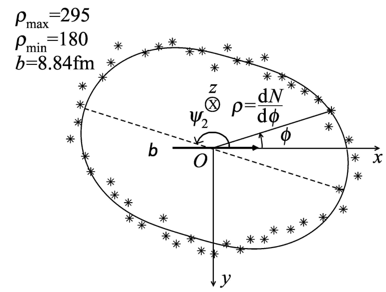

Fig. 3. Example of angular distribution obtained with NeXSPheRIO at RHIC.

This rises the question of how to compare in general results for elliptical flow obtained in standard hydrodynamics, i.e. v_2^{b}, with data. As a first rough approximation, one might argue that the problem can be ignored: what matters is to have the right amount of push, not the right direction. On the other side, one may not want to ignore the problem since something in missing in the description. Within NeXSPeRIO[9], we found that one can relate $v_2^{\mathrm{b}}(\eta)$ and data in the following way:

$$\langle v_2^{\mathrm{b}}(\eta)\rangle \sim \langle v_2^{\mathrm{obs}}(\eta)\rangle\times\sqrt{\langle\cos(2(\psi_2^{<}-\psi_2^{>})\rangle} \equiv \langle v_2^{\mathrm{rec}}(\eta)\rangle. \quad (6)$$

We note that there is now a multiplication where there used to be a division. In Fig. 4, we show $\langle v_2^{\mathrm{rec}}(\eta)\rangle$ (dash-dotted line) and $\langle v_2^{\mathrm{b}}(\eta)\rangle$ (solid line). We see that the agreement between both methods is improved compared to Fig. 2. We have also computed the elliptic flow parameter as function of transverse momentum for charged hadrons with $0<\eta<1.5$ for the 50% most central collisions. We found that $\langle v_2^{\mathrm{b}}(p_\perp)\rangle$ computed as in Eq. (1) is well approximated by $\langle v_2^{\mathrm{rec}}(p_\perp)\rangle$ computed as in Eq. (6).

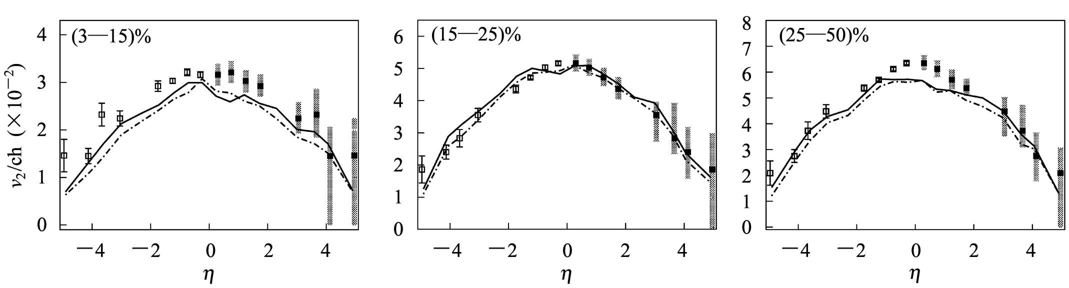

Fig. 4. Comparison between true elliptic flow v_2^{b} (solid line) and suggested method to compute reconstructed elliptic flow from data v_2^{rec} (dash-dotted) for the three Phobos centrality windows[11]. Squares represent Phobos data (black error bars are 1σ statistical errors and grey bands, systematic uncertainties at $\sim$90% confidence level).

Since our code does not have all the systematic and statistical errors that an experiment has, it remains to be checked how to apply our formula in this case.

4 Summary

In NeXSPheRIO, the impact parameter $\boldsymbol{b}$ is known so one can compare the standard hydrodynamical estimate $\langle v_2^{\rm b}\rangle$ computed with respect to $\boldsymbol{b}$ with $\langle v_2^{\rm rec}\rangle$, reconstructed in the standard experimental way, i.e. we found that

$$\langle v_2^{\rm b}\rangle \neq \langle v_2^{\rm rec}\rangle \equiv \langle v_2^{\rm obs}\rangle / \sqrt{\langle \cos[2(\psi_2^{<} - \psi_2^{>})]\rangle}.$$

On the other side, we derived and checked that for NeXSPheRIO

$$\langle v_2^{\rm b}\rangle \sim \langle v_2^{\rm rec}\rangle \equiv \langle v_2^{\rm obs}\rangle \times \sqrt{\langle \cos[2(\psi_2^{<} - \psi_2^{>})]\rangle}.$$

In general, estimates of v_2 in the reaction plane will be lower than data as seen in the works of Miller & Snellings[12] and Zhu, Bleicher & Stöcker[13], in agreement with ours[9]. Therefore more work should be done on how to compare hydrodynamical estimates with data if we want to establish thermalization, viscosity, initial conditions, etc, as attempted for example in Refs. [15—17].

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