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Fluctuations and the Phase Transition in a Chiral Model with Polyakov Loops*

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Abstract We explore the NJL model with Polyakov loops for a system of three colors and two flavors within the mean-field approximation, where both chiral symmetry and confinement are taken into account. We focus on the phase structure of the model and study the chiral and Polyakov loop susceptibilities.

Key words NJL model, chiral symmetry, phase transition, fluctuation

1 Introduction

Low-energy phenomena of QCD have been studied in various effective models based on chiral symmetry. However, the relation of spontaneous chiral symmetry breaking and confinement remains an open issue. Recently, color degrees of freedom were introduced in the Nambu-Jona-Lasinio (NJL) Lagrangian[1, 2] through an effective gluon potential expressed in terms of Polyakov loops (PNJL model)[3, 4]. The basic ingredients of the model are constituent quarks and the Polyakov loop, which is an order parameter of the $Z(3)$ symmetry of QCD in the heavy quark limit. The model has a non-vanishing coupling of the constituent quarks to the Polyakov loop and mimics confinement in the sense that only three-quark states contribute to the thermodynamics in the low-temperature phase. Hence, the PNJL model yields a better description of QCD thermodynamics near the phase transition than the NJL model. Furthermore, due to the symmetries of the Lagrangian, the model belongs to the same universality class as that expected for QCD. Thus, the model can be considered as a testing ground for the critical phenomena related to the breaking of the global $Z(3)$ and chiral symmetries.

It has been shown that the PNJL model, formulated at finite temperature and finite quark chemical potential, well reproduces some of the thermodynamical observables computed within lattice gauge theory (LGT). The properties of the equation of state[4], the in-medium modification of meson masses[5] as well as the validity and applicability of the Taylor expansion in quark chemical potential used in LGT were recently addressed within the PNJL model[6—9]. In Ref. [10] the model was extended to a system with finite isospin chemical potential and pion condensation was studied.

Enhanced fluctuations are characteristic for phase transitions. Thus, the exploration of fluctuations is a promising tool for probing the phase structure of a system. The phase boundaries can be identified by the response of the fluctuations to changes in the thermodynamic parameters. In this article we describe the susceptibilities of the order parameters and their properties in the PNJL model following Ref. [11].

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2 Nambu-Jona-Lasinio model with Polyakov loops

An extension of the NJL Lagrangian by coupling the quarks to a uniform temporal background gauge field, which manifests itself entirely in the Polyakov loop, has been proposed to account for interactions with the color gauge field in effective chiral models[3, 4]. The PNJL Lagrangian for three colors (N_c=3) and two flavors (N_f=2) with non-local four-fermion interactions is given by

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\gamma^{\mu}D_{\mu}-\hat{m})\psi+\bar{\psi}\hat{\mu}\gamma_0\psi-\mathscr{U}(\Phi[A],\bar{\Phi}[A];T)+ \frac{G_{\mathrm{S}}}{2}\left[(\bar{q}(x)q(x))^2+(\bar{q}(x)\mathrm{i}\gamma_5\boldsymbol{\tau}q(x))^2\right], \tag{1}$$

where $\hat{m}=\mathrm{diag}(m_{\mathrm{u}},m_{\mathrm{d}})$ is the current quark mass, $\hat{\mu}=\mathrm{diag}(\mu_{\mathrm{u}},\mu_{\mathrm{d}})$ is the quark chemical potential and $\boldsymbol{\tau}$ are the Pauli matrices. We assume isospin symmetry and take $m_{\mathrm{u}}=m_{\mathrm{d}}\equiv m_0$ and $\mu_{\mathrm{u}}=\mu_{\mathrm{d}}\equiv\mu$.

Here we have introduced non-local interactions which are controlled by a form factor in order to deal with the ultraviolet singularities that appear in the loop integrations. In coordinate space, the form factor $F(x)$ for the non-local current-current interaction reads:

$$q(x)=\int \mathrm{d}^4yF(x-y)\psi(y). \tag{2}$$

A possible choice for the regulator is in momentum space given by[12]:

$$f^2(p)=\frac{1}{1+(p/\Lambda)^{2\alpha}}, \tag{3}$$

where $f(p)$ is the Fourier transform of the form factor $F(x)$ and p is the three-momentum. The NJL sector is controlled by four parameters: the strength of four-fermion interaction G_{S}, the current quark mass m_0 and the constants α and Λ, which characterize the range of the non-locality. These parameters are determined in vacuum, for a given α, by requiring that the experimental values of the pion parameters and the quark condensate are reproduced.

The interaction between the effective gluon field and the quarks in the PNJL Lagrangian is implemented (1) by means of a covariant derivative

$$D_{\mu}=\partial_{\mu}-\mathrm{i}A_{\mu}, \qquad A_{\mu}=\delta_{\mu 0}A^0, \tag{4}$$

where we introduce the standard notation $A_{\mu}=gA_{\mu}^{\mathrm{a}}\frac{\lambda^{\mathrm{a}}}{2}$. Here g is the color $SU(3)$ gauge coupling constant and λ^{a} are the Gell-Mann matrices.

