Transient Beam Loading Effects in Standing Wave Cavities of Linear Accelerators

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Abstract In modern high energy accelerators, with the increase of charge in a bunch or a bunch train, the induced transient beam loading voltages become higher and higher when the beams pass through the standing wave cavities. But in the usual analysis, people usually pay more attention to the steady state instead of the transient state beam loading. In this paper, the transient nature of beam loading and the cavity's frequency changing behavior seen by the RF power generator are studied, and then the optimum detuning conditions in two cases are derived. In the first case, the resonant cavity's frequency can be tuned to meet the in-phase condition between the RF power generator current and the cavity voltage during the passage of beams. While in the second case, only few bunches in the bunch train and the cavity's resonant frequency is fixed during the passage of the bunch train. At last, the beam loading effects in the prebuncher of BEPCⅡ pre-injector and the two SHBs of BEPCⅡ future pre-injector are studied.

Key words transient beam loading, equivalent circuit model, optimum detuning

1 Introduction

In modern high energy accelerators, in order to lower the power dissipation in the cavity walls, one usually chooses very large unloaded and loaded quality factors in designing and operating the accelerating cavities. For example, the unloaded and loaded quality factors of superconducting cavities are usually very large. While a larger loaded quality factor gives a longer cavity filling time, sometimes the filling time might reach the level of milliseconds[1], which makes it possible to tune the cavity's frequency by the cavity's tuner timely to meet the in-phase condition between the RF power generator current and the cavity voltage during the passage of charged particle beams. Then, more good bunches with better performance can be obtained and the RF power system's efficiency can also be improved.

On the other hand, when there are only few bunches in a bunch train, if the beam loading cannot reach steady state and the cavity's resonant frequency cannot be tuned during the passage of the bunch train, in order to improve the RF power system's efficiency and make the phase difference between the generator current and the cavity voltage seen by every bunch as small as possible, the optimum resonant frequency of the cavity can be found and set beforehand in the usual operation.

In the above-mentioned two cases, studies on the transient nature of the beam loading voltage induced by a bunch train and the beam-cavity interaction become important. What we should keep in mind in this paper is that the RF power generator is assumed to be a matched generator, which means there is a circulator or isolator just between the RF power generator and the accelerating cavity, so any power which is reflected from the cavity and travels backward to the RF power generator will be absorbed. In addition,

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due to the frequent use of phasor diagram in the analysis of beam loading, all of the voltages and currents are expressed as phasors, and we indicate a phasor by a tilde.

2 Transient beam loading voltage

2.1 Single bunch induced beam loading voltage

If we define the beam induced cavity voltage to be the voltage seen by another point charge, then according to the equivalent parallel RLC resonator circuit model shown in Fig. 1 and the fundamental theorem of beam loading, the net cavity voltage induced by a point charge $\mathrm{d}q$ can be represented as[2—4]

$$\tilde{V}_{\mathrm{d}q}(t)=\begin{cases}0 & t<0\\ -\alpha R_{\mathrm{L}}\mathrm{d}q/2 & t=0\\ -\alpha R_{\mathrm{L}}\mathrm{d}q\mathrm{e}^{\mathrm{i}(\exp(\mathrm{i}\theta)\omega_{\mathrm{r}}t+\theta)}/\cos\theta & t>0\end{cases} \qquad (1)$$

with $\alpha=\omega_{\mathrm{r}}/2Q_{\mathrm{L}}$, $\cos\theta=(\omega_{\mathrm{r}}^2-\alpha^2)^{1/2}/\omega_{\mathrm{r}}$, $\sin\theta=\alpha/\omega_{\mathrm{r}}$. $Q_{\mathrm{L}}=Q_0/(1+\beta_{\mathrm{c}})$ is the loaded quality factor, where Q_0 is the unloaded quality factor, β_{c} is the coupling factor; ω_{r} is the natural resonant frequency of the cavity; $R_{\mathrm{L}}=2R_{\mathrm{c}}/(1+\beta_{\mathrm{c}})$ is the effective loaded shunt impedance defined by $R_{\mathrm{c}}=V_{\mathrm{c}}^2/2P_{\mathrm{c}}$, where P_{c} is the average power dissipation in the cavity walls; t is the time distance between the point charge inducing the cavity voltage and another point charge seeing the induced cavity voltage.

In Fig. 1, i_{b} is the harmonic component of the beam current and can be got by Fourier transforming the current distribution in the bunch or bunch train; I_{g} is the RF power generator current; V_{c} is the net cavity voltage defined by $V_{\mathrm{c}}=V_0T$, where T is the transit-time factor and V_0 is the net cavity voltage without considering the transit-time factor; $R_{\mathrm{c}}/\beta_{\mathrm{c}}$ is the impedance seen by the reflected wave from the cavity's coupler or a wave emitted from the cavity.

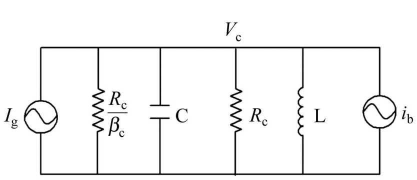

Fig. 1. Equivalent parallel circuit model representing a cavity connected to a matched generator.

