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Nuclear Fusion near Coulomb Barrier *

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Abstract By using an improved isospin dependent quantum molecular dynamics model, the fusion reaction near coulomb barrier is studied for the $^{16}O + ^{16}O$ system. The surface energy of the system is improved by a switch function method, which combines the surface energies of projectile and target with the one of compound nucleus in a proper way. The calculated fusion cross section increases obviously by using this method, which turns away the unphysical nucleon emission in the process of projectile and target approaching since the deformation effect of the surface has been embodied by the switch function method.

Key words isospin dependent quantum molecular dynamics model, switch function, surface energy

The synthesis of superheavy elements(SHEs) was and is still one of the most hot research objects in nuclear physics. The properties of SHEs were investigated both theoretically and experimentally[1—9]. There are mainly two methods for the synthsis of SHEs experimentally[3,4], cold fusion reaction with which the superheavy elements from 110 to 112 were synthesized at GSI and hot fusion reaction which was used in Dubna for the synthesis of the elements 114 and 116. The properties of SHEs were studied theoretically using the Strutinsky method[5,6], where the ground-state deformation, the fission barrier, binding energy and competition between various possible decay modes were also investigated. However, the dynamical process of the formation of SHEs is not understood well enough[7]. Recently, few attempts were undertaken to develop models for describing the fusion process and for reproducing the measured cross section data based on dinuclear system[7—9]. Microscopic transport theory such as isospin dependent quantum molecular dynamics (IQMD) which has been used successfully for studying isospin effects of heavy ion collisions at intermediate energies[10,11] is a suitable model for describing the process of SHE formation. But the model meets difficulty to study the fusion reaction near Coulomb barrier, since some unphysical nucleon emissions in the process of projectile and target approaching.

In this paper we propose a new method in which the surface term of the system is replaced by a switch function combining the surface terms of the projectile and target with the one of the compound nucleus. The deformation effect will be embodied by considering the switch function, where the deformation is caused by the nuclear and Coulomb interaction in the process of projectile and target approaching. The calculated fusion cross sections increase obviously by using the method.

In IQMD, nucleon i is represented by a coherent state of a Gaussian wave packet,

$$\psi_i(r,t) = \frac{1}{(2\pi L)^{3/4}} e^{-[r-r_i(t)]^2/4L} e^{ip_i\cdot r/\hbar}, \qquad (1)$$

where, r_i, p_i are the centers of i-th wave packet in the coordinate and momentum space, respectively, which satisfy the canonical equation of motion. L is the so-called Gaussian wave packet width (here $L = 1.72\text{fm}^2$). The total N-body wave function is assumed to be the direct product of these co-

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herent states. After Wigner transforming for Eq. (1), we get the nucleon's Wigner density distribution in the N-body phase space

$$f_i(r,p,t)=\frac{1}{(\pi\hbar)^3}\exp\Big[-\frac{[r-r_i(t)]^2}{2L}-\frac{[p-p_i(t)]^2\cdot 2L}{\hbar^2}\Big]. \tag{2}$$

The nucleons in the system move under the selfconsistently generated mean-field, and the time evolutions of r_i and p_i are governed by Hamiltonian equation of motion

$$\frac{\partial p_i}{\partial t}=-\frac{\partial H}{\partial r_i},\quad \frac{\partial r_i}{\partial t}=\frac{\partial H}{\partial p_i}, \tag{3}$$

$$H=T+U, \tag{4}$$

$$T=\sum_i\frac{p_i^2}{2m}. \tag{5}$$

The effective interaction potential energy is composed of the local interaction potential energy and Coulomb interaction potential energy,

$$U=U_{\rm loc}+U_{\rm Coul}, \tag{6}$$

and

$$U_{\rm loc}=\int V_{\rm loc}{\rm d}^3r. \tag{7}$$

$V_{\rm loc}$ is the local interaction potential energy density and can be derived directly from zero-range Skyrme interaction potential[12—14],

$$V_{\rm loc}=\frac{\alpha}{2}\frac{\rho(r)^2}{\rho_0}+\frac{\beta}{\gamma+1}\frac{\rho(r)^{r+1}}{\rho_0^r}+\frac{C_{\rm sym}}{2}\frac{(\rho_{\rm p}(r)-\rho_{\rm n}(r))^2}{\rho_0}+\frac{g}{2\rho_0}(\nabla\rho(r))^2. \tag{8}$$

