A Kinematic Fit Method for All-Photon Events*

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Abstract An improved kinematic fit method is developed for analyzing all-photon events, where the interaction point is unknown. The fitting algorithm is checked with Monte Carlo samples to ensure that the fitting program works properly. This is applied to the Monte Carlo simulated ψ(2S) decays. A higher efficiency is achieved. This method can be generally applied to analyzing all-photon events at electron-positron collider.

Key words the least-square method, all-photon event, the decay vertex

1 Introduction

Kinematic fit is used extensively for analyzing data in high-energy physics experiments. It requires energy-momentum conservation between initial and final states for one event to separate signal and background, to improve momentum resolution, and so on. In general, the coordinates of the interaction point (IP, the primary vertex) are used in the kinematic fit as known quantities.

At the Beijing Electron-Positron Collider (BEPC), the beam shape is characterized by a three-dimension Gaussian distribution. The beam length is about 5cm, and the beam radius is 0.9mm in the horizontal direction (x) and 0.05mm in the vertical direction (y). The interaction point is thus also characterized by a Gaussian distribution[1].

For the process $e^+e^- \rightarrow$ ψ(2S) →hadrons at the Beijing Spectrometer (BES)[2], the IPs can be determined by the intersection point of the multiple charged tracks. Fig. 1 shows the z coordinate distribution of those events. It's a Gaussian distribution with a standard deviation of about 4.0cm.

For a charged track, e.g., an electron, the z coordinate of the vertex is got with the information of the Main Drift Chamber (MDC); the resolution of the z coordinate in the Barrel Shower Counter (BSC) can be obtained by projecting the MDC track into the BSC without the necessity to know the IP. For example, the resolution of z coordinate in the BSC is about 2.1cm from studying the Bhabha events[3], smaller than the standard deviation of the IP in z direction.

For the events with the final state containing only neutral tracks, the z coordinate of the vertex can't be determined without the information of the MDC. So it maybe lead to a big difference if one assumes that the neutral track comes from the origin point instead of the IP.

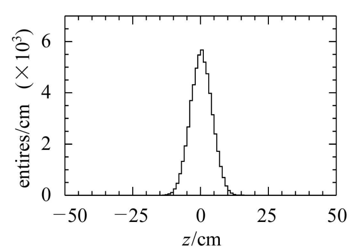

Fig. 1. The distribution of IP in the z direction measured with ψ(2S) →hadrons at BESⅡ.

In this paper, the principle of general Least-Squares estimation with constraints is expressed and

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then applied to the process $\psi(2S)\rightarrow\gamma\gamma\gamma$. At last a check is performed with Monte Carlo simulated events.

2 Principle

Let $\boldsymbol{\eta}$ be a vector of N observable, for which the first approximation values (measurements) $\boldsymbol{Y}$ with errors contained in the covariance matrix $V(\boldsymbol{Y})$ are given. In addition, a set of J unmeasurable variables $\boldsymbol{\xi}=\{\xi_1,\ldots,\xi_J\}$ are unknown. The N measurable and the J unmeasurable are related and have to satisfy a set of K constraint equations,

$$f_k(\boldsymbol{\eta},\boldsymbol{\xi})=0,\quad k=1,2,\cdots,K.$$

According to the Least-Squares Principle the best estimates of the unknown $\boldsymbol{\eta}$ and $\boldsymbol{\xi}$ should satisfy the following equations

$$\begin{cases} Q^2(\boldsymbol{\eta})=(\boldsymbol{Y}-\boldsymbol{\eta})^{\mathrm{T}}V^{-1}(\boldsymbol{Y})(\boldsymbol{Y}-\boldsymbol{\eta})=\text{minimum},\\ \boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi})=\boldsymbol{0}. \end{cases} \tag{1}$$

The problem of Eq. (1) is often resolved by the method of the Lagrangian multipliers. Introducing K additional unknowns $\boldsymbol{\lambda}=\{\lambda_1,\cdots,\lambda_k\}$, the problem is rephrased by requiring

$$Q^2(\boldsymbol{\eta},\boldsymbol{\xi},\boldsymbol{\lambda}) = (\boldsymbol{Y}-\boldsymbol{\eta})^{\mathrm{T}}V^{-1}(\boldsymbol{Y})(\boldsymbol{Y}-\boldsymbol{\eta})+ 2\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi})=\text{minimum}. \tag{2}$$

