第 31 卷 第 12 期 高 能 物 理 与 核 物 理 Vol. 31, No. 12
2007 年 12 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Dec., 2007

A Method for Measuring Event-by-Event Elliptic Flow Fluctuations with the First-Order Event Plane in Heavy-Ion Collisions

WANG Gang[1;1)] Declan Keane[2] Aihong Tang[3] Sergei A. Voloshin[4]

1 (University of California, Los Angeles, California 90095, USA)
2 (Kent State University, Kent, Ohio 44242, USA)
3 (Brookhaven National Laboratory, Upton, New York 11973, USA)
4 (Wayne State University, Detroit, Michigan 48201, USA)

Abstract A new method is presented for measuring event-by-event fluctuations of elliptic flow (v_2) using first-order event planes. By studying the event-by-event distributions of v_2 observables and first-order event-plane observables, average flow $\langle v_2\rangle$ and event-by-event fluctuations with respect to that average can be separately determined, making appropriate allowance for the effects of finite multiplicity. The relation of flow fluctuations to eccentricity fluctuations in the initial-state participant region, as well as detector acceptance effects, are discussed.

Key words elliptic flow, fluctuation, event plane, heavy-ion collision

In heavy-ion collisions, the azimuthal distributions of emitted particles can be decomposed with a Fourier expansion[1],

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi}=\frac{1}{2\pi}\left\{1+\sum_{n=1}^{\infty}2v_n\cos n(\varphi-\Psi_{\rm RP})\right\}, \qquad (1)$$

where φ and $\Psi_{\rm RP}$ denote the azimuthal angle of the particle and of the reaction plane, respectively. The Fourier coefficients,

$$v_n=\langle\cos n(\varphi-\Psi_{\rm RP})\rangle\,, \qquad (2)$$

are referred to as anisotropic flow of the nth harmonic. The second harmonic, elliptic flow, carries information on the early stage of heavy-ion collisions, and has been extensively studied. Event-by-event flow fluctuations[2—6] are of considerable interest because any fluctuation observable has potential relevance for phase transition phenomena, and because typical anisotropic flow measurements are dominated by systematic uncertainties in which flow fluctuations play a crucial role.

Most flow analyses at RHIC to date have relied on the second-order event plane, whereas in the present study, a case is presented for utilizing the first-order event plane to determine the mean elliptic flow in any sample, and to isolate the sought-after dynamical fluctuations about that mean. In RHIC experiments, first-order event planes can be obtained, for example, via the ZDC-SMD (Shower Maximum Detector)[7] or the Forward TPC[8] of the STAR detector. In the scenario envisaged here, the fluctuating anisotropies are based on measurements near midrapidity, while the first-order event plane determination utilizes detectors that are far removed in rapidity. Consequently, non-flow effects (correlations that may contribute to v_n but which are unrelated to the event reaction plane) are believed to be negligible using this method[8]. With two independent first-order event planes ψ_a and ψ_b, elliptic flow can be determined with the help of the relations

$$\begin{aligned}v_2^{\rm obs}\ =\ &\langle\cos(2\varphi-\psi_a-\psi_b)\rangle=\\&\langle\cos(2\varphi-2\Psi_{\rm RP})\rangle\,\langle\cos(2\Psi_{\rm RP}-\psi_a-\psi_b)\rangle=\\&v_2\,\langle\cos(\psi_a-\psi_b)\rangle\,,\end{aligned} \qquad (3)$$

Received 25 June 2007

1) E-mail: gwang@physics.ucla.edu

where the last factor, $\langle\cos(\psi_a-\psi_b)\rangle$ is the product of the two first-order event plane resolutions[1]. The above is based on the assumptions that the two event planes are independent, and that the distributions of ψ_a and ψ_b with respect to the true reaction plane are symmetric.