The induced voltage by a charged particle bunch can be represented as a sum over the voltages induced by all of the individual particles ($\mathrm{d}q$)

$$\tilde{V}(t)=\sum_{n=1}^{N}\mathrm{d}q\tilde{V}_{\mathrm{d}q}(t-t_{\mathrm{qn}})=-\int_{-\infty}^{t}\frac{I(t_{\mathrm{q}})\alpha R_{\mathrm{L}}}{\cos\theta}\times\left(\mathrm{e}^{\mathrm{i}(\mathrm{e}^{\mathrm{i}\theta}\omega_{\mathrm{r}}(t-t_{\mathrm{q}})+\theta)}\right)\mathrm{d}t_{\mathrm{q}}\ , \qquad (2)$$

where N is the number of particles which have passed through the cavity at time t, t_{qn} the time when the nth particle passed through the cavity, $I(t_{\mathrm{q}})$ the current through the cavity as a function of time.

For a Gaussian bunch with charge q and a characteristic length σ_{z}, one can write $I(t_{\mathrm{q}})$ as[2]

$$I(t_{\mathrm{q}})=\frac{q\beta c}{\sqrt{2\pi}\sigma_{\mathrm{z}}}\exp\left(-\frac{t_{\mathrm{q}}^2\beta^2c^2}{2\sigma_{\mathrm{z}}^2}\right)\ , \qquad (3)$$

where β is the normalized particle velocity, and c is the velocity of light. Substituting Eq. (3) into Eq. (1), one can get the voltage induced by a Gaussian bunch[3]

$$\tilde{V}_{\mathrm{gaussian}}(t)=-\frac{q\alpha R_{\mathrm{L}}}{\cos\theta}\mathrm{e}^{\mathrm{i}\theta-t^2\beta^2c^2/(2\sigma_{\mathrm{z}}^2)}\times w\left(\frac{\sigma_{\mathrm{z}}\omega_{\mathrm{r}}\mathrm{e}^{\mathrm{i}\theta}}{\sqrt{2}\beta c}-\frac{\mathrm{i}t\beta c}{\sqrt{2}\sigma_{\mathrm{z}}}\right), \qquad (4)$$

where w is the complex error function with the following definition

$$w(z)=\mathrm{e}^{-z^2}\mathrm{erf}c(-\mathrm{i}z)=\mathrm{e}^{-z^2}(1+\frac{2i}{\sqrt{\pi}}\int_0^z\mathrm{e}^{\tau^2}\mathrm{d}\tau)\ . \qquad (5)$$

In modern accelerators, the loaded quality factor Q_{L} of SW (Standing Wave) cavities is often much larger than 1, so a simplified expression for Eq. (1) can be obtained

$$\tilde{V}_{\mathrm{d}q}(t)=\begin{cases}0 & t<0\\ -\alpha R_{\mathrm{L}}\mathrm{d}q/2=-k\mathrm{d}q & t=0\\ -\alpha R_{\mathrm{L}}\mathrm{d}q\mathrm{e}^{\mathrm{i}\omega_{\mathrm{r}}t}\mathrm{e}^{-\alpha t}=2k\mathrm{d}q\mathrm{e}^{\mathrm{i}\omega_{\mathrm{r}}t}\mathrm{e}^{-\alpha t} & t>0\end{cases}\ , \qquad (6)$$

where k is the loss factor of the resonant mode.

Besides the large Q_{L}, if the bunch length is also very short and the bunch itself can be considered to be a point charge, then by simplifying Eq. (4) the beam induced voltage of a Gaussian bunch with bunch charge q can be given by

$$\tilde{V}_{\mathrm{gaussian}}(t)=\begin{cases}0 & t<0\\ -\alpha R_{\mathrm{L}}q\mathrm{e}^{-\omega_{\mathrm{r}}^2\sigma_{\mathrm{z}}^2/(2\beta^2c^2)}/2 & t=0\\ -\alpha R_{\mathrm{L}}q\mathrm{e}^{\mathrm{i}\omega_{\mathrm{r}}t}\mathrm{e}^{-\omega_{\mathrm{r}}^2\sigma_{\mathrm{z}}^2/(2\beta^2c^2)}\mathrm{e}^{-\alpha t} & t>0\end{cases}\ . \qquad (7)$$