When the derivatives of Q^2 with respect to all $N+J+K$ unknowns are put equal to zero, a set of equations written in vector form are deduced as follows

$$V^{-1}(\boldsymbol{Y}-\boldsymbol{\eta})-F_\eta^{\mathrm{T}}\boldsymbol{\lambda}=\boldsymbol{0}, \tag{3}$$

$$F_\xi^{\mathrm{T}}\boldsymbol{\lambda}=\boldsymbol{0}, \tag{4}$$

$$\boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi})=\boldsymbol{0}. \tag{5}$$

where the matrices F_η (of dimension $K\times N$) and F_ξ (of dimension $K\times J$) are defined by

$$(F_\eta)_{k_i}=\frac{\partial f_k}{\partial\eta_i},\qquad (F_\xi)_{k_j}=\frac{\partial f_k}{\partial\xi_j}. \tag{6}$$

The solution of the set of Eqs. (3)—(5) for the $N+J+K$ unknowns must in the general case be found by iterations.

A Taylor expansion of the constraint Eq. (5) is performed in the point $(\boldsymbol{\eta}^\nu,\boldsymbol{\xi}^\nu)$. When the terms of second and higher orders are neglected, it can be written as

$$\boldsymbol{f}^\nu+F_\eta^\nu(\boldsymbol{\eta}^{\nu+1}-\boldsymbol{\eta}^\nu)+F_\xi^\nu(\boldsymbol{\xi}^{\nu+1}-\boldsymbol{\xi}^\nu)=0, \tag{7}$$

where all superscripts ν indicate that f^ν, F_η^ν, F_ξ^ν are to be evaluated at the point $(\boldsymbol{\eta}^\nu,\boldsymbol{\xi}^\nu)$. Eqs. (3) and (4) now read

$$V^{-1}(\boldsymbol{\eta}^{\nu+1}-\boldsymbol{Y})+(F_\eta^{\mathrm{T}})^\nu\boldsymbol{\lambda}^{\nu+1}=\boldsymbol{0}, \tag{8}$$

$$(F_\xi^{\mathrm{T}})^\nu\boldsymbol{\lambda}^{\nu+1}=\boldsymbol{0}. \tag{9}$$

With Eqs. (7), (8) and (9), all unknowns of the (ν+1)-th iteration can be deduced by the quantities of the preceding iteration.

When the notations are introduced below

$$\boldsymbol{r}\equiv\boldsymbol{f}^\nu+F_\eta^\nu(\boldsymbol{Y}-\boldsymbol{\eta}^\nu), \tag{10}$$

$$S\equiv F_\eta^\nu V(F_\eta^{\mathrm{T}})^\nu, \tag{11}$$

S is a symmetric matrix of dimension $K\times K$, in succession

$$\boldsymbol{\xi}^{\nu+1}=\boldsymbol{\xi}^\nu-(F_\xi^{\mathrm{T}}S^{-1}F_\xi)^{-1}F_\xi^{\mathrm{T}}S^{-1}\boldsymbol{r}, \tag{12}$$

$$\boldsymbol{\lambda}^{\nu+1}=S^{-1}\left[\boldsymbol{r}+F_\xi(\boldsymbol{\xi}^{\nu+1}-\boldsymbol{\xi}^\nu)\right], \tag{13}$$

$$\boldsymbol{\eta}^{\nu+1}=\boldsymbol{Y}-VF_\eta^{\mathrm{T}}\boldsymbol{\lambda}^{\nu+1}. \tag{14}$$

With the new values for $\boldsymbol{\xi}^{\nu+1}$, $\boldsymbol{\lambda}^{\nu+1}$ and $\boldsymbol{\eta}^{\nu+1}$, the value of function $Q^2(\nu+1)$ for the (ν+1)-th iteration is calculated and compared to the previous value $Q^2(\nu)$. The iteration should be continued until a satisfactory solution has been found[4, 5].

3 Application

Now let us apply the previous formulation to the kinematic analysis of the following decay channel

$$\psi(2S)\rightarrow\gamma_1\gamma_2\gamma_3.$$

There are 9 observable parameters

$$\boldsymbol{\eta}=\{\phi_1,\tan\Lambda_1,\sqrt{E_1},\phi_2,\tan\Lambda_2,\sqrt{E_2},\phi_3,\tan\Lambda_3,\sqrt{E_3}\},$$

where ϕ_i is the azimuthal angle, $\tan\Lambda_i=\cot\theta_i$ with θ_i being the polar angle, and E_i the deposited energy of the photon i. They have corresponding measurements $\boldsymbol{Y}$ and errors contained in the covariance matrix $V(\boldsymbol{Y})$. The decay vertex of $\psi(2S)$ from where the photons come is unmeasurable for all neutral events at BESⅡ. But fortunately the x, y coordinates of the decay vertex are very small and are known very precisely, thus the x, y coordinates are corrected with their mean values in order to simplify the problem. One unmeasurable variable is the z coordinate of the decay vertex of $\psi(2S)$, i.e., $\boldsymbol{\xi}=\{z_{vtx}\}$.