We introduce two event-by-event quantities

$$c_2^{(\mathrm{k})} \equiv \langle\cos 2(\varphi-\Psi_{\mathrm{RP};k})\rangle, \tag{4}$$

$$s_2^{(\mathrm{k})} \equiv \langle\sin 2(\varphi-\Psi_{\mathrm{RP};k})\rangle, \tag{5}$$

where index k denotes the kth event. Using the equality

$$\begin{aligned}\cos\,(2\varphi-\psi_{a;k}-\psi_{b;k}) &= \cos[2(\varphi-\Psi_{\mathrm{RP};k})-\\ &(\psi_{a;k}-\Psi_{\mathrm{RP};k})-(\psi_{b;k}-\Psi_{\mathrm{RP};k})] =\\ &\cos 2(\varphi-\Psi_{\mathrm{RP};k})\cos(\Delta\psi_{a;k}+\Delta\psi_{b;k})+\\ &\sin 2(\varphi-\Psi_{\mathrm{RP};k})\sin(\Delta\psi_{a;k}+\Delta\psi_{b;k}),\end{aligned} \tag{6}$$

where $\Delta\psi_{a;k}=\psi_{a;k}-\Psi_{\mathrm{RP};k}$ and $\Delta\psi_{b;k}=\psi_{b;k}-\Psi_{\mathrm{RP};k}$, and averaging over all particles in a single event, we define

$$\begin{aligned}c_2^{\mathrm{obs;k}} &\equiv \langle\cos(2\varphi-\psi_{a;k}-\psi_{b;k})\rangle =\\ &c_2^{(\mathrm{k})}\cos(\Delta\psi_{a;k}+\Delta\psi_{b;k})+\\ &s_2^{(\mathrm{k})}\sin(\Delta\psi_{a;k}+\Delta\psi_{b;k}),\end{aligned} \tag{7}$$

$$\begin{aligned}s_2^{\mathrm{obs;k}} &\equiv \langle\sin(2\varphi-\psi_{a;k}-\psi_{b;k})\rangle =\\ &s_2^{(\mathrm{k})}\cos(\Delta\psi_{a;k}+\Delta\psi_{b;k})-\\ &c_2^{(\mathrm{k})}\sin(\Delta\psi_{a;k}+\Delta\psi_{b;k}).\end{aligned} \tag{8}$$

Note that, averaged over all events, $\langle c_2^{\mathrm{obs;k}}\rangle=v_2^{\mathrm{obs}}$, $\langle s_2^{\mathrm{obs;k}}\rangle=0$, $\langle c_2^{(\mathrm{k})}\rangle=v_2$, and $\langle s_2^{(\mathrm{k})}\rangle=0$.

Each quantity in Eq. (7) is an instance (or element) of its own distribution (or set)[9]. Thus the distribution of $c_2^{\mathrm{obs;k}}$ is related to the distributions of $c_2^{(\mathrm{k})}$, $s_2^{(\mathrm{k})}$, and $(\Delta\psi_{a;k}+\Delta\psi_{b;k})$ according to

$$\begin{aligned}\{c_2^{\mathrm{obs;k}}\} &= \{c_2^{(\mathrm{k})}\}\otimes\{\cos(\Delta\psi_{a;k}+\Delta\psi_{b;k})\}\oplus\\ &\{s_2^{(\mathrm{k})}\}\otimes\{\sin(\Delta\psi_{a;k}+\Delta\psi_{b;k})\},\end{aligned} \tag{9}$$

where the operators $\otimes$ and $\oplus$ demonstrate the relationship between the instances of the distributions, instead of between the distributions themselves[9]. Here we assume that the distribution of the first-order event planes $\{\Delta\psi_{a;k}+\Delta\psi_{b;k}\}$ is independent of $\{c_2^{(\mathrm{k})}\}$ and $\{s_2^{(\mathrm{k})}\}$. This assumption is usually not valid for second-order event planes, and this illustrates one of the advantages of the present approach. We further assume that $\{\Delta\psi_{a;k}\}$ and $\{\Delta\psi_{b;k}\}$ are both symmetric around zero, so that $\{\Delta\psi_{a;k}+\Delta\psi_{b;k}\}$ and $\{\Delta\psi_{a;k}-\Delta\psi_{b;k}\}$ are identical. Thus Eq. (9) becomes