2.2 Bunch train induced beam loading voltage

The transient beam loading voltage induced by a bunch train can be obtained by adding up all of the beam loading voltages induced by the bunches passed,

$$\tilde{V}_{\rm b}(t) = -\sum_{n}^{N}\frac{Q(n)\alpha_{\rm n}R_{\rm L}}{\cos\theta_{\rm n}}{\rm e}^{{\rm i}\theta_{\rm n}-(t-t_{\rm bn})^2\beta^2c^2/(2\sigma_{\rm z}^2)}\times w\left(\frac{\sigma_{\rm z}\omega_{\rm rn}{\rm e}^{{\rm i}\theta_{\rm n}}}{\sqrt{2}\beta c}-\frac{{\rm i}(t-t_{\rm bn})\beta c}{\sqrt{2}\sigma_{\rm z}}\right), \tag{8}$$

with $\alpha_{\rm n}=\omega_{\rm rn}/2Q_{\rm L}$, $\cos\theta_{\rm n}=(\omega_{\rm rn}^2-\alpha_{\rm n}^2)^{1/2}/\omega_{\rm rn}$, $\sin\theta_{\rm n}=\alpha_{\rm n}/\omega_{\rm rn}$; where $\omega_{\rm rn}$ and $t_{\rm bn}$ are the cavity's natural resonant frequency and the time when the nth bunch passes through the cavity, N is the number of bunches which have passed through the cavity at time t, and $Q(n)$ is the charge contained in the nth bunch.

When $Q_{\rm L}$ is much larger than 1 and the bunch length is very short, the simplified version of Eq. (8) can be represented as

$$\tilde{V}_b(t)=\begin{cases}-\sum\limits_{n}^{N}Q(n)\alpha_{\rm n}R_{\rm L}{\rm e}^{-\omega_{\rm rn}^2\sigma_z^2/(2\beta^2c^2)}{\rm e}^{{\rm i}\omega_{\rm rn}(t-t_{\rm bn})}\times {\rm e}^{-\alpha_{\rm n}(t-t_{\rm bn})} & t>t_{\rm bN}\\ -\frac{1}{2}Q(n)\alpha_{\rm N}R_{\rm L}{\rm e}^{-\omega_{\rm rN}^2\sigma_z^2/(2\beta^2c^2)}-\sum\limits_{n}^{N-1}Q(n)\alpha_{\rm n}R_{\rm L}{\rm e}^{-\omega_{\rm rn}^2\sigma_z^2/(2\beta^2c^2)}{\rm e}^{{\rm i}\omega_{\rm rn}(t-t_{\rm bn})}\times {\rm e}^{-\alpha_{\rm n}(t-t_{\rm bn})} & t=t_{\rm bN}\\ -\sum\limits_{n}^{N-1}Q(n)\alpha_{\rm n}R_{\rm L}{\rm e}^{-\omega_{\rm rn}^2\sigma_z^2/(2\beta^2c^2)}{\rm e}^{{\rm i}\omega_{\rm rn}(t-t_{\rm bn})}\times {\rm e}^{-\alpha_{\rm n}(t-t_{\rm bn})} & t<t_{\rm bN}\end{cases}. \tag{9}$$

3 Cavity's frequency changing behaviour seen by the RF generator

We should clarify two things before going to the following contents:

a) During the passage of bunches, the beam loading can only change the cavity's frequency seen by the RF power generator instead of the cavity's natural resonant frequency, which is given by $\omega_{\rm r}=1/\sqrt{LC}$ in the equivalent parallel RLC model.

b) In the usual operations[3—6], in order to meet the in-phase condition between the RF power generator current and the cavity voltage seen by each bunch, the cavity's natural resonant frequency is usually tuned to another natural resonant frequency, which is often called beam loading induced frequency detuning. But in this section, the detuning caused by beam loading means the cavity's frequency changing seen by the RF power generator, which is different from those meanings in many articles and books[2—6].

3.1 Detuning caused by beam loading with $\boldsymbol{\omega_{\rm r}=\omega_{\rm g}}$

When the cavity's natural resonant frequency $\omega_{\rm r}$ is fixed and equal to the RF power generator's frequency $\omega_{\rm g}$ during the passage of particle beams, the cavity impedance loaded by the external resistance in the equivalent parallel circuit of Fig. 1 is given by

$$\tilde{Z}_L=R_{\rm c}/(1+\beta_{\rm c}) . \tag{10}$$

Then the generator current and the generator induced voltage can be given by

$$\tilde{i}_{\rm g}=i_{\rm g}{\rm e}^{{\rm i}(\omega_{\rm g}t+\theta)}=i_{\rm g}{\rm e}^{{\rm i}(\omega_{\rm r}t+\theta)} , \tag{11}$$

$$\tilde{V}_{\rm g} = \tilde{i}_{\rm g}\tilde{Z}_{\rm L}=i_{\rm g}R_{\rm c}{\rm e}^{{\rm i}(\omega_{\rm g}t+\theta)}/(1+\beta_{\rm c})= i_{\rm g}R_{\rm c}{\rm e}^{{\rm i}(\omega_{\rm r}t+\theta)}/(1+\beta_{\rm c}), \tag{12}$$

where θ is the phase of the generator current relative to the phase of the beam current $i_{\rm b}$, which is usually equal to the design phase of synchronous particles. Using Eq. (9), one can get the transient bunch train induced beam loading voltage $\tilde{V}_{\rm b}$ by setting the natural resonant frequency seen by every bunch equal to frequency $\omega_{\rm r}$. Then the equivalent beam image current $\tilde{i}_{\rm b}$ corresponding to $\tilde{V}_{\rm b}$ is

$$\tilde{i}_{\rm b}=-i_{\rm b}=\tilde{V}_{\rm b}/\tilde{Z}_{\rm L}=(1+\beta_{\rm c})\tilde{V}_{\rm b}/R_{\rm c}. \tag{13}$$