By using

$$\langle F\rangle_i=\int\rho_i(r)F{\rm d}^3r, \tag{9}$$

the local interaction potential energy can be written as,

$$U_{\rm loc}=\frac{\alpha}{2}\sum_i\langle\frac{\rho}{\rho_0}\rangle_i+\frac{\beta}{\gamma+1}\sum_i\langle\frac{\rho^\gamma}{\rho_0^\gamma}\rangle_i+\frac{C_{\rm sym}}{2}\int\frac{(\rho_{\rm p}-\rho_{\rm n})^2}{\rho_0}{\rm d}^3r+\int\frac{g}{2\rho_0}(\nabla\rho)^2{\rm d}^3r, \tag{10}$$

where the former two terms, the third term and the last term correspond to the volume energy, the symmetry energy and the surface energy, respectively. The interaction density $\rho_i(r)$ is given by,

$$\rho_i(r_i)=\frac{1}{(4\pi L)^{3/2}}\sum_j\exp[-(r_i-r_j)^2/4L]. \tag{11}$$

The Coulomb potential energy is obtained from the form,

$$U_{\rm Coul}=\frac{1}{2}\sum_{i\neq j}\int\rho_i(r)\frac{{\rm e}^2}{|r-r'|}\rho_j(r'){\rm d}^3r{\rm d}^3r'. \tag{12}$$

The symmetry term and the Coulomb potential energy can be written by the following sum forms:

$$U_{\rm sym}=\frac{C_{\rm sym}}{2\rho_0}\sum_{i\neq j}t_{iz}t_{jz}\frac{1}{4(\pi L)^{3/2}}\exp\Big[-\frac{(r_i-r_j)^2}{4L}\Big], \tag{13}$$

$$U_{\rm Coul}=\frac{{\rm e}^2}{4}\sum_{i\neq j}\frac{1}{r_{ij}}(1+t_{iz})(1+t_{jz}){\rm erf}(r_{ij}/\sqrt{4L}), \tag{14}$$

where the t_{iz} is the z-th component of the isospin degree of freedom for the i-th nucleon, which is equal to 1 and -1 for proton and neutron, respectively. The relative distance $r_{ij}=|r_i-r_j|$. The $C_{\rm sym}$ is the symmetry strength. The parameter sets for these equations are shown in Table 1.

Table 1. The parameter sets of the model.

α/GeV	β/GeV	γ	$C_{\rm sym}$/GeV	g/GeV·fm^2	ρ_0/fm^{-3}
−0.356	0.303	7/6	0.032	0.12	0.165

The α,β,γ are taken corresponding to the soft equation of state, which give the incompressibility coefficient of order of 200MeV for a saturation density 0.165fm^{-3}.

In our model, the surface term of the system is improved by switch method which connects the surface energies of projectile and target with the one of compound nucleus. The switch function is expressed as:

$$S=C_0+C_1\frac{R-R_{\rm low}}{R_{\rm up}-R_{\rm low}}+C_2\Big(\frac{R-R_{\rm low}}{R_{\rm up}-R_{\rm low}}\Big)^2+C_3\Big(\frac{R-R_{\rm low}}{R_{\rm up}-R_{\rm low}}\Big)^3+C_4\Big(\frac{R-R_{\rm low}}{R_{\rm up}-R_{\rm low}}\Big)^4+C_5\Big(\frac{R-R_{\rm low}}{R_{\rm up}-R_{\rm low}}\Big)^5. \tag{15}$$

R is the distance of the centers between projectile and target. $R_{\rm up}$ and $R_{\rm low}$ are the distance between centers at initial time and final time at which the compound nucleus is formed ($R_{\rm up}=15$fm and $R_{\rm low}=0$fm) respectively. The parameters C_0, C_1, C_2, C_3, C_4, C_5 are taken to be 0, 0, 0, 10, −15, 6, respectively, which assures the continuity of the surface energy and its first derivative. The surface interaction energy is expressed as

$$U^{\rm surf}_{\rm system}=(U^{\rm surf}_{\rm proj}+U^{\rm surf}_{\rm targ})S+U^{\rm surf}_{\rm comp}(1-S), \tag{16}$$

here $U_i^{\rm surf}$ denotes the suface term of the i-th system. In our model, the standard Yukawa potential is replaced by the gradient potential.