$$\begin{aligned}\{c_2^{\mathrm{obs;k}}\} &= \{c_2^{(\mathrm{k})}\}\otimes\{\cos\Delta\psi_{ab;k}\}\oplus\\ &\{s_2^{(\mathrm{k})}\}\otimes\{\sin\Delta\psi_{ab;k}\},\end{aligned} \tag{10}$$

where $\Delta\psi_{ab;k}=\Delta\psi_{a;k}-\Delta\psi_{b;k}=\psi_{a;k}-\psi_{b;k}$. Note that the cosine part and the sine part on the r.h.s. of Eq. (10) are not independent. However, in the procedure for estimating the mean and RMS values of v_2 (all that is required in the present analysis), the cross term vanishes due to symmetry, so we can treat these two parts as independent. The distribution $\{s_2^{\mathrm{obs;k}}\}$ can be treated in a similar way.

In the following discussion, we use the same convention as in Ref. [9], where $E\{x\}$ and $\sigma\{x\}$ denote the mean (or expectation value) and the RMS (or standard deviation), respectively, of the distribution $\{x\}$. Eq. (10) and its sin-counterpart can be rewritten and resolved with respect to mean values of $\{c_2^{(\mathrm{k})}\}$ and $\{s_2^{(\mathrm{k})}\}$:

$$\begin{aligned}E\{c_2^{(\mathrm{k})}\} &= \big[E\{c_2^{\mathrm{obs;k}}\}E\{\cos\Delta\psi_{ab;k}\}-\\ &E\{s_2^{\mathrm{obs;k}}\}E\{\sin\Delta\psi_{ab;k}\}\big]\big/\\ &\big[E^2\{\cos\Delta\psi_{ab;k}\}+E^2\{\sin\Delta\psi_{ab;k}\}\big],\end{aligned} \tag{11}$$

$$\begin{aligned}E\{s_2^{(\mathrm{k})}\} &= \big[E\{s_2^{\mathrm{obs;k}}\}E\{\cos\Delta\psi_{ab;k}\}+\\ &E\{c_2^{\mathrm{obs;k}}\}E\{\sin\Delta\psi_{ab;k}\}\big]\big/\\ &\big[E^2\{\cos\Delta\psi_{ab;k}\}+E^2\{\sin\Delta\psi_{ab;k}\}\big].\end{aligned} \tag{12}$$

Everything on the r.h.s. of Eq. (11) and (12) is an experimental observable. In the case $E\{\sin\Delta\psi_{ab;k}\}\ll E\{\cos\Delta\psi_{ab;k},\}$ Eq. (11) and (12) reduce to

$$E\{c_2^{(\mathrm{k})}\} = E\{c_2^{\mathrm{obs;k}}\}/E\{\cos\Delta\psi_{ab;k}\}, \tag{13}$$

$$E\{s_2^{(\mathrm{k})}\} = E\{s_2^{\mathrm{obs;k}}\}/E\{\cos\Delta\psi_{ab;k}\}. \tag{14}$$

Normally, $E\{\cos\Delta\psi_{ab;k}\}$ is regarded as a correction for the event plane resolution.