Since $\tilde{V}_{\rm c}=\tilde{V}_{\rm b}+\tilde{V}_{\rm g}$, the total cavity impedance seen by the generator can be written as

$$\tilde{Z}_{\rm total} = \frac{\tilde{V}_{\rm c}}{\tilde{i}_{\rm g}}=\frac{\tilde{V}_{\rm b}+\tilde{V}_{\rm g}}{\tilde{i}_{\rm g}}= \left(1+\frac{\tilde{V}_{\rm b}}{\tilde{V}_{\rm g}}\right)\tilde{Z}_{\rm L}=\left(1+\frac{\tilde{V}_{\rm b}}{\tilde{V}_{\rm g}}\right)\frac{R_{\rm c}}{1+\beta_{\rm c}}. \tag{14}$$

Then the angular frequency $\omega_{\rm gs}$ seen by the RF power generator can be obtained by the following relation,

$$\frac{{\rm Im}(\tilde{Z}_{\rm total})}{{\rm Re}(\tilde{Z}_{\rm total})}=\tan\left(-Q_{\rm L}\left(\frac{\omega_{\rm g}}{\omega_{\rm gs}}-\frac{\omega_{\rm gs}}{\omega_{\rm g}}\right)\right), \tag{15}$$

where ${\rm Im}(\tilde{x})$ is the imaginary part of $\tilde{x}$, and ${\rm Re}(\tilde{x})$

the real part of $\tilde{x}$. When the deviation of $\omega_{\rm gs}$ from $\omega_{\rm g}$ is small, Eq. (15) can be approximated as

$$\frac{{\rm Im}(\tilde{Z}_{\rm total})}{{\rm Re}(\tilde{Z}_{\rm total})}=\tan(-2Q_{\rm L}\frac{\omega_{\rm g}-\omega_{\rm gs}}{\omega_{\rm g}}). \tag{16}$$

From Eqs. (9), (12), (14) and (16), we can find that if $\theta<0$, i.e. the bunch is injected earlier than the crest of the accelerating wave, $\omega_{\rm gs}<\omega_{\rm g}$, while if $\theta>0$, i.e. the bunch is injected later than the crest of the accelerating wave, $\omega_{\rm gs}>\omega_{\rm g}$. On the other hand, if the bunch train consists of many bunches, then with the injection of bunch train, not only the accelerating phases and the cavity voltages seen by every bunch of the bunch train are changed, which will deteriorate the beam qualities, but also the RF power generator's efficiency is decreased, so this kind of case should be mitigated by tuning the cavity's natural resonant frequency $\omega_{\rm r}$ to a different frequency from $\omega_{\rm g}$.

3.2 Detuning caused by beam loading with $\boldsymbol{\omega_{\rm r}\neq\omega_{\rm g}}$

If the cavity's natural resonant frequency $\omega_{\rm r}$ is fixed and different from the RF power generator's frequency $\omega_{\rm g}$ during the passage of the bunch train, the equations in Section 3.1 should be modified.

The cavity impedance loaded by the external resistance is

$$\tilde{Z}_{\rm L}=\frac{R_{\rm c}}{(1+\beta_{\rm c})\left(1+{\rm i}Q_{\rm L}\left(\frac{\omega_{\rm g}}{\omega_{\rm r}}-\frac{\omega_{\rm r}}{\omega_{\rm g}}\right)\right)}\approx \frac{R_{\rm c}}{(1+\beta_{\rm c})(1+2{\rm i}Q_{\rm L}\delta\omega/\omega_{\rm r})}=\frac{R_{\rm c}}{1+\beta_{\rm c}}\cos\psi{\rm e}^{{\rm i}\psi}, \tag{17}$$

with $\delta\omega=\omega_{\rm g}-\omega_{\rm r}$; where ψ is the usually defined detuning angle, which is given by

$$\tan\psi=-2Q_{\rm L}\left(\frac{\omega_{\rm g}}{\omega_{\rm r}}-\frac{\omega_{\rm r}}{\omega_{\rm g}}\right)=-2Q_{\rm L}\frac{\delta\omega}{\omega_{\rm r}}. \tag{18}$$