The neutron and proton density distributions for the initial projectile and target nuclei are determined from the Skyrme-Hartree-Fock method[11]. Using IQMD method to study the heavy-ion collision at low energies near coulomb barrier is necessary to construct the stable nucleus. because

this ensures no nucleon emission before projectile and target contact each other. The stability can retain over 800fm/c as the Fig. 1. The calculated binding energy and root mean square radii are -8.18 MeV and 2.58 fm, respectively, which are consistent with the experimental data -7.98 MeV and 2.64fm[15] quantatively.

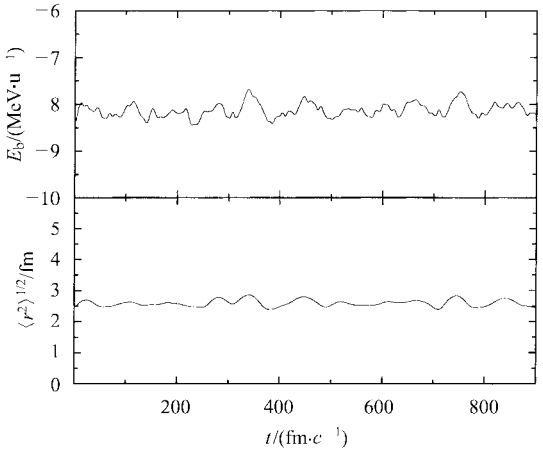

Fig. 1. The time evolution of the root mean square radius (rms) and the binding energy (E_b) of the ^{16}O nucleus.

The relation of the surface energy and the distance between the two centers can be seen from the Fig. 2. The distance between the centers at initial time is taken to be R = 15fm. The line with solid circles denotes the surface energy with switch function calculated by Liquid Drop Model (LDM), the lines with solid squares and solid diamonds are the surface energy with switch function calculated by Yukawa and density gradient method, respectively, and the lines with open squares and open diamonds represent the surface energy calculated by IQMD with and without including switch function, repectively. It is clear that the surface energies calculated with three different methods (LDM, IQMD and switch function) are almost the same at initial time (R = 15fm), however, the surface energies calculated by IQMD increase obviously after projectile and target touch each other (R = 6fm). We can also see from the figure the switch function method is more reasonable, which the surface energies decrease gradually until to the compound nucleus formed and can bound suitably these unphysical nucleon emissions.

As a test for the switch function, Fig. 3 gives the interaction surface energy (solid line) $E_{int} = E_{pt} - E_p - E_t$ and the work $W = \int (E_{pt} - E_p - E_t) S'(R) dR$ (dashed line) derived from the switch function as well as the discrepancy

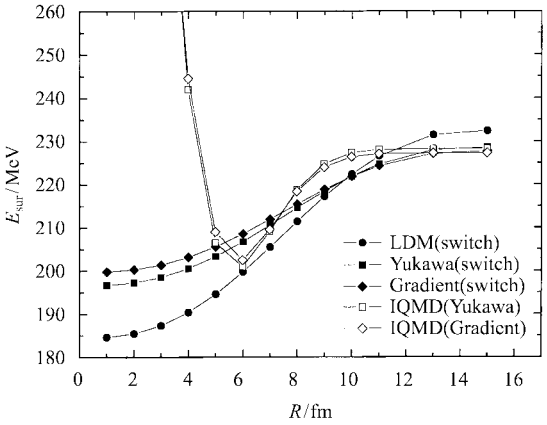

Fig. 2. The surface energies of the system calculated with different methods as a function of the distance between centers.

(dotted line) $\Delta E = E_{int} - W$. It is clear that, the variance of the interaction surface energy with the distance between the centers is more visible than the work before projectile and target touching each other (R = 6fm), at the same time, since considering the switch function, the spherical form of the nucleus ^{16}O can be retained reasonably. With further approaching until to 4fm, the interaction surface energy and the work decrease rapidly, this is because the neck dispersion lead to neck density and the density gradient decreasing gradually. However, for the distance less than 4fm, the density and the density gradient increase apparently since the overlapping region expansion, which result in a rapid increase of the interaction surface energy and work. From the figure, we can see that the interaction energy and the work are almost the same at initial time and at final time (R < 3fm) at which the compound nucleus is formed. Simultaneously, the difference between E_{int} and W in the intermediate process is converted into the volume energy and the kinetic energy etc.