The effective potential $\mathscr{U}$ of the gluon field in Eq. (1) is expressed in terms of the traced Polyakov loop Φ and its conjugate $\bar{\Phi}$

$$\Phi=\frac{1}{N_{\mathrm{c}}}\mathrm{Tr}_{\mathrm{c}}L, \qquad \bar{\Phi}=\frac{1}{N_{\mathrm{c}}}\mathrm{Tr}_{\mathrm{c}}L^{\dagger}, \tag{5}$$

where L is a matrix in color space related to the gauge field by

$$L(\boldsymbol{x})=\mathscr{P}\exp\left[\mathrm{i}\int_0^{\beta}\mathrm{d}\tau A_4(\boldsymbol{x},\tau)\right], \tag{6}$$

with $\mathscr{P}$ being the path (Euclidean time) ordering, and $\beta=1/T$ with $A_4=\mathrm{i}A_0$. In the heavy quark mass limit, QCD has the $Z(3)$ center symmetry which is spontaneously broken in the high-temperature phase. The thermal expectation value of the Polyakov loop $\langle\Phi\rangle$ acts as an order parameter of the $Z(3)$ symmetry. Consequently, $\langle\Phi\rangle=0$ at low temperatures in the confined phase and $\langle\Phi\rangle\neq 0$ at high temperatures corresponding to the deconfined phase.

The effective potential $\mathscr{U}(\Phi,\bar{\Phi})$ of the gluon field is expressed in terms of the Polyakov loops so as to preserve the $Z(3)$ symmetry of the pure gauge theory[13]. We adopt an effective potential of the following form[4]:

$$\frac{\mathscr{U}(\Phi,\bar{\Phi};T)}{T^4}=-\frac{b_2(T)}{2}\bar{\Phi}\Phi-\frac{b_3}{6}(\Phi^3+\bar{\Phi}^3)+\frac{b_4}{4}(\bar{\Phi}\Phi)^2, \tag{7}$$

with

$$b_2(T)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2+a_3\left(\frac{T_0}{T}\right)^3. \tag{8}$$

The coefficients T_0, a_i and b_i are fixed by requiring that the equation of state obtained in pure gauge theory on the lattice is reproduced. In particular, at T_0=270MeV the model reproduces the first order deconfinement phase transition of the pure gauge theory.

3 Susceptibilities and the phase structure

In the PNJL model the constituent quarks and the Polyakov loops are effective fields related with the order parameters for the chiral and $Z(3)$ symmetry breaking. In LGT the susceptibilities associated with these fields show clear signals of the phase transitions[14].

In Fig. 1 we show the chiral $\chi_{\rm mm}$ and Polyakov loop $\chi_{l\bar{l}} = \langle\bar{\Phi}\Phi\rangle - \langle\bar{\Phi}\rangle\langle\Phi\rangle$ susceptibilities computed at $\mu = 0$ in the PNJL model in the chiral limit within the mean field approximation.

The chiral susceptibility exhibits a very narrow divergent peak at the chiral critical temperature $T_{\rm ch}$, while the peak of $\chi_{l\bar{l}}$ is much broader and the susceptibility remains finite at all temperatures. This is due to the explicit breaking of the $Z(3)$ symmetry by the presence of quark fields in the PNJL Lagrangian. Nevertheless, $\chi_{l\bar{l}}$ still exhibits a peak structure that can be considered as a signal for the deconfinement transition in this model. One finds the interference of $\chi_{l\bar{l}}$ with $\chi_{\rm mm}$ in Fig. 1: At the chiral transition, $T = T_{\rm ch}$, there is a narrow structure in $\chi_{l\bar{l}}$. We stress that this feature is not related with the deconfinement transition, but expresses the coupling of quarks to the Polyakov loops. Thus, for the parameters used in the model, the deconfinement transition, signaled by the broad bump in $\chi_{l\bar{l}}$, sets in earlier than the chiral transition at vanishing net quark density.

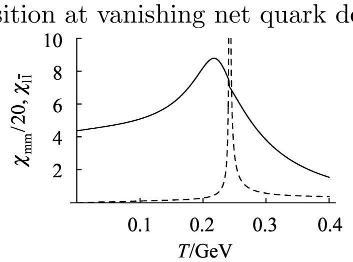

Fig. 1. The chiral $\chi_{\rm mm}$ (dashed-line) and the Polyakov loop $\chi_{l\bar{l}}$ (solid-line) susceptibilities in the chiral limit as functions of temperature T for $\mu = 0$.

The peak positions of the $\chi_{\rm mm}$ and $\chi_{l\bar{l}}$ susceptibilities determine the phase boundaries in the (T, μ)-plane. At finite chemical potential, there is a shift of the chiral phase transition to lower temperatures. In Fig. 2 we show the resulting phase diagram for the PNJL model. The boundary lines of deconfinement and chiral symmetry restoration do not coincide. There is only one common point in the phase diagram where the two transitions appear simultaneously.

Recent LGT results both at vanishing and at finite quark chemical potential show that deconfinement and chiral symmetry restoration appear in QCD along the same critical line[14]. In general it is possible to choose the PNJL model parameters such that the both critical temperatures coincide at $\mu = 0$.