The generator current, the generator-induced voltage, the beam image current, and the total impedance seen by the generator can be written as

$$\tilde{i}_{\rm g}=i_{\rm g}{\rm e}^{{\rm i}(\omega_{\rm g}t+\theta)}, \tag{19}$$

$$\tilde{V}_{\rm g}=\tilde{i}_{\rm g}\tilde{Z}_{\rm L}=\frac{i_{\rm g}R_{\rm c}}{1+\beta_{\rm c}}\cos\psi{\rm e}^{{\rm i}\omega_{\rm g}t}{\rm e}^{{\rm i}(\psi+\theta)}, \tag{20}$$

$$\tilde{i}_{\rm b}=-i_{\rm b}=\frac{\tilde{V}_{\rm b}}{\tilde{Z}_{\rm L}}=\frac{(1+\beta_{\rm c})\tilde{V}_{\rm b}}{R_{\rm c}\cos\psi}{\rm e}^{-{\rm i}\psi}, \tag{21}$$

$$\tilde{Z}_{\rm total}=\frac{\tilde{V}_{\rm c}}{\tilde{i}_{\rm g}}=\frac{\tilde{V}_{\rm b}+\tilde{V}_{\rm g}}{\tilde{i}_{\rm g}}=\left(1+\frac{\tilde{V}_{\rm b}}{\tilde{V}_{\rm g}}\right)\tilde{Z}_{\rm L}=\left(1+\frac{\tilde{V}_{\rm b}}{\tilde{V}_{\rm g}}\right)\frac{R_{\rm c}}{1+\beta_{\rm c}}\cos\psi{\rm e}^{{\rm i}\psi}. \tag{22}$$

Since the RF power provided by the generator is $P_{\rm g}=\left|\tilde{i}_{\rm g}\right|^2R_{\rm c}/8\beta$, we can obtain

$$\tilde{Z}_{\rm total}=\frac{R_{\rm c}}{1+\beta_{\rm c}}\cos\psi{\rm e}^{{\rm i}\psi}+\sqrt{\frac{R_{\rm c}}{8\beta P_{\rm g}}}\tilde{V}_{\rm b}{\rm e}^{-{\rm i}(\omega_{\rm g}t+\theta)}. \tag{23}$$

Using the same method as Section 3.1, we can get the beam induced cavity voltage $\tilde{V}_{\rm b}$. By inserting $\tilde{V}_{\rm b}$ into Eq. (23) and substituting the obtained total impedance $\tilde{Z}_{\rm total}$ into Eqs. (15) or (16), one can obtain the cavity's resonant frequency seen by the generator.

If the bunch train is long enough to let the beam loading reach a steady state, and the cavity's optimum detuning frequency for the steady state beam loading compensation is $\omega_{\rm optimum}$, we can get the following conclusions on the behavior of the cavity's frequency seen by the generator during the passage of the bunch train by analyzing Eqs. (9), (15), (16) and (23),

a) $\theta<0$ and $\omega_{\rm r}<\omega_{\rm g}$. $\omega_{\rm gs}$ will always be lower than $\omega_{\rm g}$.

b) $\theta<0$ and $\omega_{\rm optimum}>\omega_{\rm r}>\omega_{\rm g}$. At the start, $\omega_{\rm gs}$ is higher than the generator's frequency $\omega_{\rm g}$, but with the injection of the bunch train, $\omega_{\rm gs}$ will equal to $\omega_{\rm g}$ at times, after this, $\omega_{\rm gs}$ will be lower than $\omega_{\rm g}$.

c) $\theta<0$ and $\omega_{\rm r}>\omega_{\rm optimum}>\omega_{\rm g}$. $\omega_{\rm gs}$ will always be higher than $\omega_{\rm g}$.

d) $\theta>0$ and $\omega_{\rm r}>\omega_{\rm g}$. $\omega_{\rm gs}$ will always be higher than $\omega_{\rm g}$.

e) $\theta>0$ and $\omega_{\rm optimum}<\omega_{\rm r}<\omega_{\rm g}$. At the start, $\omega_{\rm gs}$ is lower than the generator's frequency $\omega_{\rm g}$, but with the injection of the bunch train, $\omega_{\rm gs}$ will equal to $\omega_{\rm g}$ at times, after this, $\omega_{\rm gs}$ will be higher than $\omega_{\rm g}$.

f) $\theta>0$ and $\omega_{\rm r}<\omega_{\rm optimum}<\omega_{\rm g}$. $\omega_{\rm gs}$ will be always lower than $\omega_{\rm g}$.

4 Optimum detuning

In the usual operations[3—6], if we want to minimize the cavity voltages' differences seen by the bunches in the bunch train and improve the RF power system's efficiency by mitigating the beam loading effects, the usually adopted way is to tune the natural resonant frequency of the cavity.

4.1 Optimum detuning with varying ω_r

If we know the frequency tuning speed v and assume the tuning is linear, the cavity's natural resonant frequency ω_{rn} seen by the nth bunch can be represented as,

$$\omega_{rn} = \omega_{r1} + (t_{bn} - t_{b1})v, \tag{24}$$

where ω_{r1} is the frequency seen by the 1st bunch, which is an unknown value and needs to be found and optimized, and t_{b1} the time when the 1st bunch passes through the cavity. Then the beam loading voltage seen by the Nth bunch is,

$$\tilde{V}_{bN} = -\frac{1}{2}Q(N)\alpha_N R_L e^{-(\omega_{r1}+(t_{bN}-t_{b1})v)^2\sigma_z^2/(2\beta^2c^2)} - \sum_n^{N-1} Q(n)\alpha_n R_L e^{\frac{(\omega_{r1}+(t_{bn}-t_{b1})v)^2\sigma_z^2}{2\beta^2c^2}} \times e^{i(\omega_{r1}+(t_{bn}-t_{b1})v)(t_{bN}-t_{bn})} e^{-\alpha_n(t_{bN}-t_{bn})}. \tag{25}$$