The RMS values of the v_2 distributions in Eq. (10)

and its sin-counterpart are calculable as per

$$\sigma^2\{c_2^{(k)}\} = (VE\{\cos^2\Delta\psi_{ab;k}\} - SE\{\sin^2\Delta\psi_{ab;k}\})/E\{\cos 2\Delta\psi_{ab;k}\}, \quad (15)$$

$$\sigma^2\{s_2^{(k)}\} = (SE\{\cos^2\Delta\psi_{ab;k}\} - VE\{\sin^2\Delta\psi_{ab;k}\})/E\{\cos 2\Delta\psi_{ab;k}\}. \quad (16)$$

where

$$V = \sigma^2\{c_2^{\mathrm{obs;k}}\} - E^2\{c_2^{(k)}\}\ \sigma^2\{\cos\Delta\psi_{ab;k}\} - E^2\{s_2^{(k)}\}\ \sigma^2\{\sin\Delta\psi_{ab;k}\}, \quad (17)$$

$$S = \sigma^2\{s_2^{\mathrm{obs;k}}\} - E^2\{c_2^{(k)}\}\ \sigma^2\{\sin\Delta\psi_{ab;k}\} - E^2\{s_2^{(k)}\}\ \sigma^2\{\cos\Delta\psi_{ab;k}\}. \quad (18)$$

The denominator in Eqs. (15) and (16), $E\{\cos 2\Delta\psi_{ab;k}\}$, is related to the second-order event plane resolutions derived from the first-order event planes.

The observed fluctuations $\sigma\{c_2^{(k)}\}$ and $\sigma\{s_2^{(k)}\}$ each have two contributions: dynamical and statistical fluctuations.

$$\sigma^2\{c_2^{(k)}\} = \sigma^2_{\mathrm{dyn}}\{c_2^{(k)}\} + \sigma^2_{\mathrm{stat}}\{c_2^{(k)}\} = \sigma^2_{\mathrm{dyn}}\{c_2^{(k)}\} + 0.5(1+v_4-2E^2\{c_2^{(k)}\})/M, \quad (19)$$

$$\sigma^2\{s_2^{(k)}\} = \sigma^2_{\mathrm{dyn}}\{s_2^{(k)}\} + \sigma^2_{\mathrm{stat}}\{s_2^{(k)}\} = \sigma^2_{\mathrm{dyn}}\{s_2^{(k)}\} + 0.5(1-v_4-2E^2\{s_2^{(k)}\})/M, \quad (20)$$

where M denotes multiplicity, and the v_4 term arises from setting $n=2$ in the following equalities:

$$\langle\cos^2 n(\varphi-\Psi_{\mathrm{RP}})\rangle = 0.5(1+v_{2n}), \quad (21)$$

$$\langle\sin^2 n(\varphi-\Psi_{\mathrm{RP}})\rangle = 0.5(1-v_{2n}). \quad (22)$$

v_4 is usually negligible compared with 1.

Although Eq. (13) has minimal, if any, non-flow contributions, Eqs. (15) and (16) are influenced by non-flow effects, since two-particle correlation is involved in the definition of flow fluctuations. If we assume that there are the same or similar amounts of non-flow contributions in $\sigma_{\mathrm{dyn}}\{c_2^{(k)}\}$ and $\sigma_{\mathrm{dyn}}\{s_2^{(k)}\}$, then in principle we can greatly suppress non-flow contributions with the difference:

$$\sigma^2_{\mathrm{dyn}}\{c_2^{(k)}\} - \sigma^2_{\mathrm{dyn}}\{s_2^{(k)}\} = \frac{V-S}{E\{\cos 2\Delta\psi_{ab;k}\}} - \frac{(v_4 - E^2\{c_2^{(k)}\} + E^2\{s_2^{(k)}\})}{M}. \quad (23)$$

In the case where the distribution $\{\varphi-\Psi_{\mathrm{RP};k}\}$ is symmetric around zero, $E\{s_2^{(k)}\}$ vanishes, and we have

$$\sigma^2_{\mathrm{dyn}}\{c_2^{(k)}\} = \frac{V-S}{E\{\cos 2\Delta\psi_{ab;k}\}} - \frac{(v_4 - E^2\{c_2^{(k)}\})}{M}. \quad (24)$$