According to the momentum and energy conservation, four constraint equations are satisfied in the kinematic fit, they are

$$\begin{aligned} f_1 &= p_{1x}+p_{2x}+p_{3x}=0, \\ f_2 &= p_{1y}+p_{2y}+p_{3y}=0, \\ f_3 &= p_{1z}+p_{2z}+p_{3z}=0, \\ f_4 &= E_1+E_2+E_3-E_{\rm cm}=0, \end{aligned}$$

where

$$\begin{aligned} p_{ix} &= E_i\cos\Lambda_i\cos\phi_i, \\ p_{iy} &= E_i\cos\Lambda_i\sin\phi_i, \\ p_{iz} &= E_i\sin\Lambda_i. \end{aligned}$$

The matrices F_η (of dimension 4×9) and F_ξ (4×1) are obtained from the derivatives of four constraint equations $f_k(k=1,2,3,4)$ with respect to the observable parameters $\boldsymbol{\eta}$ and the unmeasurable $\boldsymbol{\xi}$. They are:

$$F_\eta=\begin{pmatrix} -p_{1y} & a_1p_{1x} & b_1p_{1x} & -p_{2y} & a_2p_{2x} & b_2p_{2x} & -p_{3y} & a_3p_{1x} & b_3p_{3x} \\ p_{1x} & a_1p_{1y} & b_1p_{1y} & p_{2x} & a_2p_{2y} & b_2p_{2y} & p_{1x} & a_3p_{1y} & b_3p_{3y} \\ 0 & a_1p_{1z}+p_{1xy} & b_1p_{1z} & 0 & a_2p_{2z}+p_{2xy} & b_2p_{2z} & 0 & a_3p_{3z}+p_{3xy} & b_3p_{3z} \\ 0 & 0 & 2\sqrt{E_1} & 0 & 0 & 2\sqrt{E_2} & 0 & 0 & 2\sqrt{E_3} \end{pmatrix},$$

where

$$a_i=-\tan\Lambda_i/(1.0+\tan^2\Lambda_i),$$

$$b_i=2/\sqrt{E_i},\qquad (i=1,2,3)$$

and

$$F_\xi=\begin{pmatrix} \sum\limits_{i=1}^{3}\dfrac{R_iE_i\cos\phi_i(z_i-z_{vtx})}{\sqrt{[R_i^2+(z_i-z_{vtx})^2]^3}} \\ \sum\limits_{i=1}^{3}\dfrac{R_iE_i\sin\phi_i(z_i-z_{vtx})}{\sqrt{[R_i^2+(z_i-z_{vtx})^2]^3}} \\ \sum\limits_{i=1}^{3}\dfrac{-R_i^2E_i}{\sqrt{[R_i^2+(z_i-z_{vtx})^2]^3}} \\ 0 \end{pmatrix},$$

where R_i is the distance from IP to the first hit layer in ϕ plane, z_i is the z coordinate of the shower at the first hit layer.

To start the iteration the measurements are taken as the initial $\boldsymbol{\eta}^0$,

$$\begin{aligned} \boldsymbol{\eta}^0 = \{&\phi_1^0,\tan\Lambda_1^0,\sqrt{E_1^0},\phi_2^0,\tan\Lambda_2^0, \\ &\sqrt{E_2^0},\phi_3^0,\tan\Lambda_3^0,\sqrt{E_3^0}\}. \end{aligned}$$

For $\boldsymbol{\xi}^0$, the initial value is taken as $\mathbf{0}$, i.e. the origin point. Thus the initial value of the vector $\boldsymbol{r}$ from Eq. (10) is

$$\boldsymbol{r}^0=\boldsymbol{f}^0=(f_1^0,f_2^0,f_3^0,f_4^0).$$

Inserting the approximations $(\boldsymbol{\eta}^0,\boldsymbol{\xi}^0)$, F_η^0 and F_ξ^0 can be found. The 4×4 matrix S is obtained from Eq. (11),

$$S=F_\eta^0V(F_\eta^0)^{\rm T}.$$

Inverting this matrix, the values of $\boldsymbol{\xi}^1,\boldsymbol{\lambda}^1,\boldsymbol{\eta}^1$ are found in succession from Eqs. (12)—(14). Q^2 is calculated with these first estimates, and the process is continued. When the difference of the four momenta between two iterations is small enough (less than 10^{-4} of the center mass energy), the desired minimum is reached. The values of $\boldsymbol{\xi},\boldsymbol{\lambda},\boldsymbol{\eta}$ in the final iteration are the solution of the kinematic fit.