Using Eqs. (17) and (20), we can obtain the generator induced voltage seen by the Nth bunch,

$$\tilde{V}_{gN} = \tilde{i}_g \tilde{Z}_{LN} = \frac{i_g R_c}{1+\beta_c}\cos\psi_N e^{i\omega_g t_{bN}} e^{i(\psi_N+\theta)} = \frac{\sqrt{8\beta R_c P_g}}{1+\beta_c}\cos\psi_N e^{i\omega_g t_{bN}} e^{i(\psi_N+\theta)}, \tag{26}$$

where Z_{LN} and ψ_N can be obtained by the following equations,

$$\tilde{Z}_{LN} = \frac{R_c}{(1+\beta_c)\left(1+iQ_L\left(\frac{\omega_g}{\omega_{r1}+(t_{bN}-t_{b1})v} - \frac{\omega_{r1}+(t_{bN}-t_{b1})v}{\omega_g}\right)\right)} \approx \frac{R_c}{(1+\beta_c)(1+2iQ_L\delta\omega_N)/(\omega_{r1}+(t_{bN}-t_{b1})v)} = \frac{R_c}{1+\beta_c}\cos\psi_N e^{i\psi_N}, \tag{27}$$

with $\delta\omega_N = \omega_g - (\omega_{r1} - (t_{bN} - t_{b1})v)$.

$$\tan\psi_N = -2Q_L \frac{\delta\omega_N}{\omega_{r1} + (t_{bN} - t_{b1})v}. \tag{28}$$

Then we can know the accelerating phase seen by the Nth bunch,

$$\tan(\phi_{sN}) = \frac{\mathrm{Im}(\tilde{V}_{bN} + \tilde{V}_{gN})}{\mathrm{Re}(\tilde{V}_{bN} + \tilde{V}_{gN})}. \tag{29}$$

After we obtain all of the accelerating phases seen by all of the bunches, we can obtain the optimized starting frequency ω_{r1} by the following equations,

$$\frac{d}{d\omega_{r1}} \sum_{N=1}^{M} Q(N)(\phi_{sN} - \phi_{sdesign})^2 = 0, \tag{30}$$

or

$$\frac{d}{d\omega_{r1}} \sum_{N=1}^{M} Q(N)(\omega_{gsN} - \omega_g)^2 = 0, \tag{31}$$

where M is the number of bunches in the bunch train, $\phi_{sdesign}$ the designed accelerating phase, and ω_{gsN} the cavity's resonant frequency seen by the generator when the Nth bunch passes through the cavity.

4.2 Optimum detuning with fixed ω_r

If there are only several bunches in the bunch train and the cavity's detuned resonant frequency is ω_r, then the beam loading voltage and the generator induced voltage seen by the Nth bunch can be given by

$$\tilde{V}_{gN} = \tilde{i}_g \tilde{Z}_L = \frac{\sqrt{8\beta_c R_c P_g}}{1+\beta_c}\cos\psi e^{i\omega_g(t_{bN}-t_{b1})} e^{i(\psi+\theta)}, \tag{32}$$

$$\tilde{V}_{bN} = -\frac{1}{2}Q(N)\alpha R_L e^{-\omega_r^2\sigma_z^2/(2\beta^2c^2)} - \sum_n^{N-1} Q(n)\alpha R_L e^{-\frac{\omega_r^2\sigma_z^2}{2\beta^2c^2}} e^{i\omega_r(t_{bN}-t_{bn})} e^{-\alpha_n(t_{bN}-t_{bn})}. \tag{33}$$

The effective accelerating phase seen by the Nth bunch can be obtained by Eq. (29), then the optimized detuning frequency ω_r can be found by replacing ω_{r1} with ω_r in Eq. (30) or Eq. (31) and solving the corresponding equation.

5 Beam loading effects in BEPCⅡ

According to the classical mechanism of the beam loading, usually if there is no modulation of the injected beam current at the cavity's frequency, there is no way for a voltage to be induced, so there should be no beam loading in the first bunching cavity of linear accelerators if the injected beam is a DC (Direct Current) beam. However, it has already been observed that a DC beam can induce a detuning caused by beam loading in the CTF3 accelerator, which might be due to the fact that there is some visible beam modulation in the downstream part of the bunching cavity[5]. In this section, we will use the methods described in this paper to estimate the beam loading detuning effects in the bunching cavities of the BEPCⅡ present pre-injector and the BEPCⅡ future pre-injector.