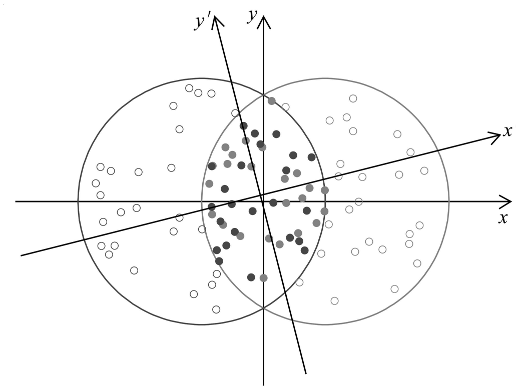

Fig. 1. (color online) Schematic representation, in the plane transverse to the beam (z) direction, of a collision between two identical nuclei. The x- and y-axes are drawn as per the standard convention. The solid circles illustrate a possible configuration of the participant nucleons. Due to the fluctuation of this event, the overlap zone is shifted and tilted with respect to the (x,y) frame. x' and y' are the principal axes of inertia of the solid circles.

In heavy-ion collisions, due to the finite number of participants, the overlap zone could be translated and rotated with respect to the conventional coordinate system, as illustrated in Fig. 1. As a result, the final produced particles are symmetrically distributed about the x' axis, instead of the x axis. In this paper, we neglect the translation, and only consider the rotation of the overlap zone. We define the angle between x' and x direction to be $\Delta\psi_k$ for the kth event, and $c_2^{(k)}$ becomes

$$c_2^{(k)} = c_2'^{(k)}\cos 2\Delta\psi_k - s_2'^{(k)}\sin 2\Delta\psi_k\,, \quad (25)$$

where $c_2'^{(k)}$ and $s_2'^{(k)}$ are measured along the x' axis. $s_2^{(k)}$ can be treated in the same way. The mean values of $\{c_2^{(k)}\}$ and $\{s_2^{(k)}\}$ are related to those of $\{c_2'^{(k)}\}$ by

$$E\{c_2^{(k)}\} = E\{c_2'^{(k)}\}E\{\cos 2\Delta\psi_k\}, \quad (26)$$

$$E\{s_2^{(k)}\} = E\{c_2'^{(k)}\}E\{\sin 2\Delta\psi_k\}. \quad (27)$$

$E\{s_2'^{(k)}\}$ goes to zero, due to symmetry. Therefore $E\{c_2^{(k)}\}$ is always less than or equal to $E\{c_2'^{(k)}\}$. If the distribution $\{\Delta\psi_k\}$ is symmetric around zero, then

$E\{s_2^{(\mathrm{k})}\}$ vanishes. $\{\Delta\psi_k\}$ can be simulated, for example, in a Glauber calculation[10].

To suppress the non-flow contributions, in principle we can follow the same idea as in Eq. (23),

$$E\{(c_2^{(\mathrm{k})})^2-(s_2^{(\mathrm{k})})^2\}= E\{(c_2^{\prime(\mathrm{k})})^2-(s_2^{\prime(\mathrm{k})})^2\}E\{\cos 4\Delta\psi_k\}. \quad (28)$$

where the distribution $\{\Delta\psi_k\}$ is assumed to be symmetric around zero. Due to symmetry, we set $E\{s_2^{(\mathrm{k})}\}$, $E\{s_2^{\prime(\mathrm{k})}\}$ and $\sigma_{\mathrm{dyn}}\{s_2^{\prime(\mathrm{k})}\}$ to zero, and arrive at the fluctuation of interest

$$\sigma^2_{\mathrm{dyn}}\{c_2^{\prime(\mathrm{k})}\} = \left(E^2\{c_2^{(\mathrm{k})}\}+\frac{V-S}{E\{\cos 2\Delta\psi_{ab;k}\}}\right)\Big/ E\{\cos 4\Delta\psi_k\}-E^2\{c_2^{\prime(\mathrm{k})}\}- (v_4^{\prime}-E^2\{c_2^{\prime(k)}\})/M\,. \quad (29)$$