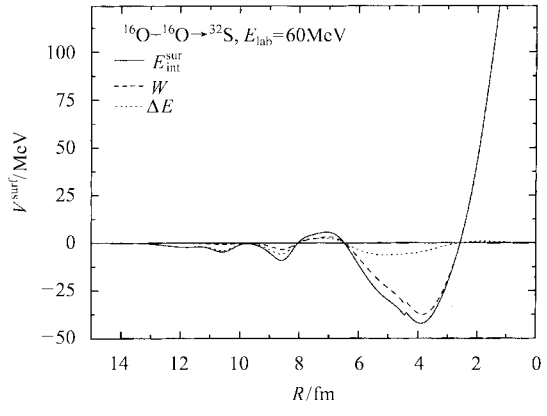

Fig. 3. The interaction surface energy (—), the work (----) and their variance (····) versus the distance between centers.

The fusion cross section σ_{fus} is obtained by following expression:

$$\sigma_{fus} = 2\pi\int db \cdot bP_f(b), \tag{17}$$

where $P_f(b)$ stands for the fusion probability for the impact parameter b. $P_f(b)$ is estimated by $\hat{P}_f(b)$, which is expressed as

$$\hat{P}_f = \frac{n}{N}, \tag{18}$$

where n is the number of complete fusion events and N the number of total events. The reliability of the estimation is given in following

$$|P_f(b) - \hat{P}_f(b)| \leqslant \Delta P_f, \tag{19}$$

$$\Delta P_f = 1.64 \cdot \left[\frac{n(N-n)}{N^3}\right]^{1/2}. \tag{20}$$

In our calculation, the total events are over 2000, the relative error of the fusion probability $\Delta P_f/P_f$ is very small. The fusion cross sections are shown in Fig. 4, which don't include the events of nucleon emissions in the fusion reaction. The closed circles with error bars represent the experimental data, the open circles with error bars denote the calculated ones with considering the switch function and the pentagrams with error bars are the calculated ones by the old IQMD model. The experimental data are taken from Refs. [16,17]. We can see from the figure that, the calculated fusion cross sections increase apparently (increasing about a factor of 2 or 3 comparing the old IQMD calculations) after using the switch function method. Maruyama et al. took into account the kinetic energy term of the momentum variance of wave packets to the Hamiltonian[18], they found that the calculated fusion cross section increases obviously in a version of QMD with the variance of the wave-packet. Wang Ning et al. calculated the fusion cross sections for both $^{40}Ca + ^{90}Zr$ and $^{40}Ca + ^{96}Zr$ reactions near Coulomb barrier[19,20], they found that the calculated cross sections were in good agreement with the experimental results, but they included events with emitting several nucleons (less than 6 nucleons) prior to the formation of compound nucleus. Without those events included, the calculated fusion cross sections would be much smaller than the experimental values.

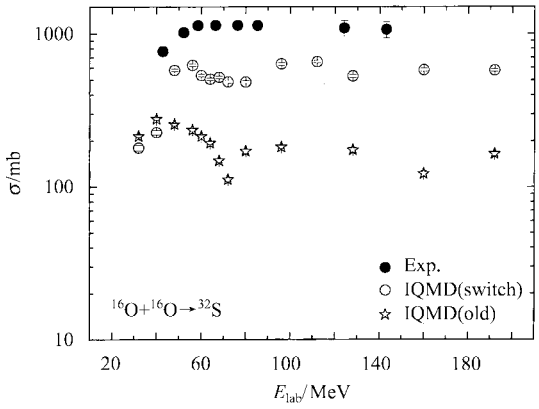

Fig. 4. Comparison of the calculated fusion cross section by considering the switch function method (○) and no considering (☆) with the experimental data (●).

In summary, within the framework of the improved isospin dependent molecular dynamics model we calculated the fusion cross section in the $^{16}O + ^{16}O$ system and found that the fusion cross sections increase apparently. Shell effect is very important in heavy-ion collision at the energies near Coulomb barrier, especially on the studies of superheavy nucleus synthesis. How to take into account this effect in dynamical calculation is still an open problem.

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库仑位垒附近熔合反应的研究*

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摘要　利用改进的量子分子动力学研究了$^{16}O+^{16}O$库仑位垒附近的熔合反应,用连接函数的方法对体系的表面能做了改进.利用这种方法避免了非物理的核子发射,熔合截面有了明显的增加,这是因为体系的表面形变效应可以通过连接函数来体现,从而抑制了弹靶在熔合过程中非物理的核子发射.

关键词　量子分子动力学模型　连接函数　表面能

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