5.1 Beam loading effects in BEPCⅡ prebuncher

For the BEPCⅡ present pre-injector, there are five bunches in the bunch train, the bunch charges from the 1st bunch to the 5th bunch are 1.0nC, 4.9nC, 5.8nC, 3.8nC and 0.35nC, respectively. The bunch interval and the bunch train interval are 350ps and 56.02ns, respectively. Before the beam enters the prebuncher, it is a DC beam. At the exit of the prebuncher, the bunch's characteristic length σ_z and the normalized velocity β are about 5.7mm and 0.63, respectively. The typical parameters of the prebuncher are[7, 8]: $f = 2856$MHz, $Q_0 = 1000$, $\beta_c = 1.04$, $r_s = 2R_c = 80$kΩ, $P_g = 20$kW. As the prebuncher is used to realize energy modulation, the value of θ can be known to be $-90°$, which is also the value of the designed accelerating phase.

a) Case of one bunch train

Using the above-mentioned parameters of the prebuncher, we can get some un-optimized and optimized results shown in Table 1 and Table 2 for the case of only one bunch train. For the un-optimized results, the cavity's natural resonant frequency is 2856MHz, while for the optimized results, it is 2856.36MHz.

Table 1. Un-optimized results for the beam loading effects in BEPCⅡ prebuncher (one bunch train).

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	1.0	2855.98	40.001	−90.45	−0.31
2nd	4.9	2855.84	40.057	−93.06	−2.14
3rd	5.8	2855.61	40.368	−97.74	−5.44
4th	3.8	2855.40	40.867	−101.8	−8.37
5th	0.35	2855.31	41.136	−103.5	−9.60

In Table 1 and Table 2, f_{gs} is the cavity's resonant frequency seen by the generator when each bunch passes through the cavity; V_c and ϕ_s are the amplitude of the cavity voltage and the synchronous phase seen by each bunch, respectively.

Table 2. Optimized results for the beam loading effects in BEPCⅡ prebuncher (one bunch train).

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	1.0	2856.34	39.654	−83.40	+4.56
2nd	4.9	2856.20	39.486	−86.04	+2.73
3rd	5.8	2855.96	39.398	−90.83	−0.57
4th	3.8	2855.74	39.555	−95.08	−0.35
5th	0.35	2855.65	39.690	−96.85	−4.73

Comparing Table 1 and Table 2, we can see that in the optimized case not only the accelerating phase and the effective cavity voltage seen by each bunch are closer to the design values of 0° and 0kV, but also the differences between the resonant frequencies seen by the generator and the cavity's natural resonant frequency are smaller than those of the un-optimized case. Actually, only when the beam moves to the downstream part of the cavity, can the modulation on the beams become visible, but in our calculation the bunch is assumed to be a modulated bunch before coming into the cavity, so the detuning effect might be overestimated. In this case, we can think the real detuning is one third of the calculated detuning, so the optimized cavity's natural resonant frequency might be 2856.12MHz. Correspondingly, the accelerating phase and the effective cavity voltage will be more close to the design values. On the other hand, as the bandwidth of the prebuncher is about 5.47MHz, the detuning effect usually cannot be seen and felt in the usual operation.

b) Case of two bunch trains

Since BEPCⅡ will adopt a two-bunch acceleration scheme to double the positron injection rate into the storage ring in the future[9], we also studied the detuning effect in the prebuncher for the case of two bunch trains.

For the un-optimized results, the cavity's natural resonant frequency is fixed at 2856MHz, while for the optimized results, it should be fixed at 2856.49MHz. But due to the overestimation of the detuning effect, we can think the real detuning is one third of the calculated detuning, so the optimized cavity's natural resonant frequency might be 2856.16MHz.

Comparing Table 3 and Table 4, besides the same conclusions we got for the case of one bunch train, we can get another two important conclusions:

Table 3. Un-optimized results for the beam loading effects in BEPCⅡ prebuncher (two bunch trains).

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1-1st	1.0	2855.98	40.001	−90.45	−0.31
1-2nd	4.9	2855.84	40.057	−93.06	−2.14
1-3rd	5.8	2855.61	40.368	−97.74	−5.44
1-4th	3.8	2855.40	40.867	−101.8	−8.37
1-5th	0.35	2855.31	41.136	−103.5	−9.60
2-1st	1.0	2855.72	40.188	−95.54	−3.88
2-2nd	4.9	2855.59	40.402	−98.09	−5.69
2-3rd	5.8	2855.36	40.992	−102.6	−8.96
2-4th	3.8	2855.16	41.726	−106.5	−11.9
2-5th	0.35	2855.08	42.085	−108.1	−13.1

Table 4. Optimized results for the beam loading effects in BEPCⅡ prebuncher (two bunch trains).

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	1.0	2856.46	39.393	−80.94	+6.21
2nd	4.9	2856.33	39.147	−83.58	+4.38
3rd	5.8	2856.08	38.921	−88.41	+10.8
4th	3.8	2855.86	38.958	−92.73	−1.86
5th	0.35	2855.77	39.045	−94.53	−3.08
1st	1.0	2856.20	39.609	−86.11	+0.97
2nd	4.9	2856.06	39.607	−88.72	−0.83
3rd	5.8	2855.82	39.816	−93.44	−4.11
4th	3.8	2855.62	40.229	−97.57	−7.02
5th	0.35	2855.53	40.467	−99.25	−8.22

a) The effective accelerating voltage seen by the second bunch train has a large difference from that seen by the first bunch train for both the un-optimized and optimized cases; this will not only result in an energy difference, but also lead to different bunching efficiency.

b) When the first bunch of the second bunch train traverses the prebuncher, the frequency seen by the generator is higher than the frequency seen by the generator when the last bunch of the first bunch train traverses the prebuncher, which is because the bunch train interval can be comparable with the prebuncher's filling time of $2Q_L/\omega_r = 54.7$ns. When the second bunch comes into the prebuncher, the beam loading voltage induced by the first bunch has decreased a lot and can be neglected.