Another way to measure v_2 fluctuations along the x' axis is through eccentricity. The standard definition of eccentricity for the kth event is

$$\epsilon_k \equiv \frac{\langle y_k^2-x_k^2\rangle}{\langle y_k^2+x_k^2\rangle}\,. \quad (30)$$

where x_k and y_k denote the participant coordinates. The coordinates (x,y) and (x',y') are linked by the rotation through $\Delta\psi_k$, thus the eccentricities in the two coordinate systems are related according to

$$\epsilon_k=\epsilon_k^{\prime}\cos 2\Delta\psi_k+\frac{2x_k^{\prime}y_k^{\prime}\sin 2\Delta\psi_k}{(y_k^{\prime})^2+(x_k^{\prime})^2}\,. \quad (31)$$

The mean values of $\{\epsilon_k\}$ and $\{\epsilon_k^{\prime}\}$ have the same relationship as $\{c_2^{(\mathrm{k})}\}$ and $\{c_2^{\prime(\mathrm{k})}\}$ in Eq. (26), i.e.,

$$E\{\epsilon_k\}=E\{\epsilon_k^{\prime}\}\,E\{\cos 2\Delta\psi_k\}\,. \quad (32)$$

As long as the ratio between $E\{\epsilon_k\}$ and $E\{\epsilon_k^{\prime}\}$ is known, we can calculate v_2 and its fluctuation along the x' axis.

To allow for detector imperfections, we can break down the v_2 observable into cosine and sine components.

$$v_{2\cos}^{\mathrm{obs}} \equiv \langle 2\cos 2\varphi\ \cos(\psi_a+\psi_b)\rangle \quad (33)$$

$$v_{2\sin}^{\mathrm{obs}} \equiv \langle 2\sin 2\varphi\ \sin(\psi_a+\psi_b)\rangle \quad (34)$$

In the kth event, we have

$$c_{2\cos}^{\mathrm{obs;k}} = c_2^{\mathrm{obs;k}}+\langle\cos(2\varphi+\psi_{a;k}+\psi_{b;k})\rangle = 2\langle\cos^2 2\varphi\rangle\, c_2^{\mathrm{obs;k}}+\langle\sin 4\varphi\rangle\, s_2^{\mathrm{obs;k}} \quad (35)$$

$$c_{2\sin}^{\mathrm{obs;k}} = c_2^{\mathrm{obs;k}}-\langle\cos(2\varphi+\psi_{a;k}+\psi_{b;k})\rangle = 2\langle\sin^2 2\varphi\rangle\, c_2^{\mathrm{obs;k}}-\langle\sin 4\varphi\rangle\, s_2^{\mathrm{obs;k}}. \quad (36)$$

If φ can be measured with negligible detector imperfections, then $\langle\sin 4\varphi\rangle$ should vanish, and there is no difference between $\langle\cos^2 2\varphi\rangle$ and $\langle\sin^2 2\varphi\rangle$. Otherwise, $E\{c_{2\cos}^{\mathrm{obs;k}}\}$ and $E\{c_{2\sin}^{\mathrm{obs;k}}\}$ will be different from each other, and from $E\{c_2^{\mathrm{obs;k}}.\}$ In some of the cases previously mentioned, $E\{s_2^{\mathrm{obs;k}}\}$ goes to zero, and we may consider $\langle\cos^2 2\varphi\rangle$ and $\langle\sin^2 2\varphi\rangle$ to be correction factors for the detector deficiencies.