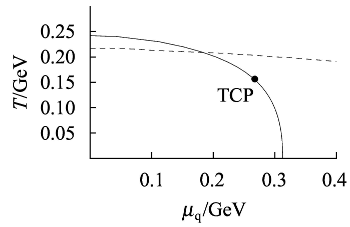

Fig. 2. The phase diagram of the PNJL model in the chiral limit. The solid (dashed) line denotes the chiral (deconfinement) phase transition respectively. The TCP (bold-point) is located at ($T_{\rm c}$=157, $\mu_{\rm c}$=266)MeV.

From Fig. 2 one finds that the slope of $T_{\rm dec}$ as a function of μ is almost flat, indicating that at low temperature the chiral phase transition should appear much earlier than deconfinement. However, there are general arguments, that the deconfinement transition should precede restoration of chiral symmetry (see Refs. [15, 16]). In view of this, it seems unlikely that at $T \simeq 0$ the chiral symmetry sets in at the lower baryon density than deconfinement. In the PNJL model, the parameters of the effective gluon potential were fixed by fitting quenched LGT calculations. Consequently, the parameters are taken as independent on μ. However, it is conceivable that the effect of dynamical quarks can modify the coefficients of this potential, thus resulting in μ-dependence of the parameters. Consequently, the slope of $T_{\rm dec}$ as a function of μ could be steeper1). Consequently, the effective Polyakov loop potential Eq. (7) should, with μ-independent coefficients, be considered as a good approximation only for $\mu/T < 1$.

While the susceptibilities $\chi_{\rm mm}$ and $\chi_{l\bar{l}}$ exhibit expected behaviors associated with the phase transitions, other Polyakov loop susceptibilities,

$$\chi_{ll} = \langle\Phi^2\rangle - \langle\Phi\rangle^2\,, \quad \chi_{\bar{l}\bar{l}} = \langle\bar{\Phi}^2\rangle - \langle\bar{\Phi}\rangle^2\,, \tag{9}$$

are negative in a broad temperature range above $T_{\rm ch}$[11]. This is in disagreement with recent lattice results, where χ_{ll} is always positive in the presence

1) Such a modification was explored in Ref. [17] where explicit μ- and $N_{\rm f}$-dependence of T_0 is extracted from the running coupling constant $\alpha_{\rm s}$, using the argument based on the renormalization group.

of dynamical quarks[14]. A possible origin of this behavior could be the approximation to the effective Polyakov loop potential used in the Eq. (7).

Recently an improved effective potential with temperature-dependent coefficients has been suggested[7]

$$\frac{\mathscr{U}(\Phi,\bar{\Phi};T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1-6\bar{\Phi}\Phi+ 4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\bar{\Phi}\Phi\right)^2\right], \tag{10}$$

where

$$a(T)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2, \quad b(T)=b_3\left(\frac{T_0}{T}\right)^3, \tag{11}$$

The polynomial in Φ and $\bar{\Phi}$, used in Eq. (7), is replaced by a logarithmic term, which accounts for the Haar measure in the group integral. The parameters in Eq. (10) were fixed to reproduce the lattice results for pure gauge QCD thermodynamics and for the behavior of the Polyakov loop. In Fig. 3 we show the χ_{ll} susceptibility calculated with the potential of Eq. (10).

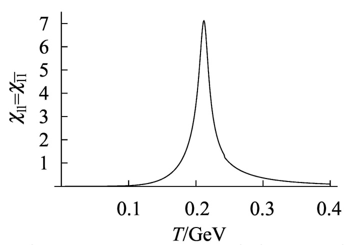

Fig. 3. The $\chi_{ll}=\chi_{\bar{l}\bar{l}}$ susceptibility in the chiral limit as a function of temperature T for $\mu=0$. The effective Polyakov loop potential Eq. (10) was used.

The improved potential indeed yields positive values for all the Polyakov loop susceptibilities. We note that the phase diagram calculated with the improved potential is similar to that obtained with the previous choice of the Polyakov loop interactions, shown in Fig. 3.

4 Summary and discussions

We introduced susceptibilities related with the three different order parameters in the PNJL model, and analyzed their properties and their behavior near the phase transitions. In particular, for the quark-antiquark and chiral density-density correlations we have discussed the interplay between the restoration of chiral symmetry and deconfinement. We observed that a coincidence of the deconfinement and chiral symmetry restoration is accidental.

We found that, within the mean field approximation and with the polynomial form of an effective gluon potential the correlations of the Polyakov loops in the quark-quark channel show an unphysical behavior, being negative in a broad parameter range. This behavior was traced back to the parameterization of the Polyakov loop potential. We argue that the $Z(\mathrm{N})$-invariance of this potential and the fit to lattice thermodynamics in the pure gluon sector is not sufficient to provide correct description of the Polyakov loop fluctuations. Actually it was pointed out[7] that the polynomial form used in this work does not possess the complete group structure of color $SU(3)$ symmetry. The improved potential of Ref. [7] yields a positive, i.e. physical, χ_{ll} susceptibility, in qualitative agreement with the LGT results.

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