5.2 Beam loading effects in BEPCⅡ future Two SHBs

In the future, in order to produce a bunch train consisting of two single bunches with an interval 56.02ns, two SHBs will be used in the BEPCⅡ pre-injector. In this section, we will consider the beam loading effects in the two SHBs.

a) Beam loading effects in SHB1

According to the beam dynamics simulation[9], the bunch's characteristic length σ_z is about 0.5ns at the entrance of SHB1, and 0.4ns at the exit, so the bunch's average characteristic length $\sigma_z = 0.45$ns is used in our calculation. Each bunch of the bunch train has a bunch charge of 15nC. The typical parameters of the SHB1 are[10]: $f = 142.8$MHz, $Q_0 = 7200$, $\beta_c \approx 1$, $r_s = 2R_c \approx 1.15\text{M}\Omega$, $P_g = 10$kW, $\beta = 0.63$, $\theta = -90°$. The Un-optimized results for the beam loading effects in the BEPCⅡ future SHB1 is shown in Table 5, there is almost no any cavity detuning effect. If we keep on calculating the cavity's optimized natural resonant frequency, we can get a result of 142.8MHz, which means that no any detuning is needed for SHB1 to get the best operating efficiency.

Table 5. Un-optimized results for the beam loading effects in BEPCⅡ future SHB1.

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	15	142.7999	107.24	−90.3	−0.50
2nd	15	142.7997	107.25	−90.8	−1.50

b) Beam loading effects in SHB2

Table 6. Un-optimized results for the beam loading effects in BEPCⅡ future SHB2.

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	13.5	571.199	144.93	90.8	−1.94
2nd	13.5	571.198	145.03	92.3	−5.83

Table 7. Optimized results for the beam loading effects in BEPCⅡ future SHB2.

bunch number	bunch charge/ nC	f_{gs}/ MHz	V_c/ kV	φ_s/ (°)	$V_c\cos\phi_s$/ kV
1st	13.5	571.201	144.83	−89.3	+1.80
2nd	13.5	571.199	144.85	−90.8	−3.31

When the bunch train reaches SHB2, the bunch's characteristic length σ_z is about 0.25ns, and each bunch has a bunch charge of 13.5nC[9]. The typical parameters of the SHB2 are[11]: $f = 571.2$MHz, $Q_0 = 12370$, $\beta_c \approx 1$, $r_s = 2R_c \approx 3.00$MΩ, $P_g = 7$kW, $\beta = 0.63$, $\theta = -90°$. The un-optimized and optimized results for SHB2 are shown in Table 6 and Table 7, respectively. The SHB2's optimized natural resonant frequency is 571.201MHz, which means that the detuning effect in SHB2 is also very small.

6 Conclusion

In this paper, the transient beam loading effects on SW cavities in linear accelerators are described, and some formulae are derived. In addition, the beam loading effects in the BEPCII prebuncher and BEPCⅡ future two SHBs are analyzed. All of the formulae given in this paper can also be used in the transient beam loading analysis of the storage ring, but may need some small modifications.

On the other hand, although cavity detuning caused by a single bunch or a bunch train consisting of only few bunches usually cannot be measured and can be neglected in all of the existing accelerators, we think it must exist and will be concerned in the future.

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直线加速器驻波腔中的瞬态束流负载效应

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摘要　在高能加速器中, 随着单个束团和束团串中电荷量的提高, 当粒子束穿过加速腔的时候, 感应出的瞬态束流负载电压也越来越高. 但是, 在通常分析束流负载的时候, 往往对稳态束流负载研究的比较多, 而对瞬态束流负载的研究要相对少一些. 本文首先对束流负载的瞬态特性和束团穿过加速腔时高频源所看到谐振腔谐振频率的变化方式进行了分析, 然后又对两种情况下谐振腔的最优失谐条件进行了讨论, 并给出了相应的解析公式. 在第1种情况下, 当粒子束穿过加速腔的时候, 谐振腔的自然谐振频率能够及时地得到调节, 从而使高频源的电流与谐振腔的腔压同相, 以提高高频源的效率; 在第2种情况下, 当粒子束穿过加速腔的时候, 谐振腔的自然谐振频率保持不变, 不能被调节. 最后, 还对BEPCⅡ现有预注入器的预聚束腔、BEPCⅡ未来预注入器的两个次谐波聚束腔中的瞬态束流负载效应进行了分析.

关键词　瞬态束流负载　等效电路模型　最优失谐

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