We can further separate the v_2 observable into four terms:

$$v_{2\cos'}^{\mathrm{obs}} \equiv \langle 4\cos 2\varphi\cos\psi_a\cos\psi_b\rangle, \quad (37)$$

$$v_{2\cos''}^{\mathrm{obs}} \equiv \langle -4\cos 2\varphi\sin\psi_a\sin\psi_b\rangle, \quad (38)$$

$$v_{2\sin'}^{\mathrm{obs}} \equiv \langle 4\sin 2\varphi\sin\psi_a\cos\psi_b\rangle, \quad (39)$$

$$v_{2\sin''}^{\mathrm{obs}} \equiv \langle 4\sin 2\varphi\cos\psi_a\sin\psi_b\rangle. \quad (40)$$

By way of example, we next consider the first term in the kth event.

$$c_{2\cos'}^{\mathrm{obs;k}} = [1+\langle\cos 4\varphi\rangle+\cos 2\psi_{a;k}+\cos 2\psi_{b;k}]\, c_2^{\mathrm{obs;k}}+ [\langle\sin 4\varphi\rangle-\sin 2\psi_{a;k}-\sin 2\psi_{b;k}]\, s_2^{\mathrm{obs;k}}\,. \quad (41)$$

If φ, ψ_a and ψ_b are all measured with negligible detector imperfections, then $E\{c_{2\cos'}^{\mathrm{obs;k}}\}=E\{c_2^{\mathrm{obs;k}}.\}$ When $\langle\cos 4\varphi\rangle$, $\langle\cos 2\psi_{a;k}\rangle$ and $\langle\cos 2\psi_{b;k}\rangle$ are all very small compared with 1, the correction factor can be approximated according to

$$1+\langle\cos 4\varphi+\cos 2\psi_{a;k}+\cos 2\psi_{b;k}\rangle\approx [1+\langle\cos 4\varphi\rangle]\,[1+\langle\cos 2\psi_{a;k}\rangle]\,[1+\langle\cos 2\psi_{b;k}\rangle]= 2\langle\cos^2 2\varphi\rangle\, 2\langle\cos^2\psi_{a;k}\rangle\, 2\langle\cos^2\psi_{b;k}\rangle. \quad (42)$$

The other three terms of the v_2 observable can be treated in a similar way.

In summary, this work presents a new method for experimental analysis of elliptic flow in a scenario where the first-order event plane can be resolved. The method allows the extraction of mean v_2 and its dynamical event-by-event fluctuations, and good immunity to statistical fluctuations can be expected. Recipes have been developed to suppress non-flow contributions. Fluctuations in spatial eccentricity in the initial-state participant region have been considered, as have detector acceptance effects.

References

1 Voloshin S, ZHANG Y. Z. Phys., 1996, **C70**: 665; Poskanzer A M, Voloshin S A. Phys. Rev., 1998, **C58**: 1671

2 Adler C et al. (STAR Collaboration). Phys. Rev., 2002, **C66**: 034904

3 Adams J et al. (STAR Collaboration). Phys. Rev., 2005, **C72**: 014904

4 Miller M, Snellings R. preprint nucl-ex/0312008

5 Alver B et al. (PHOBOS Collaboration). preprint nucl-ex/0608025

6 WANG G, TANG A H, Keane D et al. preprint nucl-ex/0611001

7 Adler C et al. Nucl. Instrum. Methods, 2001, **A470**: 488; The STAR ZDC-SMD has the same structure as the STAR EEMC SMD: Allgower C E et al. Nucl. Instrum. Methods, 2003, **A499**: 740; Nucl. Instrum. Methods, 2003, **A499**: 713

8 Adams J et al. (STAR Collaboration). Phys. Rev., 2006, **C73**: 034903

9 Fraser D A S. Statistics: An Introduction. New York: John Wiley, 1960

10 Back B B et al. Phys. Rev., 2002, **C65**: 031901; Adcox K et al. Phys. Rev. Lett., 2001, **86**: 3500; Bearden I G et al. Phys. Lett., 2001, **B523**: 227; Adams J et al. (STAR Collaboration). Phys. Rev., 2004 **C70**: 054907