4 Test using Monte Carlo simulated events

The reliability of the iteration procedure was studied with Monte Carlo simulation. For the process $\psi(2S)\to\gamma\pi^0\to\gamma\gamma\gamma$, three phase space Monte Carlo samples, each of 20 000 events, were generated. For these three samples, the x and y coordinates of the vertex are fixed at the origin point, while the z coordinate is fixed to 0, 4 and 10cm, respectively, and their resolutions are set to be zero, i.e., $\sigma_x=\sigma_y=\sigma_z=0$.

Table 1 lists the input and output values of the z coordinates of the vertex and the corresponding resolutions given by using the above iteration procedures to the three Monte Carlo samples. The output values for z coordinate are consistent with the input values within 6% for every sample (Fig. 2). The resolutions got from the three Monte Carlo samples are almost the same (about 2.2cm), which reflect the position resolution of the BSC. These all indicate that the iteration procedure is correct.

Table 1. Monte Carlo input/output test of the z coordinate of the vertex. μ and σ are the mean value and the standard deviation of the distribution (unit: cm).

sample	input		output	
	μ	σ	μ	σ
1	0.0	0.0	0.05 ± 0.03	2.15 ± 0.03
2	4.0	0.0	3.77 ± 0.03	2.16 ± 0.03
3	10.0	0.0	9.45 ± 0.03	2.12 ± 0.03

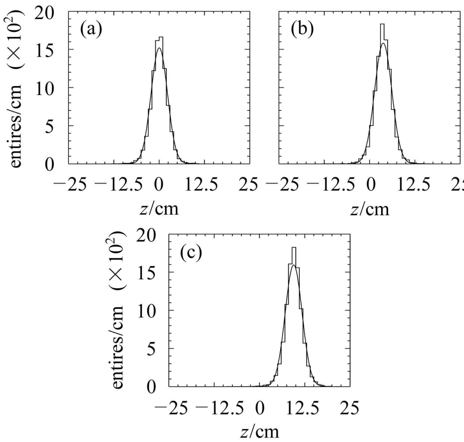

Fig. 2. The z coordinate distributions of the vertex for $\psi(2S) \to \gamma\pi^0 \to \gamma\gamma\gamma$ Monte Carlo samples. From (a) to (c), the input IPs are 0, 4, and 10cm, respectively.

A direct consequence of the improved kinematic fit (with a free parameter z_{vtx} and the energy-momentum conservation, there are 3 constraints, so it is called 3C-fit hereafter) is the increase of the efficiency as compared with the general kinematic fit where only the energy-momentum conservation (4C-fit) is required. The efficiencies of the three samples are almost the same (about 4.3%) with the 3C-fit. For Sample 2, the efficiency with the 3C-fit is 6% bigger than that with the 4C-fit. For Sample 3, only less than 1% events pass the 4C-fit. This indicates that the 4C-fit program is powerless for these events. In fact, more events are selected for J/ψ data at BESⅡ when the 3C-fit is used, which agrees with the Monte Carlo expection. Monte Carlo simulation also shows that the 3C-fit doesn't improve the mass resolution of π^0 signal.

5 Conclusion and discussion

According to the principle of general Least-Squares estimation with constraints, an improved kinematic fit method is developed for analyzing all-photon events at BESⅡ. The fitting program is checked with Monte Carlo sample to ensure that the fitting program works properly. This is applied to the Monte Carlo simulated $\psi(2S)$ decays into $\gamma\gamma\gamma$ final states, a higher efficiency is achieved. This method is useful not only for $\psi(2S)$ decays, but also for J/ψ, $\psi(3770)$ decays, not only for the analyses at BESⅡ, but also for the analyses at other high energy experiments.

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纯中性事例事例顶点 z 向坐标的最小二乘拟合*

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摘要　对于相互作用顶点 z 坐标未知的纯中性事例, 发展了一种运动学拟合的方法. 经蒙特卡罗样本检验, 拟合所用的程序是合理可靠的. 把该方法用于 $\psi' \to \gamma\pi^0 \to \gamma\gamma\gamma$ 蒙特卡罗样本, 得到了更高的选择效率. 该方法可普遍应用于正负电子对撞实验中纯中性事例的分析.

关键词　最小二乘方法　纯中性事例　衰变顶点

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